Wang Algebra and Interconnects

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Wang Algebra – 70+ Years Ago

K.T. Wang, "On a new method of analysis of electrical networks," in Memoirs 2, *Nat. Res. Inst. Eng. Academia Sinica*, pp. 1-11, 1934

S.L. Ting, "On the general properties of electrical network determinants," *Chinese J. Physics*, vol 1, pp. 18-40, 1935

C.T. Tsai, "Short cut methods of Wang algebra of network problems," *Chinese J. Physics*, vol. 3, pp. 141-181, 1939

R.J. Duffin and T.D. Morley, "Wang algebra and matriods," IEEE Trans Circuit and Systems, vol CAS-25, no 9, pp. 755-762, Sept., 1978

W.K. Chen, Graph Theory and Its Engineering Applications (ch. 5, sect. 4, "The Wang-algebra formulation"), World Scientific Publ., 1997

Wang Algebra:

XX = 0
X+X = 0
XY = YX

= *W*

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Wang Algebra

"K. T. Wang managed an electrical power plant in China, and in his spare time sought simple rules for solving the network equations. Wang's rules were published in the reference indicated below [5]. Wang could not write in English so his paper was actually written by his son, then a college student. Raoul Bott and I recognized that Wang's rules actually define an algebra. We restated the rules as three postulates for an algebra:

$$xy = yx,\ x + x = 0,\ xx = 0."$$

R.J. Duffin, "Some Problems of Mathematics and Science," Bulletin of the American Mathematical Society, Nov. 1974, p. 1060, web link: http://www.ams.org/bull/1974-80-06/S0002-9904-1974-13610-4/S0002-9904-1974-13610-4.pdf

("[5]" is the K.T. Wang reference on slide 2)

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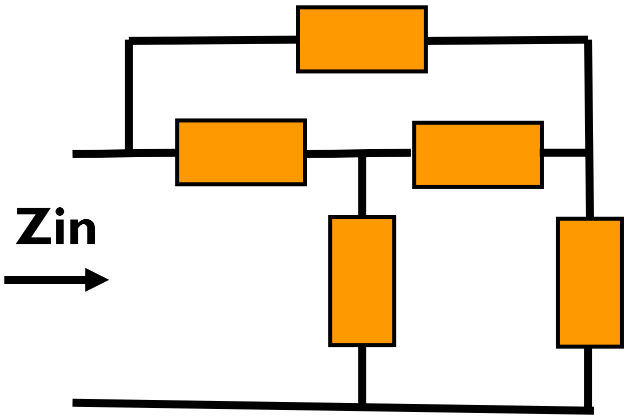
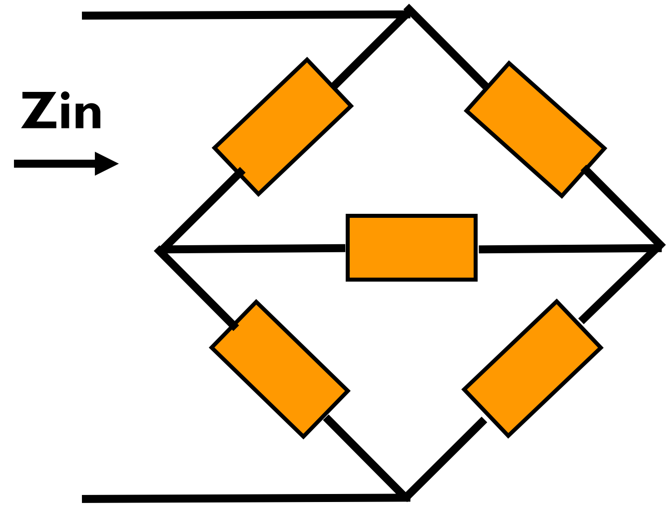
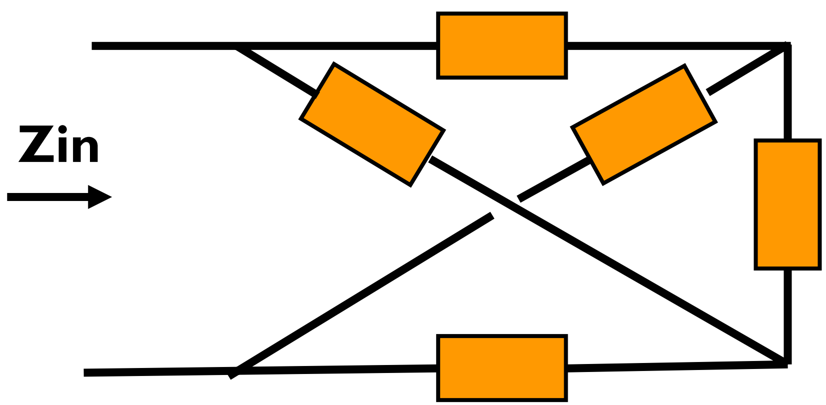
Easy General Solutions (Including Difficult Topologies)







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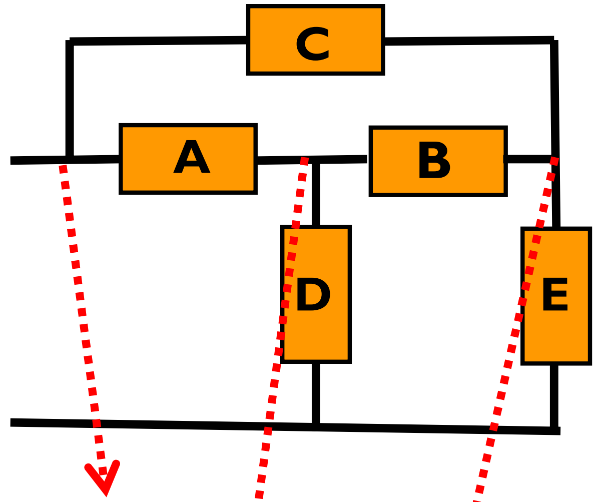
Solving [I]=[Y][V] for Zin (Traditional Method)

Zin=1/E →



Nodal Equations:

A … E are admittances

E = 1/R

$$[\mathrm{I}]=[\mathrm{Y}][\mathrm{V}]=\begin{bmatrix}\mathrm{Iin}\\0\\0\end{bmatrix}\begin{bmatrix}A+C & -A & -C\\-A & A+B+D & -B\\-C & -B & B+C+E\end{bmatrix}\begin{bmatrix}\mathrm{Vin}\\\mathrm{V_D}\\\mathrm{V_E}\end{bmatrix},$$

$$\mathrm{Zin}=\frac{\mathrm{Vin}}{\mathrm{Iin}}=\frac{\begin{vmatrix}A+B+D & -B\\-B & B+C+E\end{vmatrix}}{\begin{vmatrix}A+C & -A & -C\\-A & A+B+D & -B\\-C & -B & B+C+E\end{vmatrix}}=\frac{AB+AC+AE+BC+BD+BE+CD+DE}{ABD+ABE+ACD+ACE+ADE+BCD+BCE+CDE}$$

(18 initial terms yields 8 final denominator terms)

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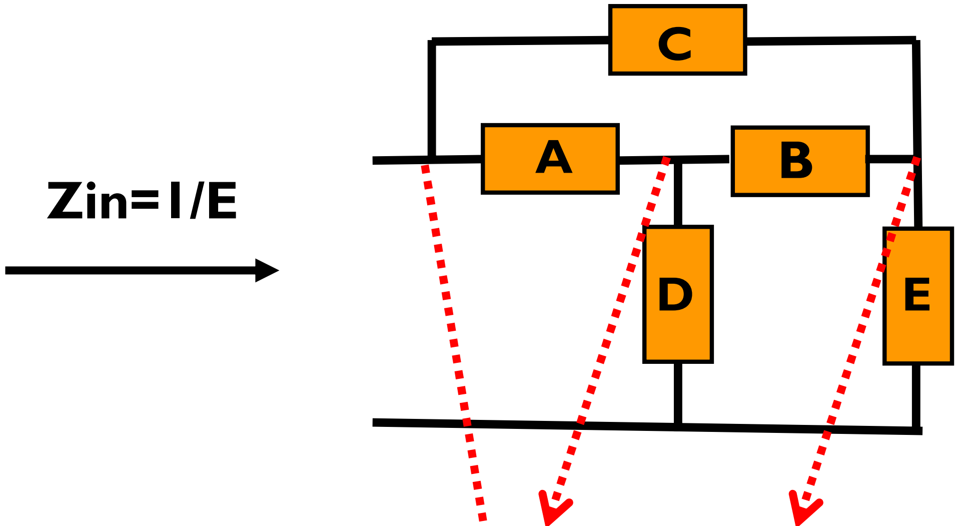
Solving [I]=[Y][V] for Zin = R (Wang Algebra)



Nodal Equations:

A … E are admittances

E = I/R

$$\text{Zin} = \frac{\text{numerator}}{\text{denominator}} = \frac{(A+B+D)*\boxed{\text{W}}*(B+C+E)}{(A+C)*\boxed{\text{W}}*(\text{numerator})} = \boxed{\frac{1}{E}}$$

XX=0 →

X+X=0 → **(after XX=0)**

$$= \frac{AB + AC + AE + \cancel{BB} + BC + BE + BD + CD + DE}{\cancel{ABC} + ABE + ABD + ACD + ADE + \cancel{ABC} + ACE + BCE + BCD + CDE}$$

$$= \boxed{\frac{AB + AC + AE + BC + BD + BE + CD + DE}{ABD + ABE + ACD + ACE + ADE + BCD + BCE + CDE}}$$

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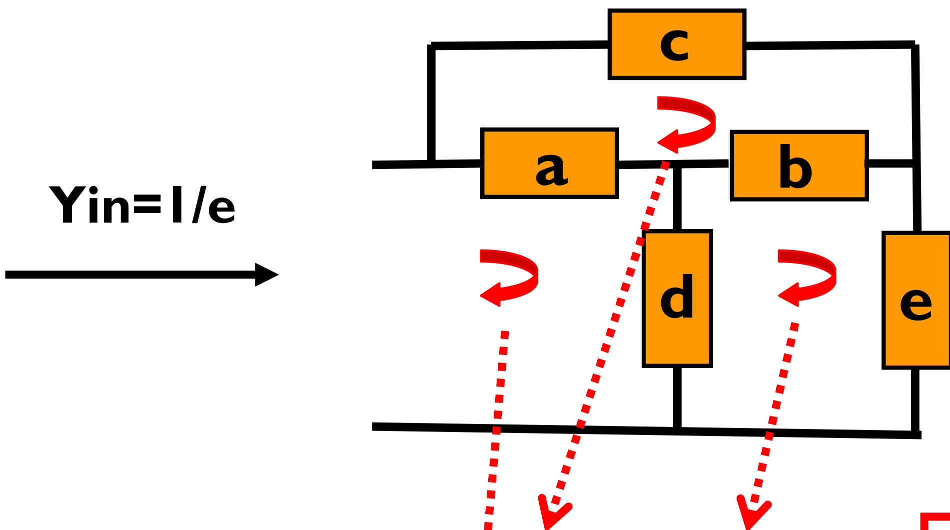
Solving [V]=[Z][I] for Zin= 1/Yin = R (Wang Algebra)



Loop Equations:

a … e are impedances

e = R

$$\text{Yin} = \frac{\text{numerator}}{\text{denominator}} = \frac{(a+b+c)*\text{W}*(b+d+e)}{(a+d)*\text{W}*(\text{numerator})} = \frac{1}{e}$$

XX=0 →

$$= \frac{ab+ad+ae+bb+bd+be+bc+cd+ce}{abd+abe+abc+acd+ace+abd+ade+bde+bcd+cde}$$

X+X=0 → **(after XX=0)**

$$= \frac{ab+ad+ae+bd+be+bc+cd+ce}{abc+abe+acd+ace+ade+bde+bcd+cde}$$

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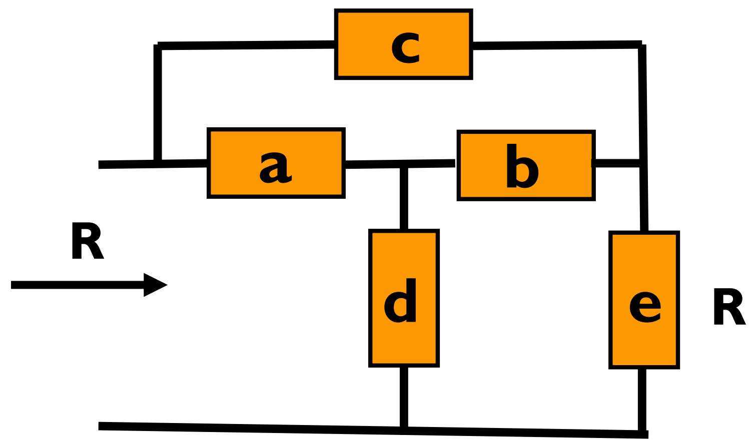
Constant R Constraint



General

$$d(a+b)+ab+R(a-b)-R^2-\frac{R^2(a+b)}{c}=0$$

Symmetric (a = b)

$$2da+a^2-R^2-\frac{2R^2a}{c}=0$$

Substitute impedances and equate powers of the Laplace variable "s" for constant R relationships

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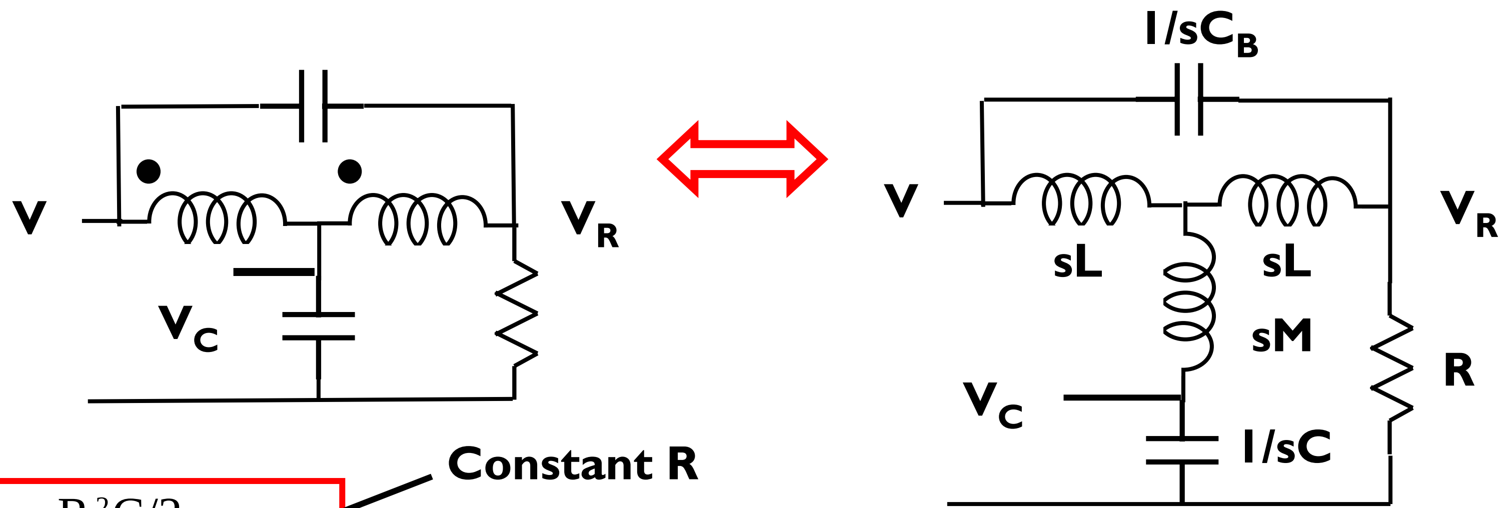
(Constant R) T-coil Example

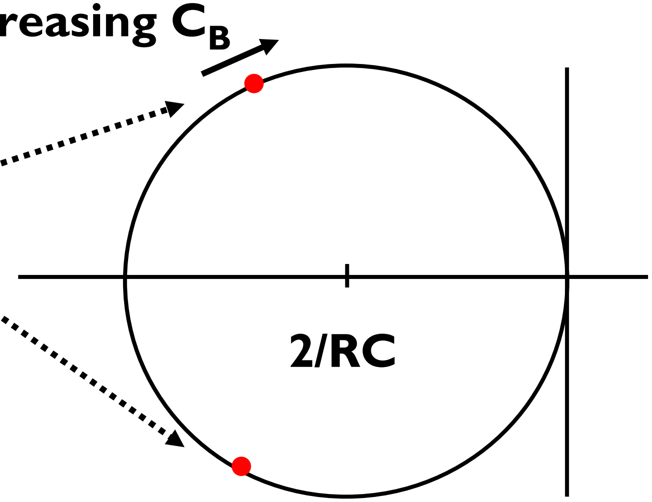


$$L = R^2C/2$$

$$M = R^2C_B - L/2$$

$$\frac{V_C}{V} = \frac{1}{R^2CC_Bs^2 + \frac{RC}{2}s + 1}$$

$$\frac{V_R}{V} = \frac{R^2CC_Bs^2 - \frac{RC}{2}s + 1}{R^2CC_Bs^2 + \frac{RC}{2}s + 1}$$



Poles 30 degrees for maximally flat envelope delay (MFED)

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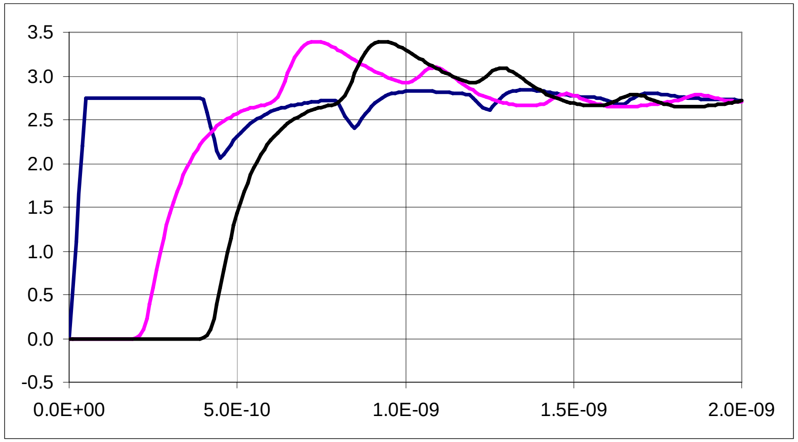
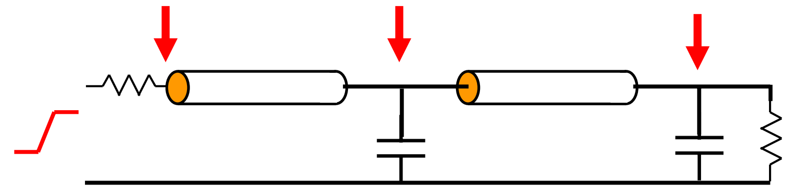
T-coil Improvement (Terminated Multi-drop Line)

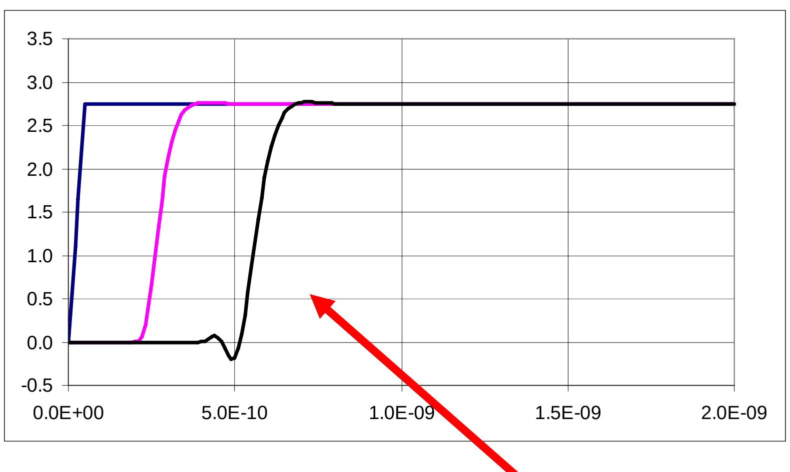


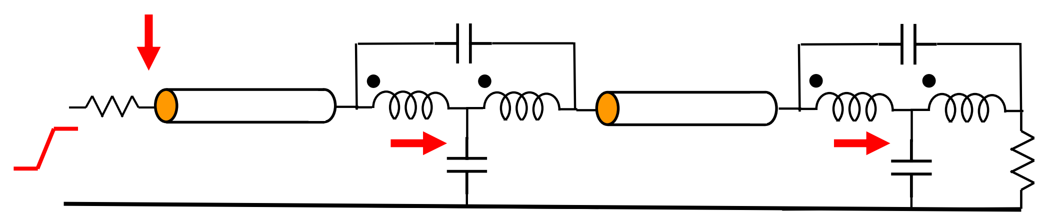


Rsource = 10 Ω, Rload = 50 Ω

C = 2 pF, TL = 50 Ω, 200 ps

Vin = 0 to 3.3 V, 50 ps ramp





Cleaner and faster responses, but with more delay

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Historical Applications (1)

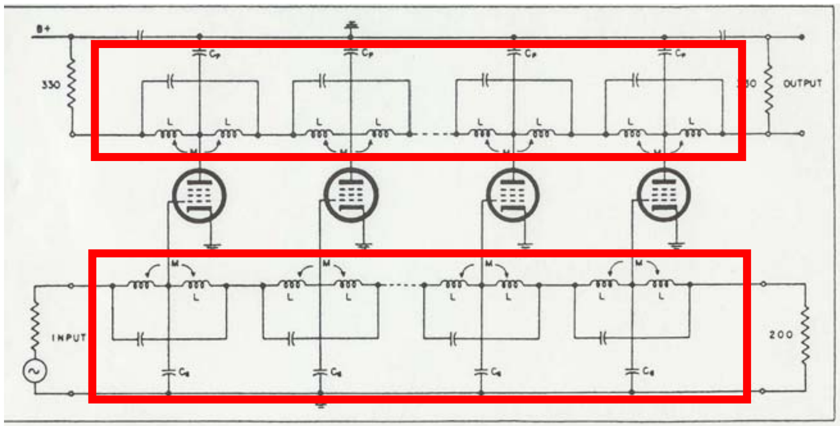
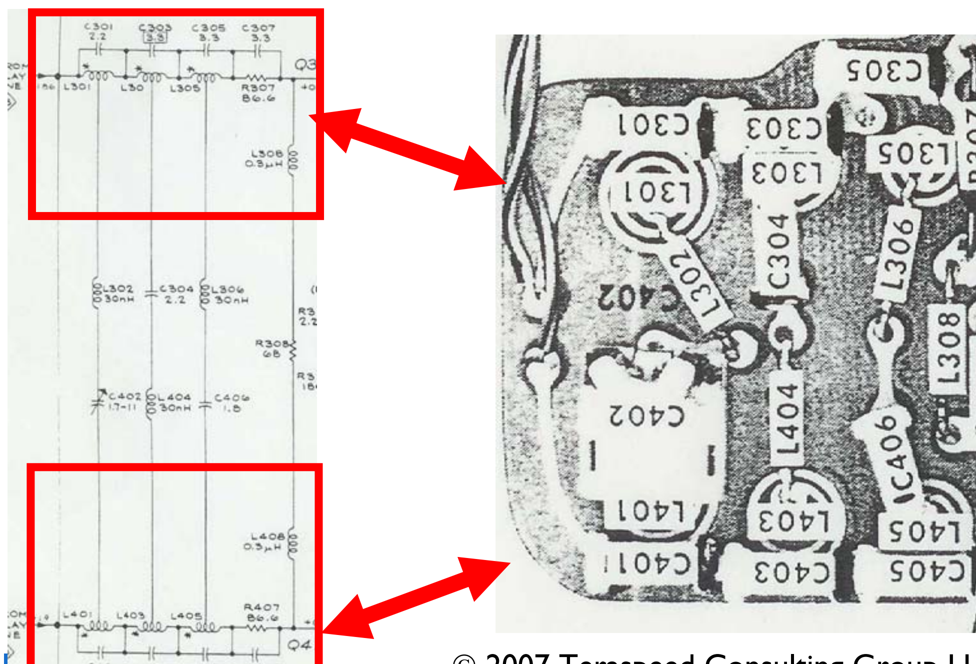


Figure 3. *Basic Amplifier Circuit Using Bridged-T Lines*

High speed (traveling wave) distributed amplifier in 1940's (Similar to GTL and source synchronous control)



Dual input delay line phase equalization using cascaded printed circuit board T-coils in 1960's

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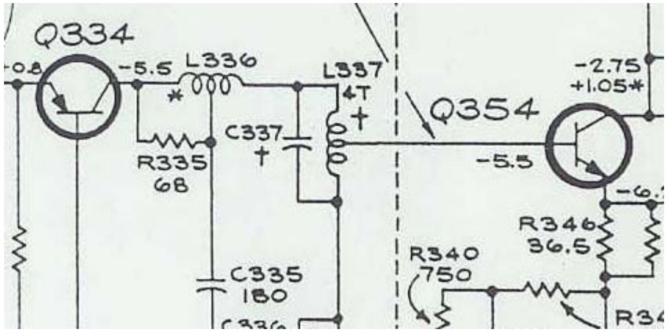
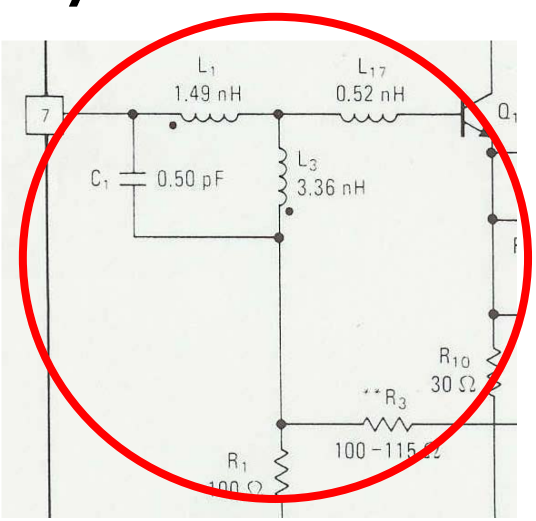
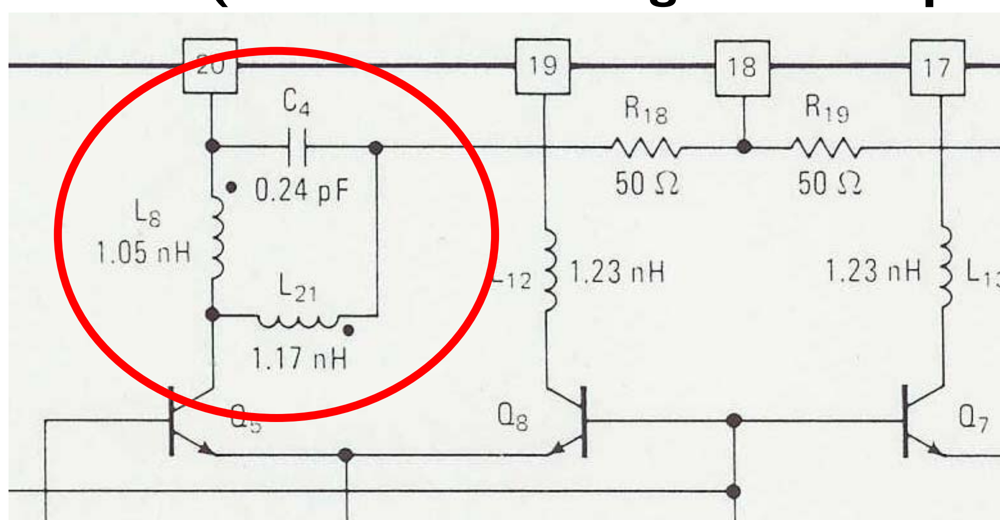
Historical Applications (2)



Parasitic bandwidth switch compensation and cascaded interstage peaking in 1960's

One-half of hybrid IC differential 50 Ω input and 50 Ω output with asymmetrical T-coils in 1970's (Current mode logic-like output)





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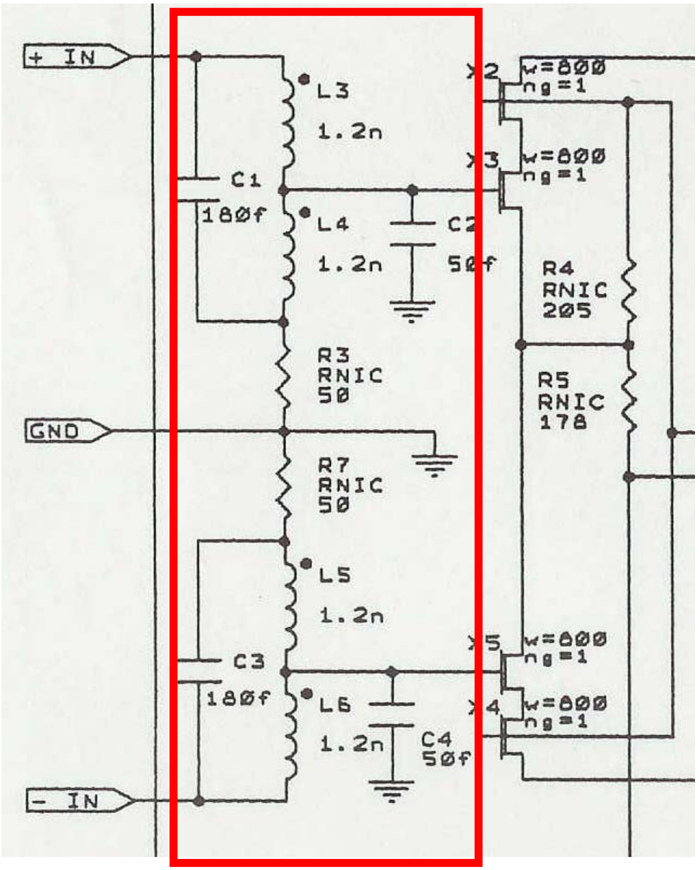
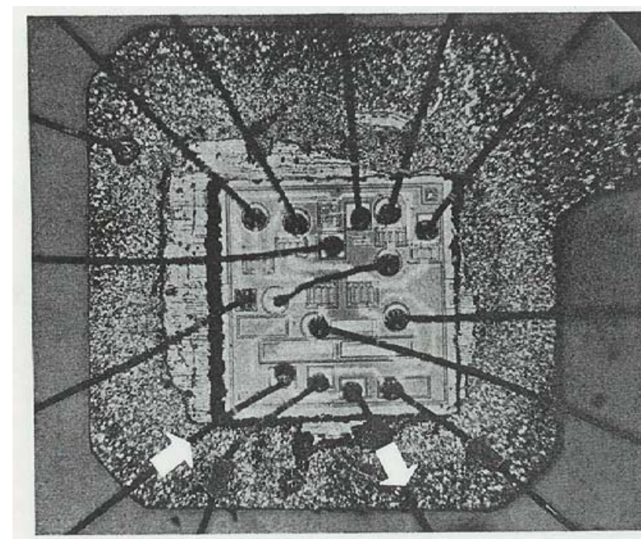
Historical Applications (3)



High speed 50 Ω input for FET hybrid IC and with metalization (not shown) for T-coils in 1990's



7. A chip trick. Tee-coil is realized by looping the input signals thro

Package bond wire compensation with T-coil trick in 1970's

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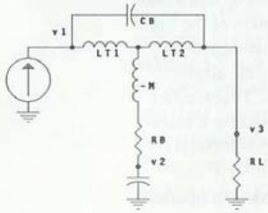
Original and General Lossy Derivations Used Wang Algebra

Two Types of Lossy Capacitor T-coils

ROSS CONSTANT-RESISTANCE T-COIL



$$L_{T1}=\frac{R_L^2 C}{2}\left(1-\frac{R_B}{R_L}\right)$$

$$L_{T2}=\frac{R_L^2 C}{2}\left(1+\frac{R_B}{R_L}\right)$$

$$C_B=\frac{C}{16\delta^2}\left(1+\frac{R_B}{R_L}\right)^2$$

$$M=\frac{R_L^2 C}{4}\left[1-\left(\frac{R_B}{R_L}\right)^2-\frac{1}{4\delta^2}\left(1+\frac{R_B}{R_L}\right)^2\right]$$

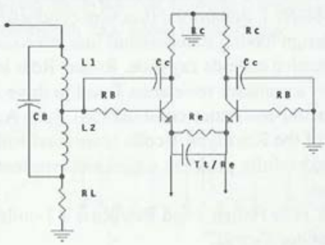
$\delta = dampingfactor$ of quadratic response

$\frac{v1}{i}=R_L$ The Constant-Resistance property

$$\frac{v2}{iin}=\frac{R_L}{1+\frac{(R_L+R_B)}{2}C s+R_L^2 C C_B s^2}$$ Two Pole Response

HALLEN MINIMUM VSWR T-COIL

For the Hallen and the Ross T-coils
$L_{total} = R_L^2 C_{total}$
As R_B gets bigger, the input coil inductance gets smaller.
With a finite R_B, the response at R_L is not allpass



$$L_{total}=R_L^2\left[\frac{T_T}{R_e}+\left(\frac{R_c}{R_e}+1\right)C_c\right]$$

$$L_1=\frac{L_{total}}{2}\left[1+\frac{1}{R_L}\left(\frac{R_e C_c T_t(R_c+2R_B)}{(T_t+R_e C_c+R_c C_c)^2}-\frac{2R_B T_t+R_e R_c C_c}{T_t+R_e C_c+R_c C_c}\right)\right]$$

$$L_2=L_{total}-L_1$$

$$C_B=\frac{1}{R_L^2}\left[\frac{R_L C_c T_t(R_c+2R_B)(L_1-L_2)}{(T_t+R_e C_c+R_c C_c)(L_1+L_2)}+\frac{2R_B R_c C_c T_t}{T_t+R_e C_c+R_c C_c}+\frac{L_1 L_2}{L_1+L_2}\right]$$

Figure 10–11.
Two Types of Lossy Capacitor T-coils.

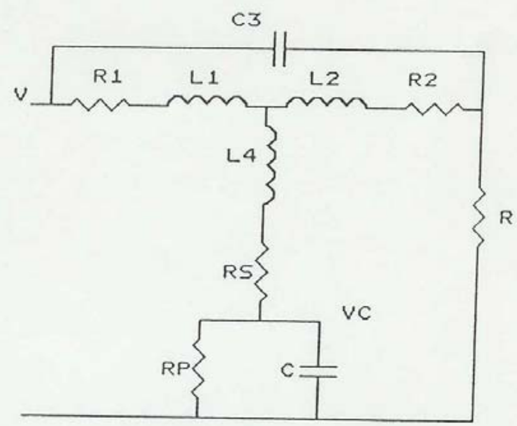


Figure 5: General asymmetrical bridged T-coil

B. Ross, "Generalizaton of T-Coil Equations," Proceeding of the Third Electrotechnical and Computer Science Conference ERK'94 (in Slovenia) Sept. 26-28, 1994, pp. 39-43.

Tuned for bipolar transistor technology

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T-coils, Interconnect, Terminators

- T-coil summary
  - Constant R provides ideal load or termination
  - MFED: 2.73 bandwidth improvement over RC
  - MFED: 0.4% overshoot for ideal step input
  - Complexity reduction (poles/zero cancellation): usually produces second order function for easier design
  - Lossless case: ideal MFED, double terminated thru path
- Interconnects and terminators
  - Older high-speed analog design ideas apply to current high-speed digital interconnects
  - Constant R target yields many benefits

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Wang Algebra

- Useful “trick” for easy calculations
  - Regular algebra plus xx=0, x+x=0 for on-going simplification
  - Numerator calculation first
  - Denominator from numerator result
  - Listing “xx” cancelled terms unnecessary
  - Accurate for larger, complex networks
- Used for T-coil technology advances

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