Equalizations for multi-level signal

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Improved MMSE Algorithm for DFE Optimization

Objectives

The signal in channels with high-speed designs is attenuated by channel loss, inter-symbol interference, jitter, noise and crosstalk.

Main way to recover the signal is by using equalizations, such as Feed-Forward Equalizer, Continuous Time-Linear Equalizer and Decision Feedback Equalizer (DFE).

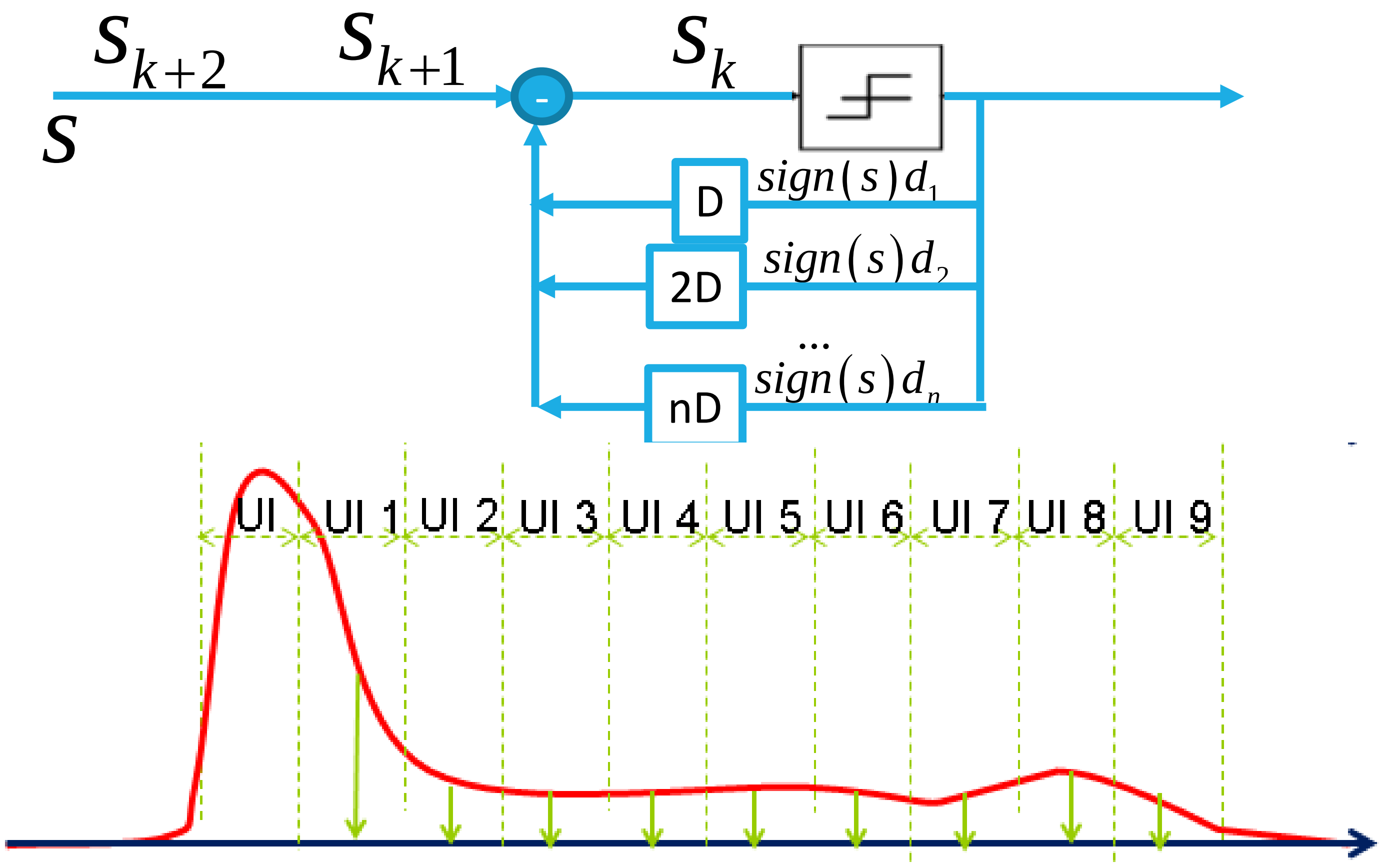
One of the important problems with high-speed design and channel simulations is developing fast optimization algorithms for each equalizer.

Result: **Improvement of MMSE optimization for DFE for both NRZ and PAM-4**

DFE Equalization

$$V_k^{DFE} = V_k - d_1 sign\left(V_{k-UI}^{DFE}\right) - d_2 sign\left(V_{k-2UI}^{DFE}\right) - \ldots - d_n sign\left(V_{k-nUI}^{DFE}\right)$$

Decision Feedback Equalizer (DFE) is a nonlinear filter that uses feedback of detected symbols to produce an estimate of the channel output. DFE feeds a sum of logic or symbol decisions back to the symbol decoder.



MMSE (Minimum Mean Square Error) optimization for DFE

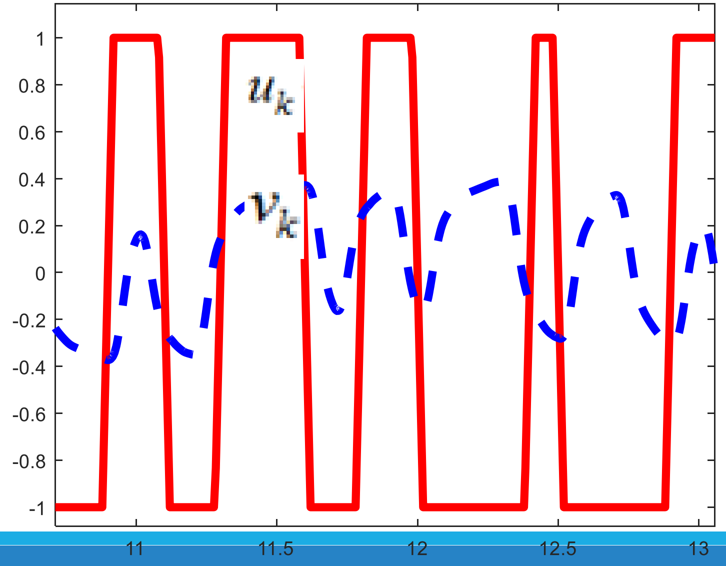
$u_k, k=1,\dots,N$ - values of input signal at the cursor location; N - number of bits in the input signal;

$v_k, k=1,\dots,N$ -corresponding values of the channel response.

Let $v_k^{DFE}, k=1,\dots,N$ be M-Tap DFE equalized response:

$$v_k^{DFE} = v_k - \sum_{i=1}^{M} sign\left(v_{k-i}^{DFE}\right) d_i, \;\; k=1,\dots,N.$$

MMSE finds such $d_i, i=1,\dots,M$ tap coefficient values, that minimizes difference error $Err = \sum_{k=1}^{N}\left(u_k - v_k^{DFE}\right)^2$ between input signal and DFE equalized signal.



The optimization problem:

$$\min_{d_i, i=1,\dots,M} \sum_{k=1}^{N}\left(u_k - \left(v_k - \sum_{i=1}^{M} s_{k-i} d_i\right)\right)^2$$

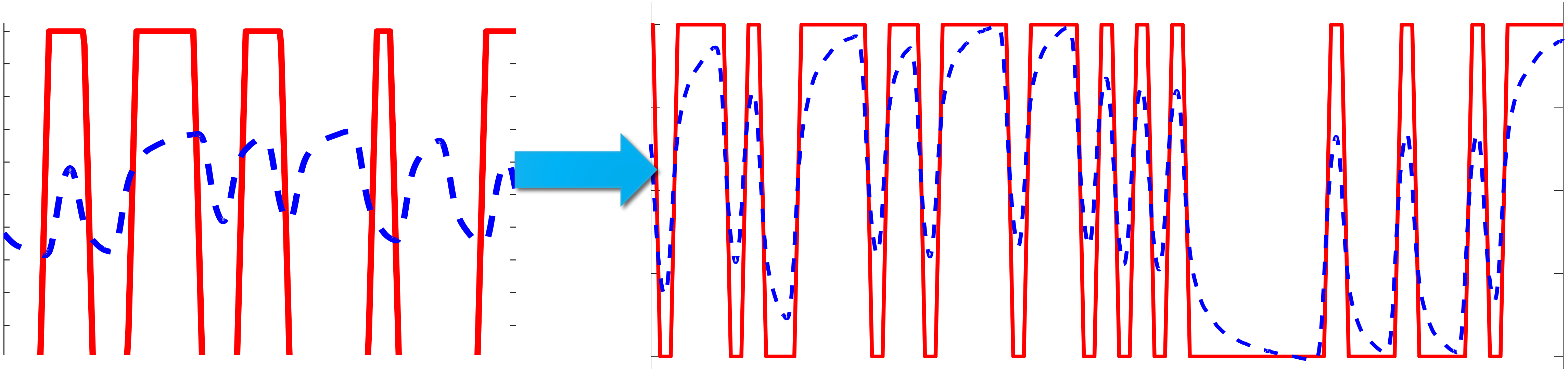
where for simplicity $s_k \equiv sign(u_k)$.

DFE Optimization Algorithm Modification

Square error minimization between the transmitted and received waveforms

$$\left[\left(V_k - d_1 sign\left(V_{k-UI}\right) - d_2 sign\left(V_{k-2UI}\right) - \ldots - d_n sign\left(V_{k-nUI}\right)\right) - V_k^{Desired}\right] \rightarrow \min$$

$V_k^{Desired}$ is a training signal, but it should be normalized - to maximum of the unequalized signal



Modification:

Introduce normalization level as an additional optimization parameter

Modified DFE Optimization Algorithm

$$\left[\left(V_k - d_1 sign\left(V_{k-UI}^{Desired}\right) - d_2 sign\left(V_{k-2UI}^{Desired}\right) - \ldots - d_n sign\left(V_{k-nUI}^{Desired}\right)\right) - aV_k^{Desired}\right] \rightarrow \min$$

$$\min_{d_i, a}\left(\sum_{k=1}^{N}\left[\left(V_k - \sum_{i=1}^{m} \boxed{d_i} sign\left(V_{k-i*UI}^{Desired}\right)\right) - \boxed{a} V_k^{Desired}\right]^2\right)$$

Optimization parameters

Advantages:

1. Normalization coefficient is also subject of optimization and now the equalized signal will stabilize near the optimal line;
2. Standard algorithm might not converge when number of taps will increase due to non-optimal normalization, but modified algorithm will always converge

Improved optimization for NRZ

$$\min_{a,d_i,i=1,..,M} Err\left(a,d_1,\ldots,d_M\right),$$

$$Err\left(a,d_1,\ldots,d_M\right)=\sum_{k=1}^{N}\left(\left(au_k-v_k\right)+\sum_{i=1}^{M}s_{k-i}d_i\right)^2.$$

To solve the optimization problem, we need to derivate $Err\left(a,d_1,\ldots,d_M\right)$ with respect to $a,d_1,\ldots,d_M$ and equal to zero. We will get the following M+1 linear set of equations with M+1 unknowns.

$$\frac{\partial Err\left(a,d_1,\ldots,d_M\right)}{\partial a}=0,$$

$$\frac{\partial Err\left(a,d_1,\ldots,d_M\right)}{\partial d_1}=0,$$

$$\vdots$$

$$\frac{\partial Err\left(a,d_1,\ldots,d_M\right)}{\partial d_M}=0.$$

Which is the same as:

$$2\sum_{k=1}^{N}\left(\left(au_k-v_k\right)+\sum_{i=1}^{M}s_{k-i}d_i\right)u_k=0,$$

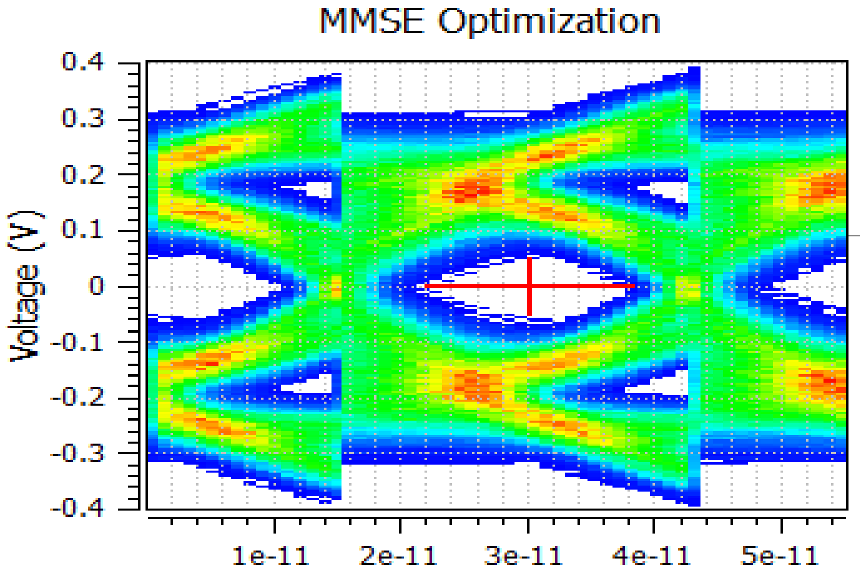
$$2\sum_{k=1}^{N}\left(\left(au_k-v_k\right)+\sum_{i=1}^{M}s_{k-i}d_i\right)s_{k-1}=0,$$

$$\vdots$$

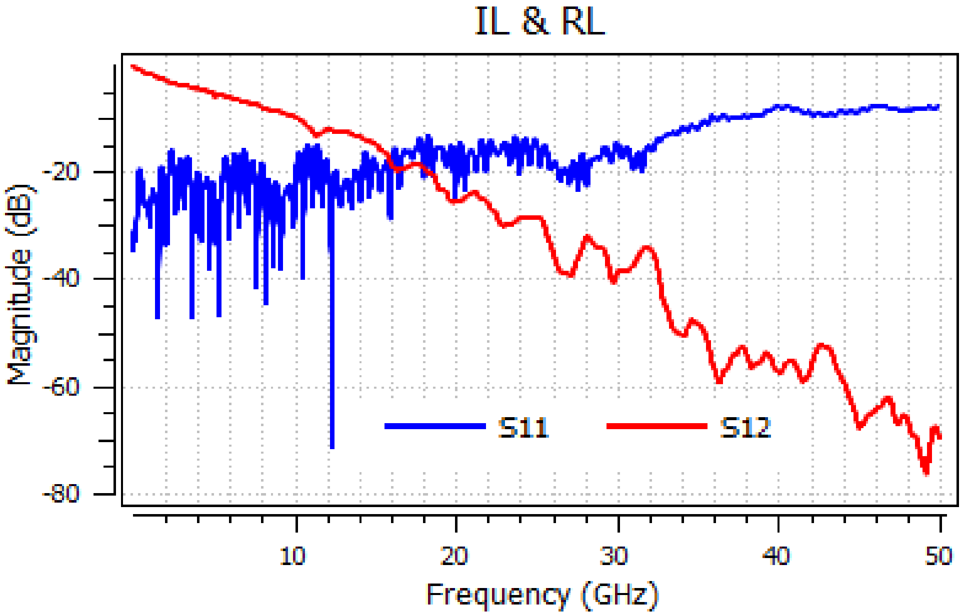
$$2\sum_{k=1}^{N}\left(\left(au_k-v_k\right)+\sum_{i=1}^{M}s_{k-i}d_i\right)s_{k-M}=0.$$

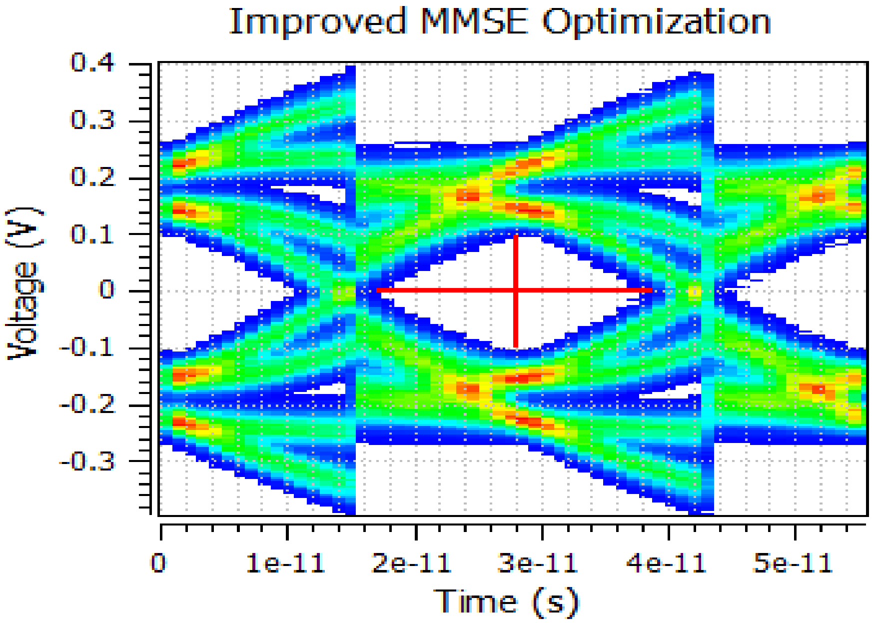
NRZ Results

Only 5-tap DFE equalizer,
35 Gbps,
PRBS15,
20% rise-fall time,



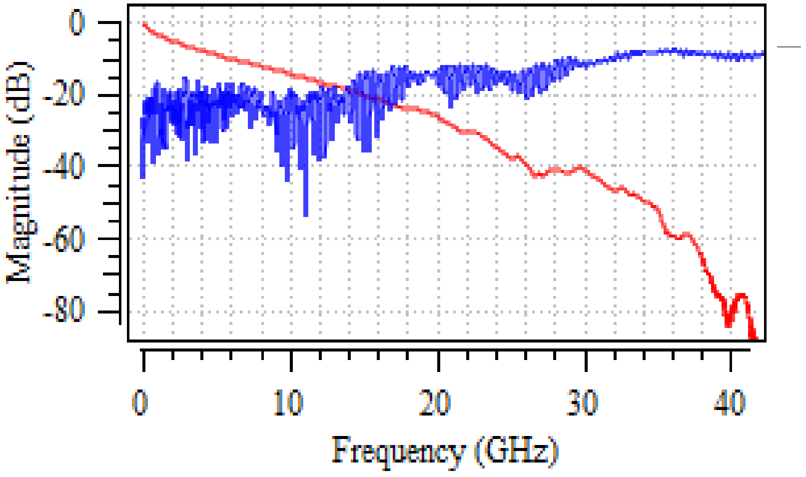
EH 101 mV;
EW - 16 ps;

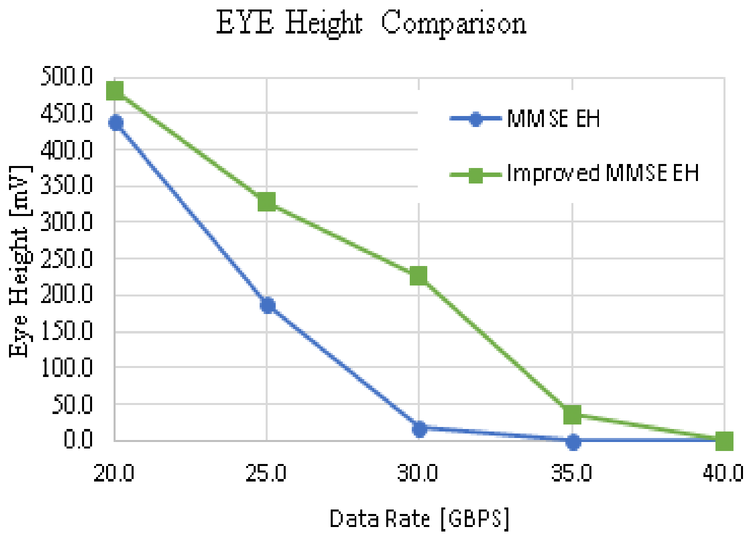


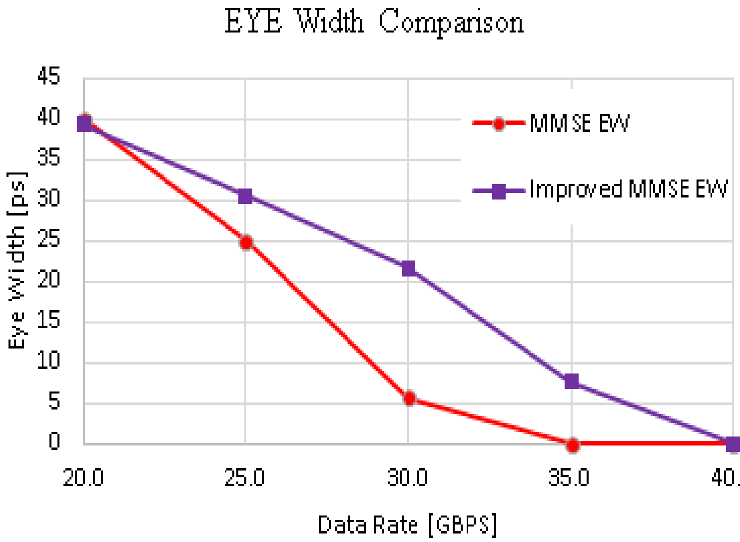


EH - 201 mV,
EW - 22ps;

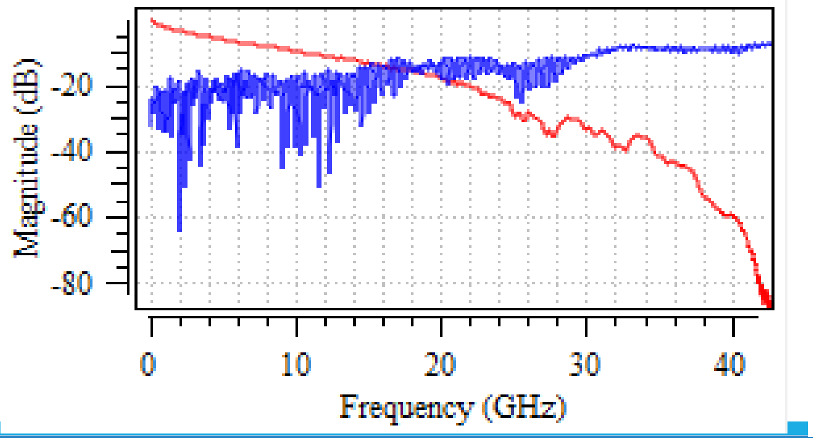
NRZ Results by datarates

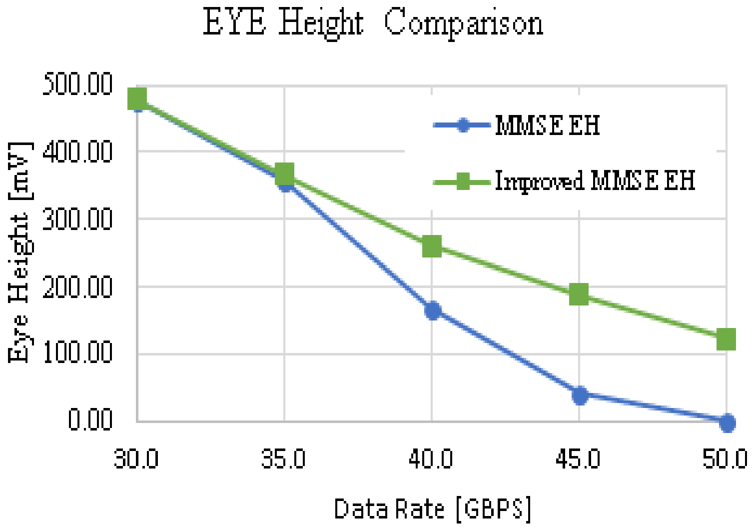


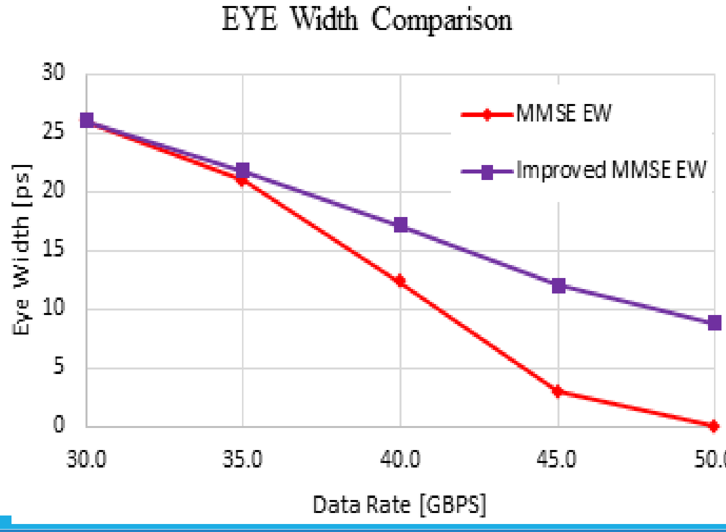




7-tap DFE

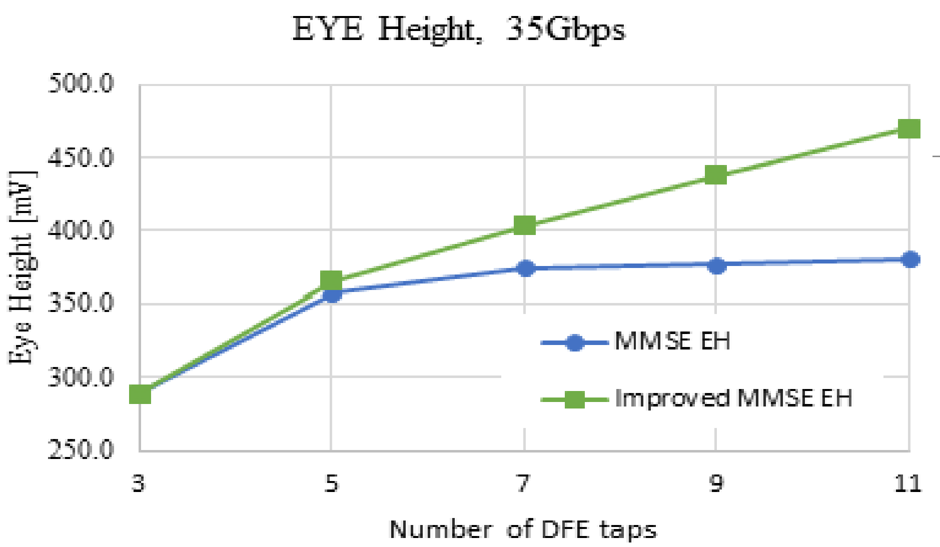


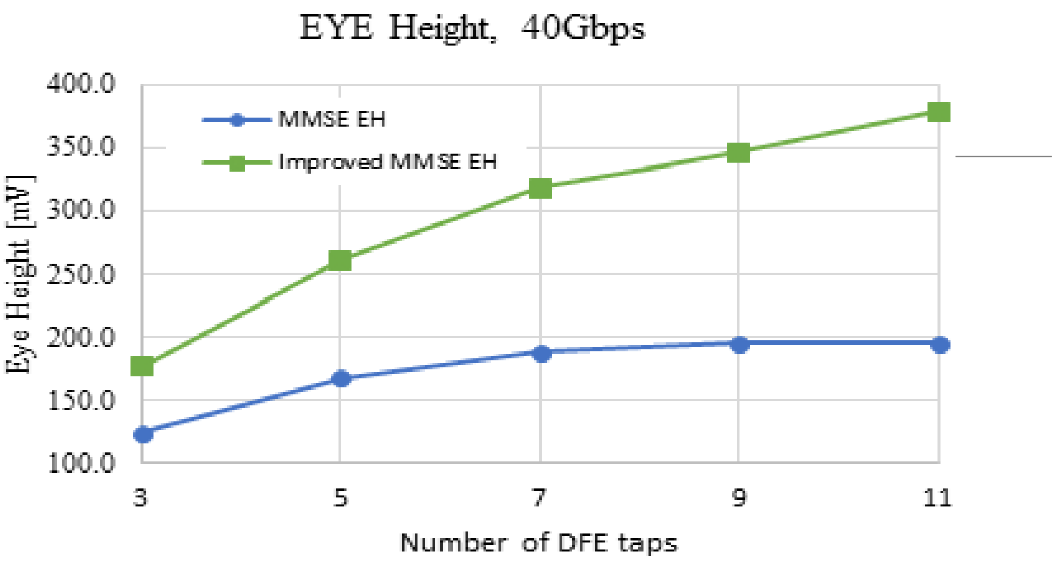


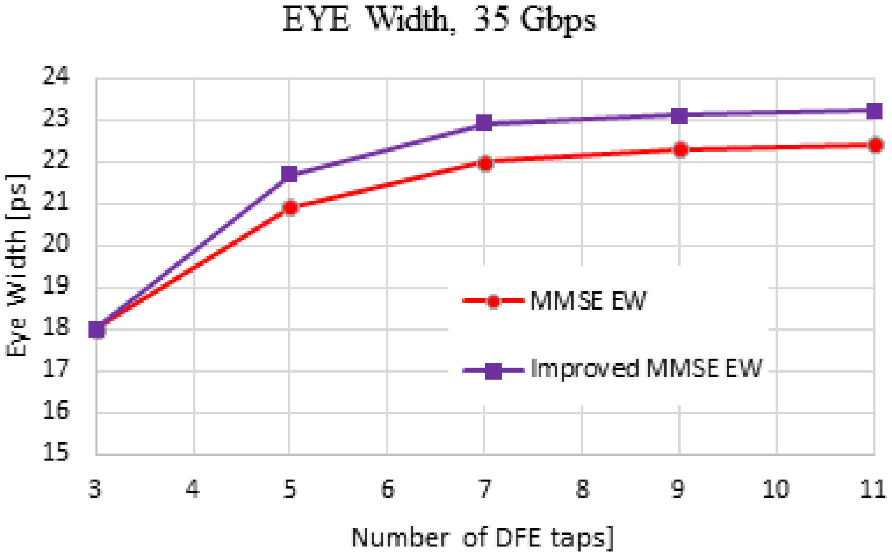


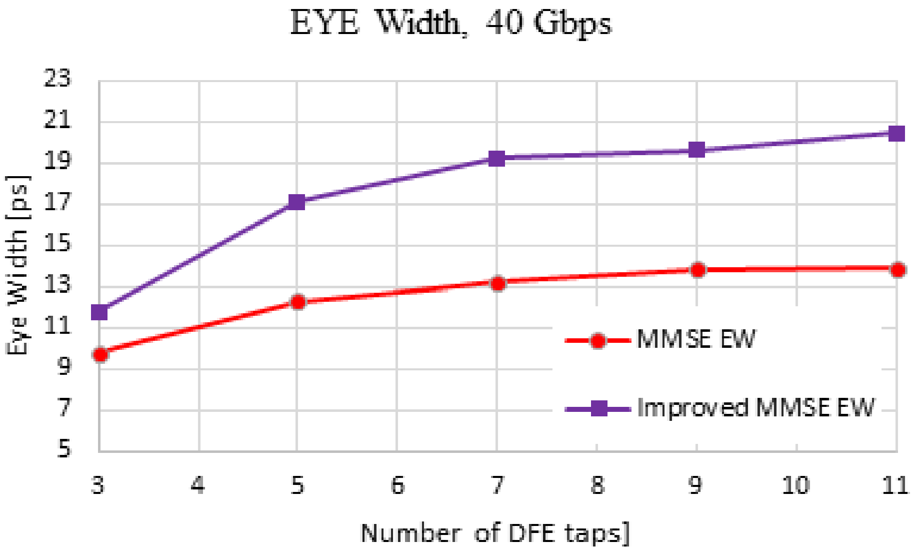
5-tap DFE

NRZ Results by increasing number of taps



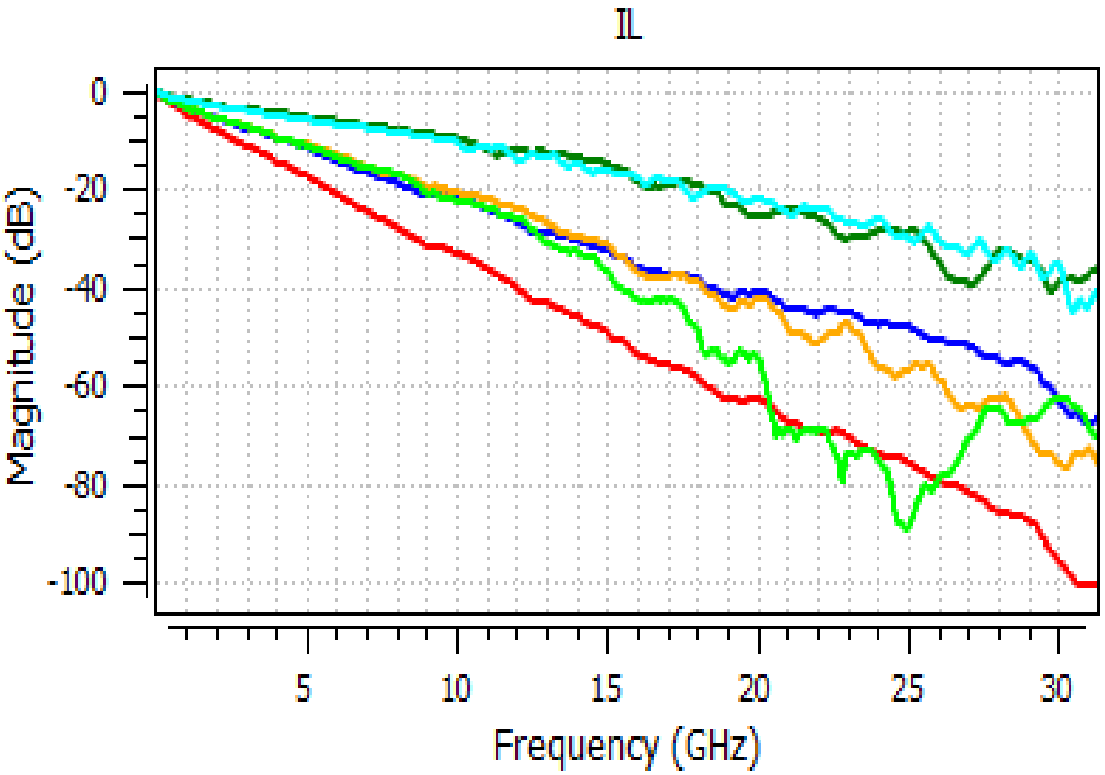


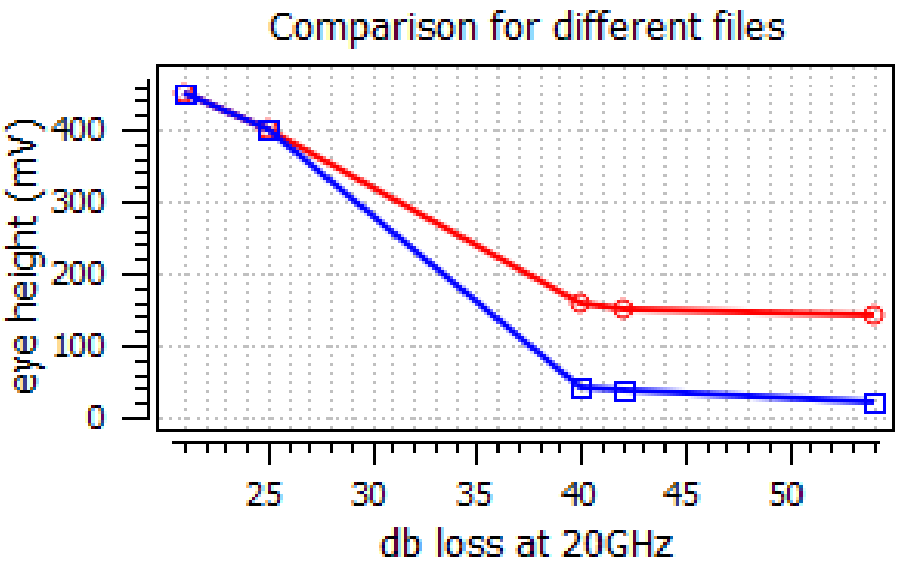


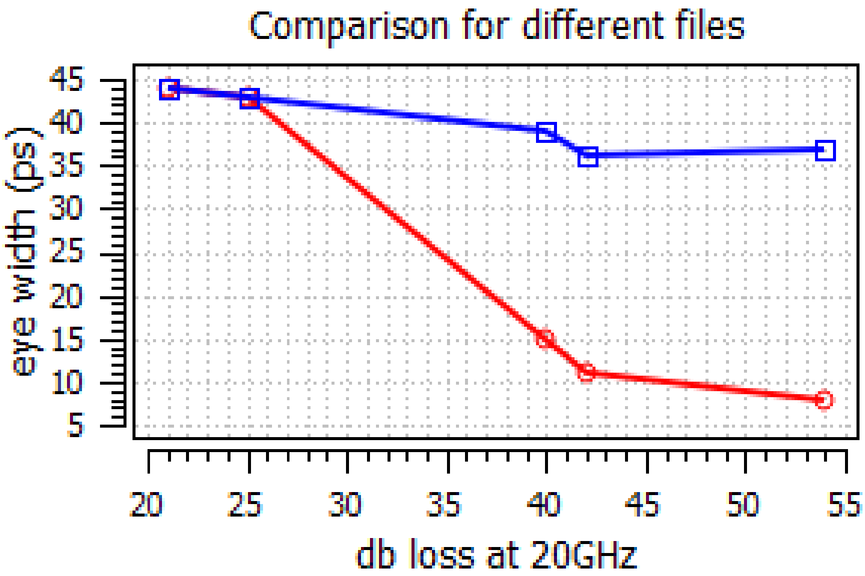


NRZ Results by Loss

Comparison of different files regarding insertion loss corresponding to 20 Gbps with 7-tap DFE with the standard and improved optimization methods;

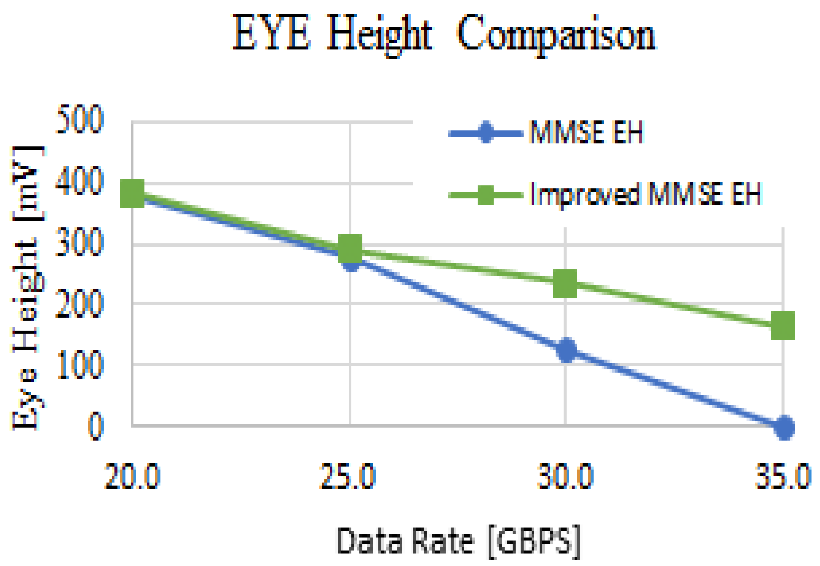


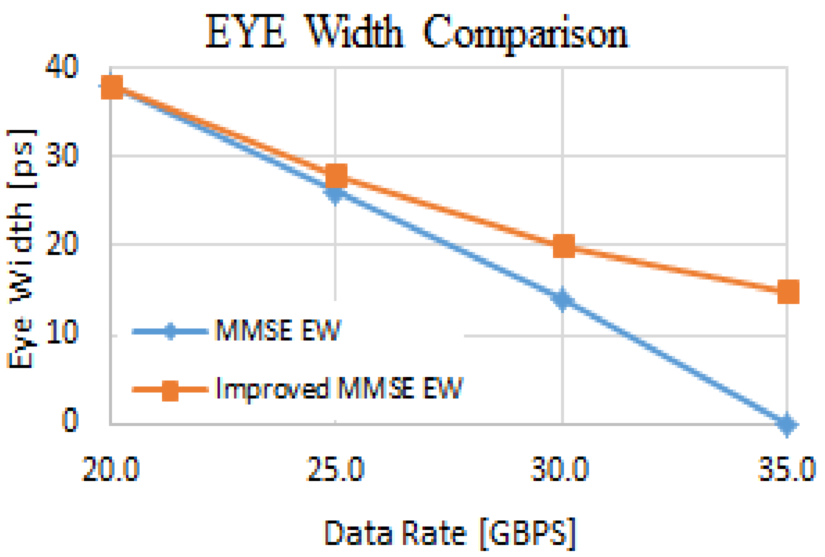




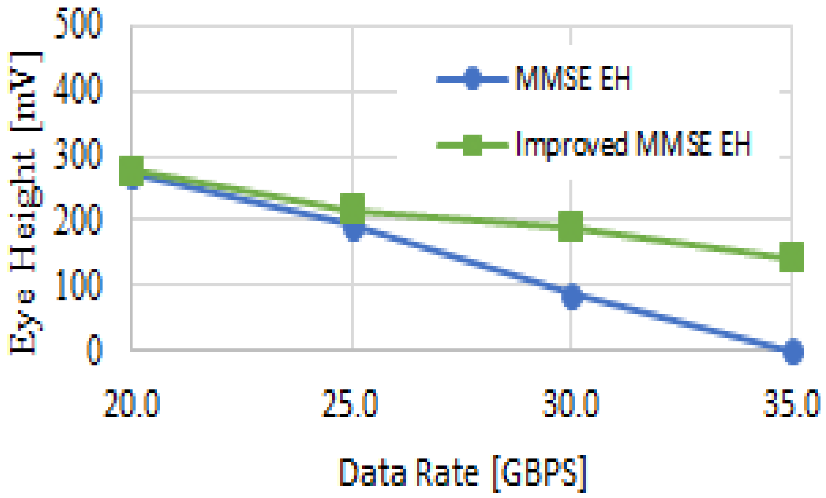
NRZ Results by FFE-DFE Combination

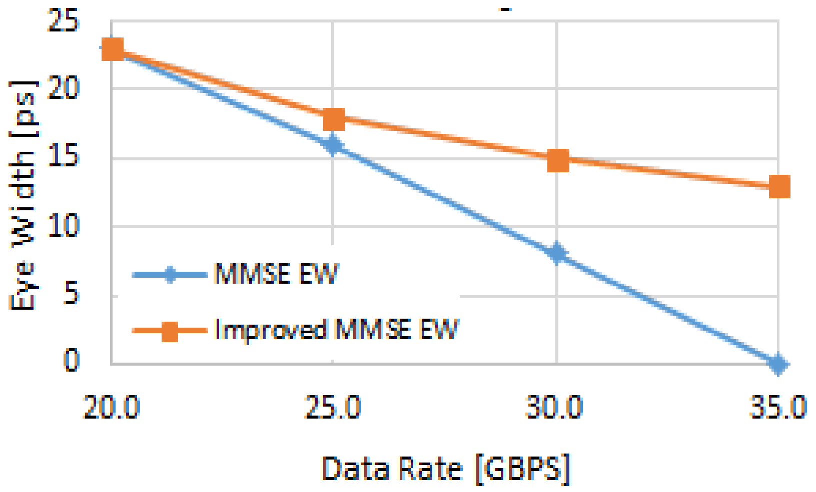
20-35 Gbps,
2-tap FFE
7-tap DFE





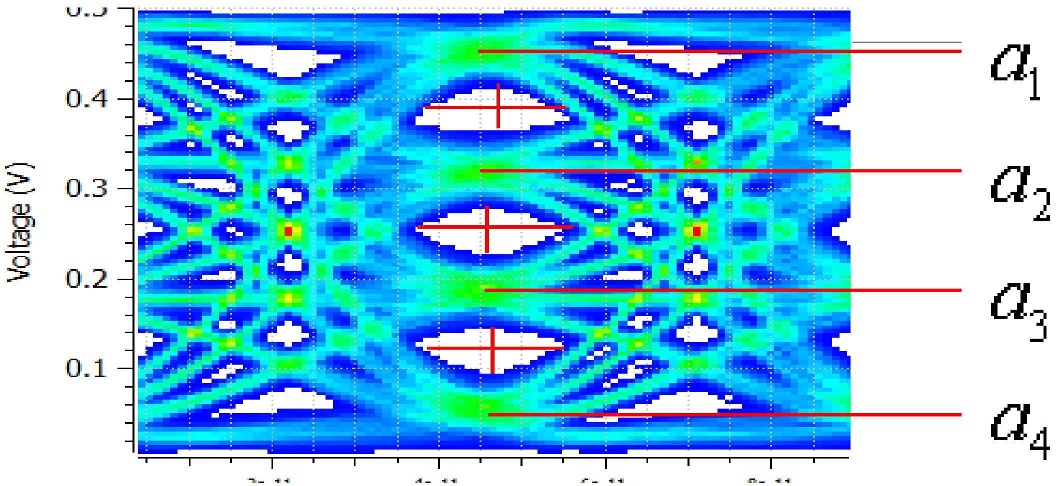
The same tests with induced 10% RMS





Improved MMSE optimization of DFE for PAM-4

Goal function for PAM-4 M-Tap DFE optimization will include four optimal signal level values $\left(a_1, a_2, a_3, a_4\right)$



and M optimal DFE tap coefficients $\left(d_1, d_2, \dots, d_M\right)$

$$\min_{a, d_i, i=1,..,M} Err\left(a_1, \dots, a_4, d_1, \dots, d_M\right),$$

$$Err = \sum_{k=1}^{N}\left(\left(\sum_{i=1}^{4} a_i u_k^{(i)} - v_k\right) + \sum_{i=1}^{M} s_{k-i} d_i\right)^2.$$

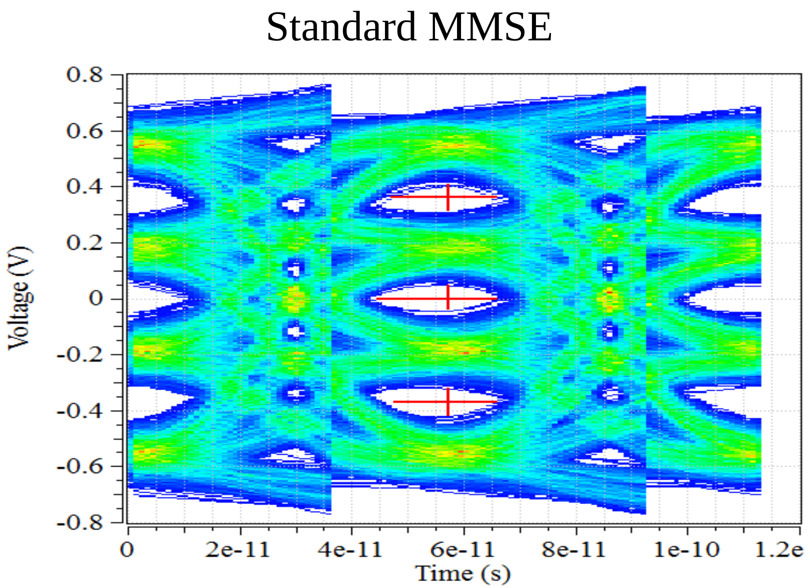
$$u_k^{(1)} = \begin{cases} -1, & if\ u_k = V_{Level1}, \\ 0, & if\ u_k \neq V_{Level}, \end{cases} \quad u_k^{(2)} = \begin{cases} -\frac{1}{3}, & if\ u_k = V_{Level2}, \\ 0, & if\ u_k \neq V_{Level2}, \end{cases}$$

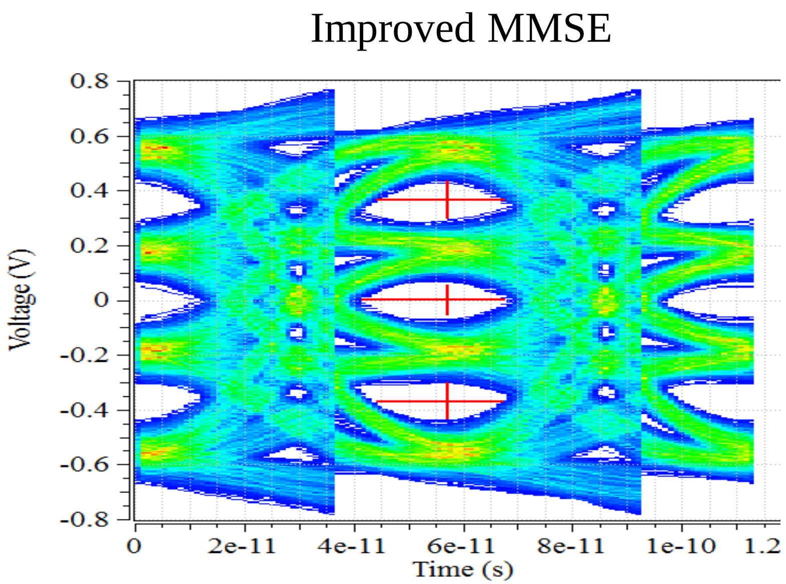
$$u_k^{(3)} = \begin{cases} \frac{1}{3}, & if\ u_k = V_{Level3}, \\ 0, & if\ u_k \neq V_{Level3}, \end{cases} \quad u_k^{(4)} = \begin{cases} 1, & if\ u_k = V_{Level4}, \\ 0, & if\ u_k \neq V_{Level4}, \end{cases}$$

$$s_k = \begin{cases} -1, & if\ u_k = V_{Level1}, \\ -\frac{1}{3}, & if\ u_k = V_{Leve2}, \\ \frac{1}{3}, & if\ u_k = V_{Leve3}, \\ 1, & if\ u_k = V_{Leve4}, \end{cases}$$

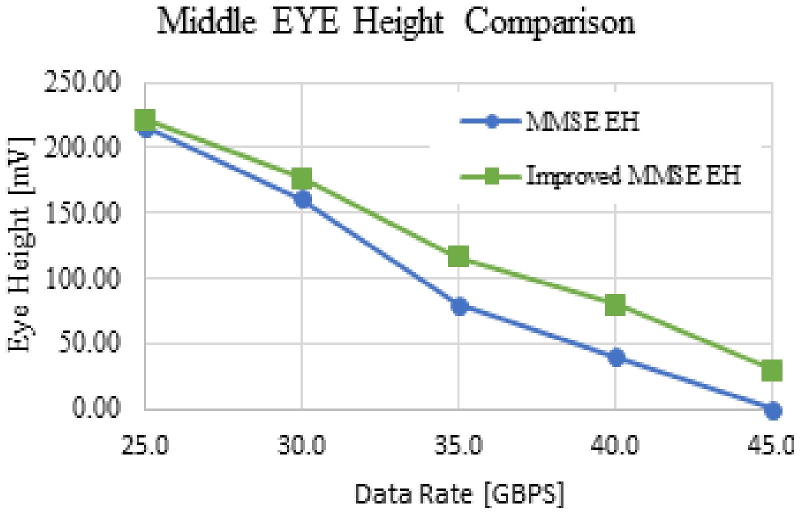
PAM-4 Results

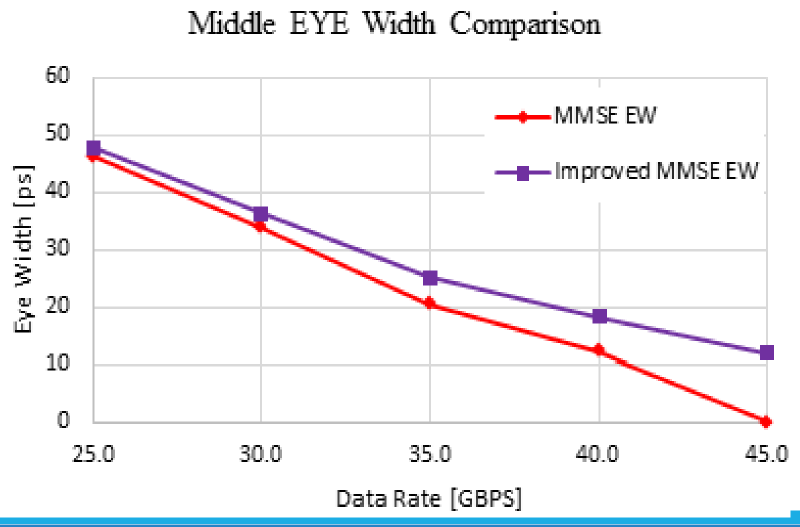
PAM-4, 35 Gbps with only 9-tap DFE equalization.





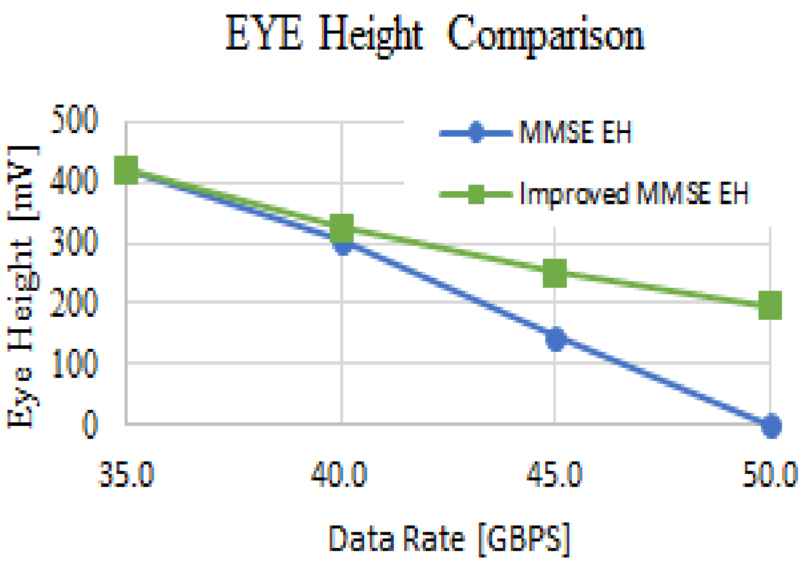
PAM-4 middle eye opening, datarates from 25 to 45 Gbps, 7-tap DFE equalization.

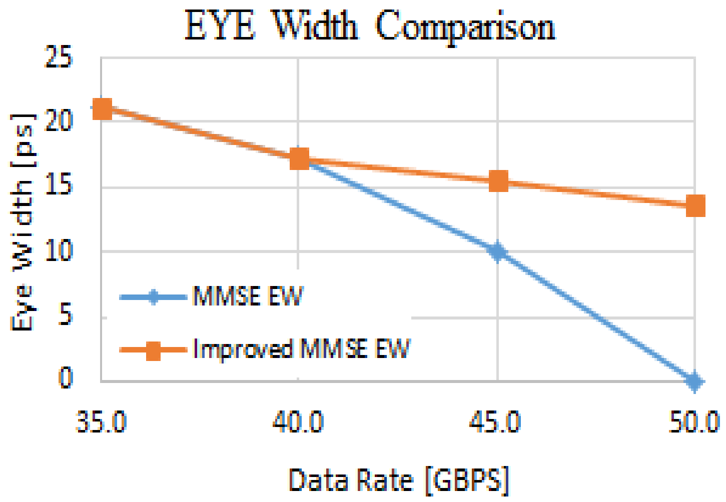




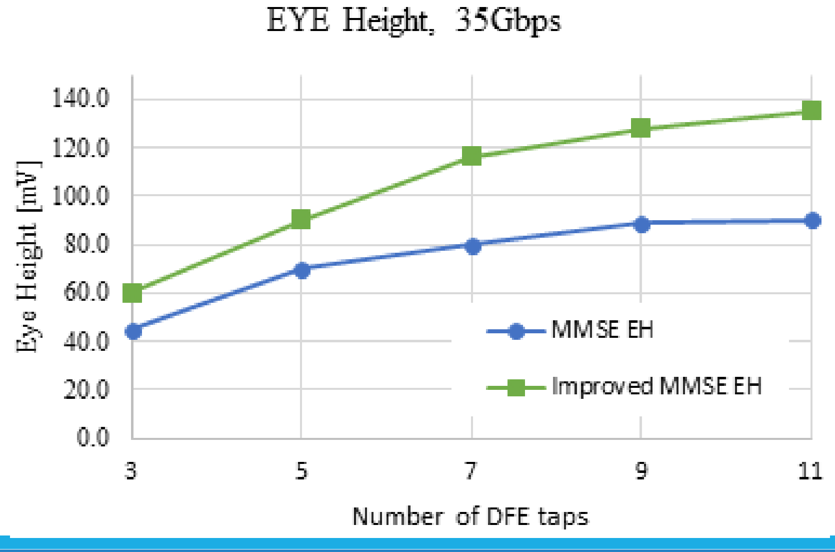
PAM-4 Results

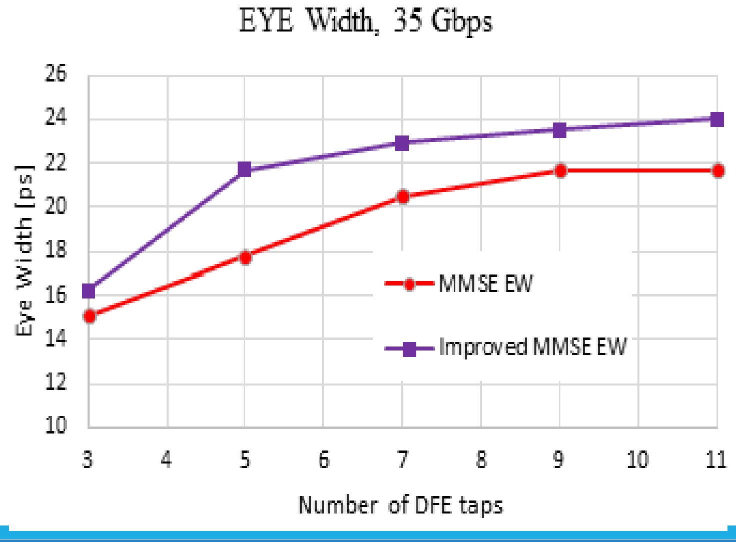
PAM-4 middle eye opening, datarates from 35 to 50 Gbps; 2-tap FFE, 7-tap DFE equalization.





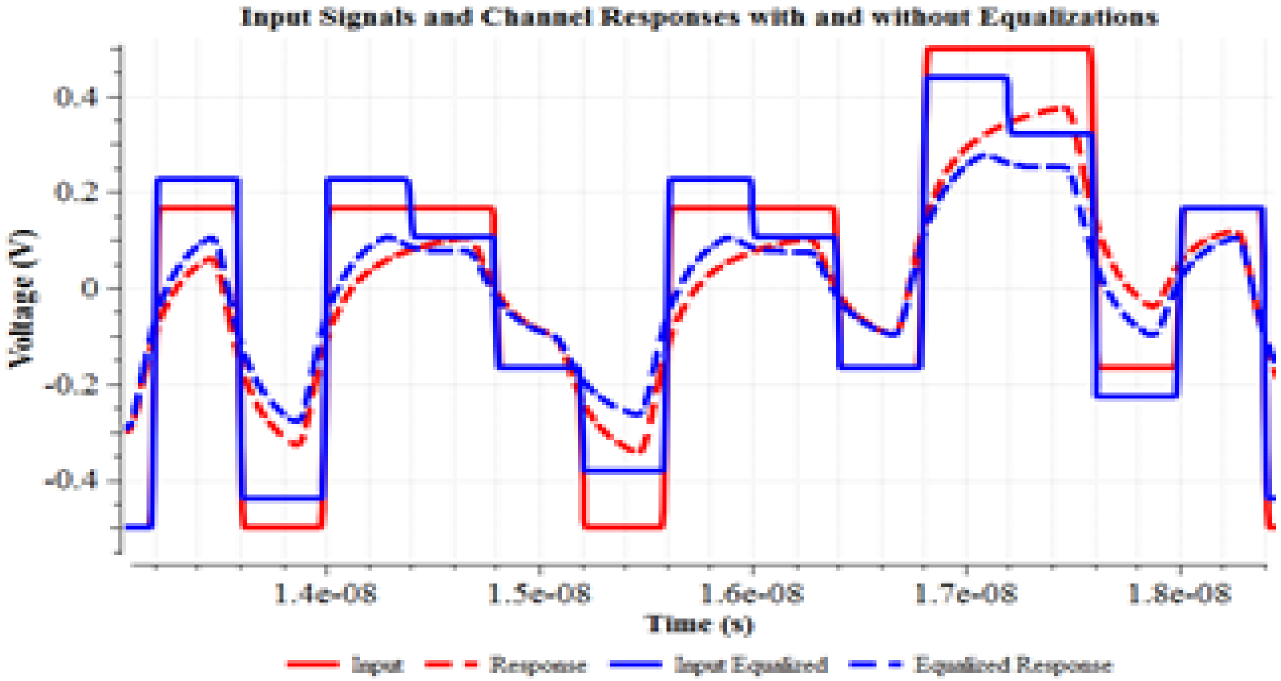
Dependence of middle eye opening on increasing number of DFE taps for PAM4, 35 Gbps

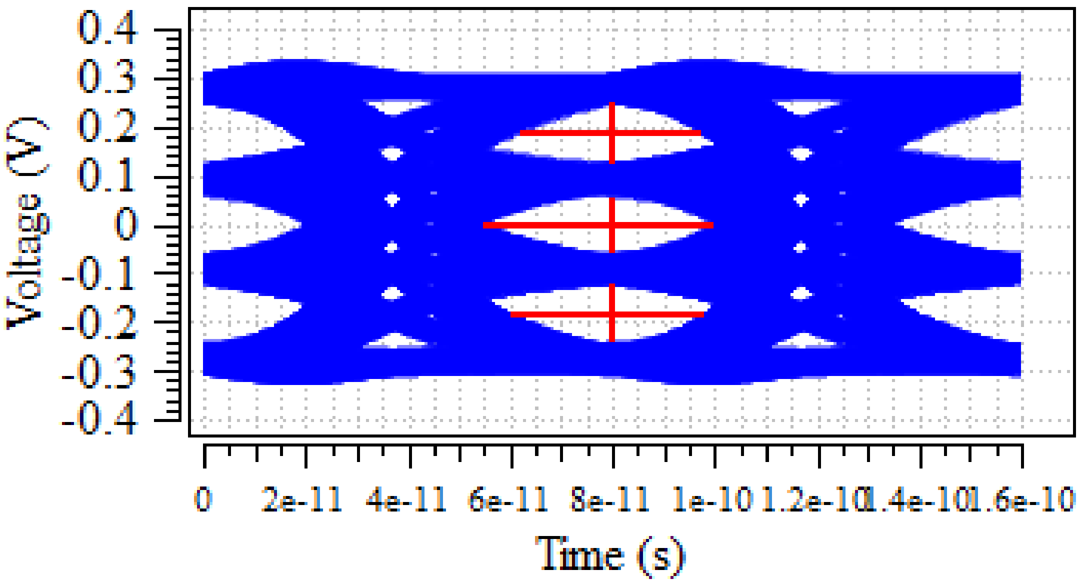




De-emphasis for PAM4 signaling

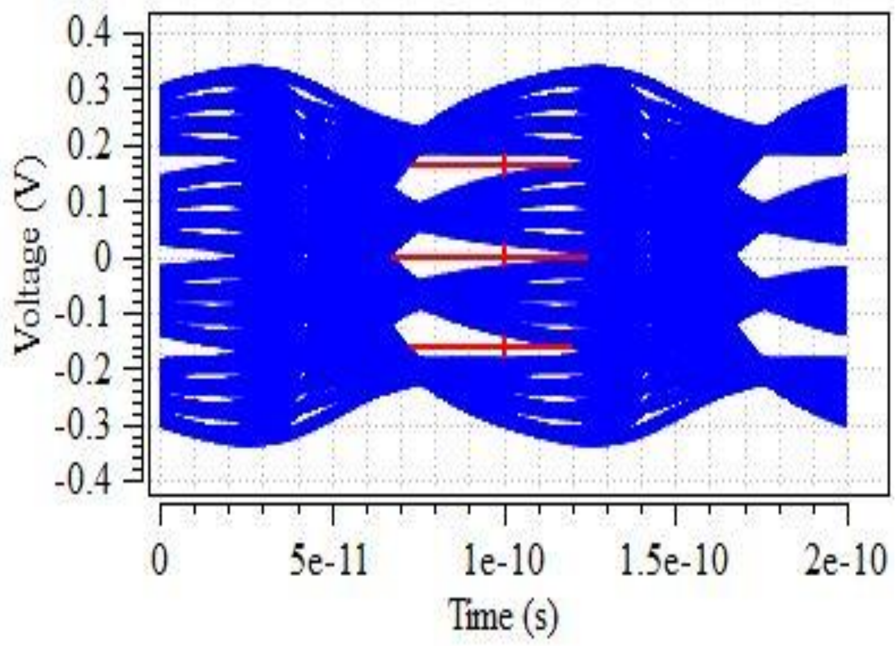
De-emphasized signal

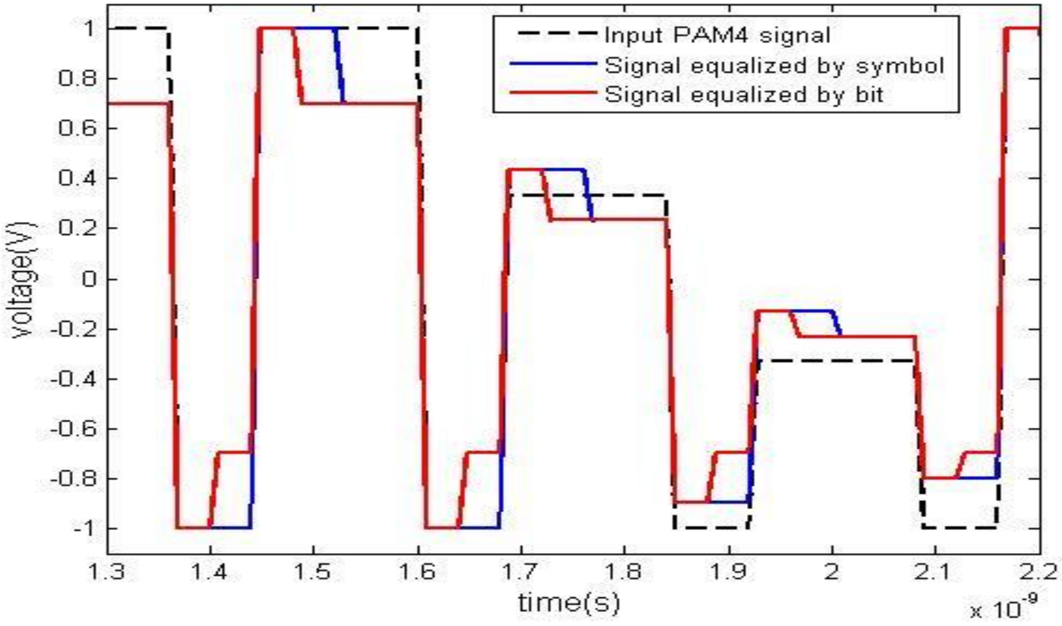




Equalizing the signal by bit

Over-equalized eye

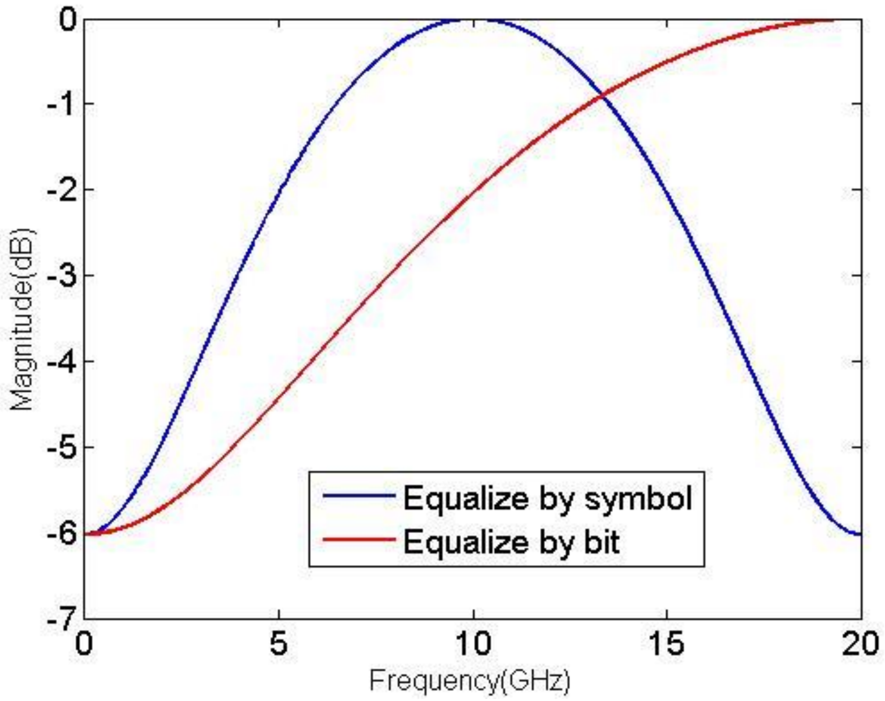




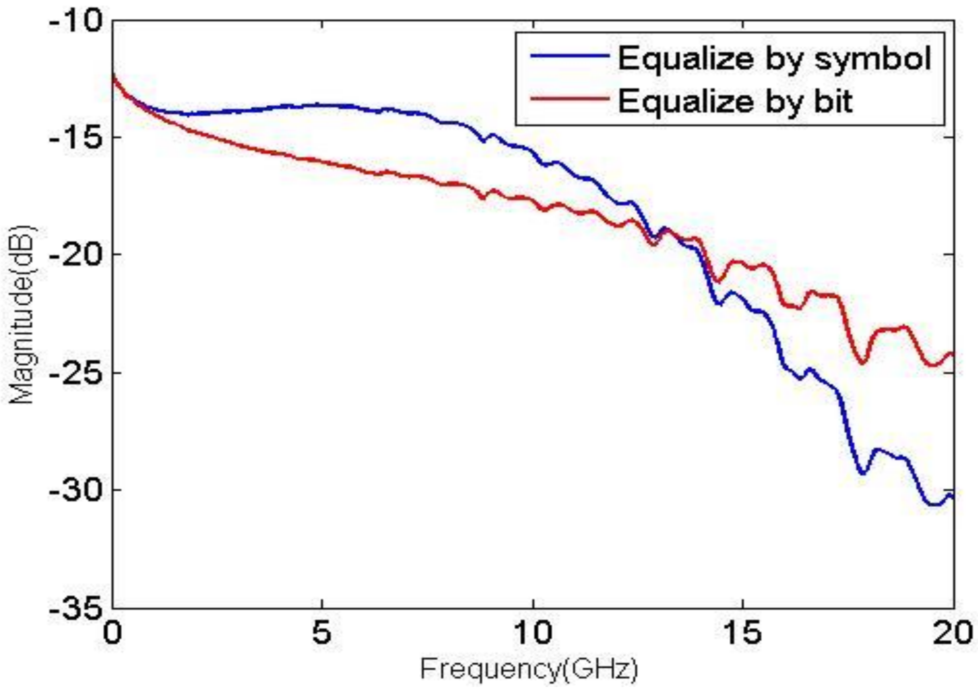
By equalizing the PAM4 signal by bit rather than symbol, the overshooting issue can be mitigated. In transitions, the signal will be emphasized to a higher or lower level by the equalizer.

Equalizing the signal by bit

Transfer function of the equalizer

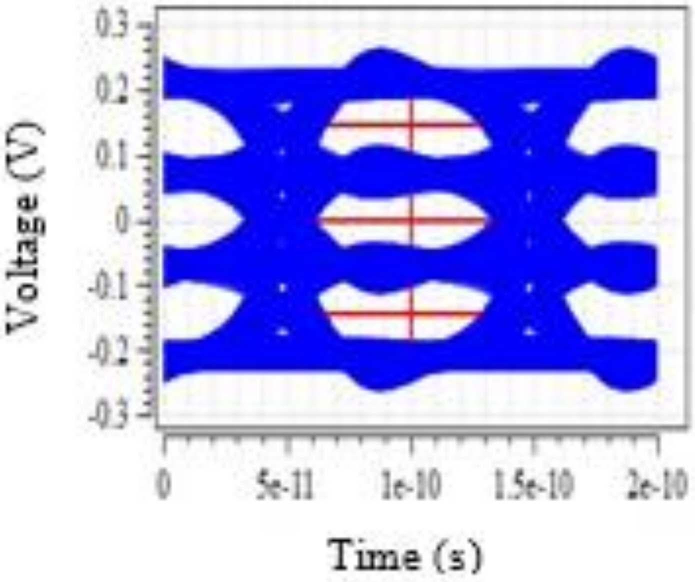


Transfer function of the equalizer combined with the test channel

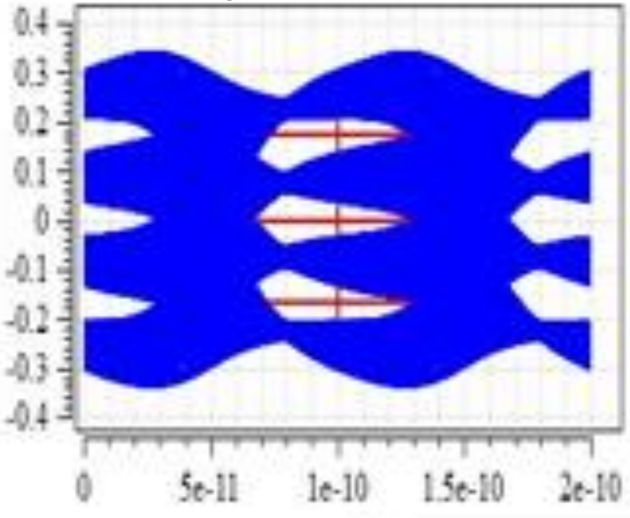


Comparison of eye diagrams

Per bit method

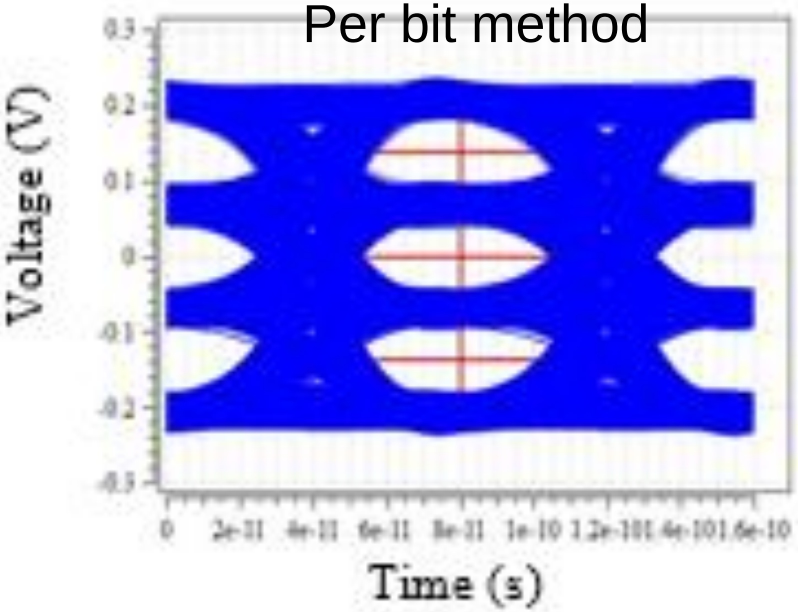


Per symbol method

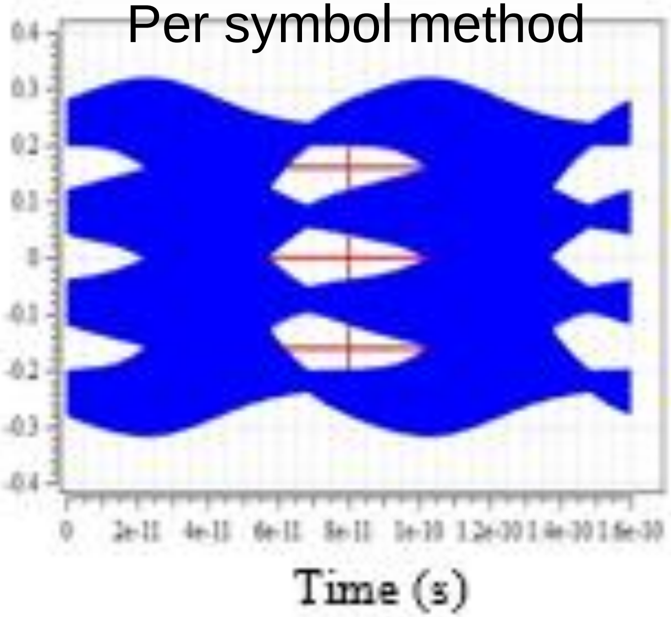


Eye diagrams for taps (0.75, -0.25) at 20Gbps

Per bit method

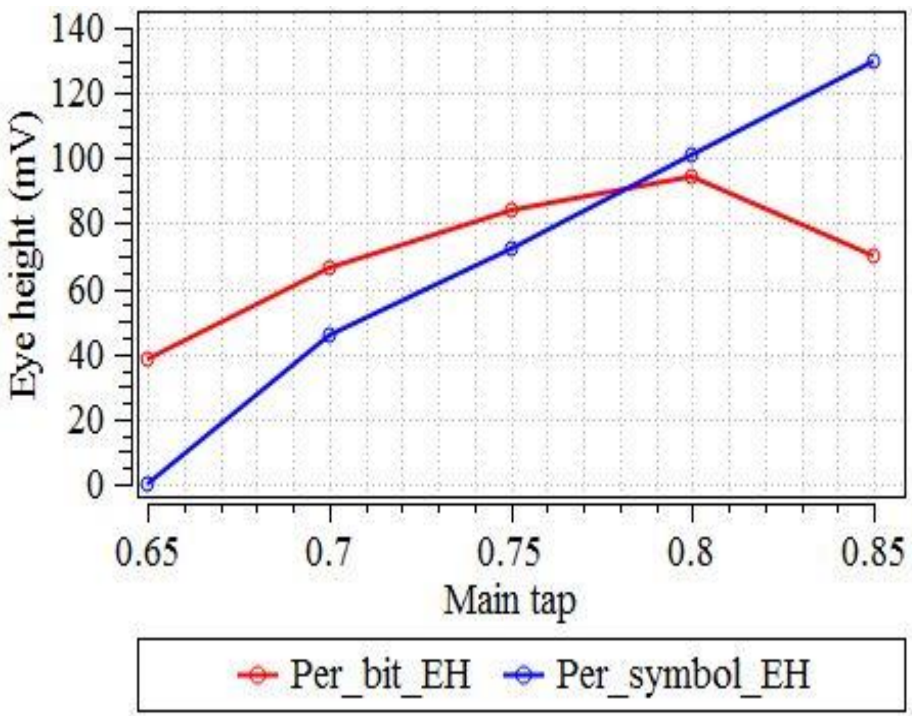


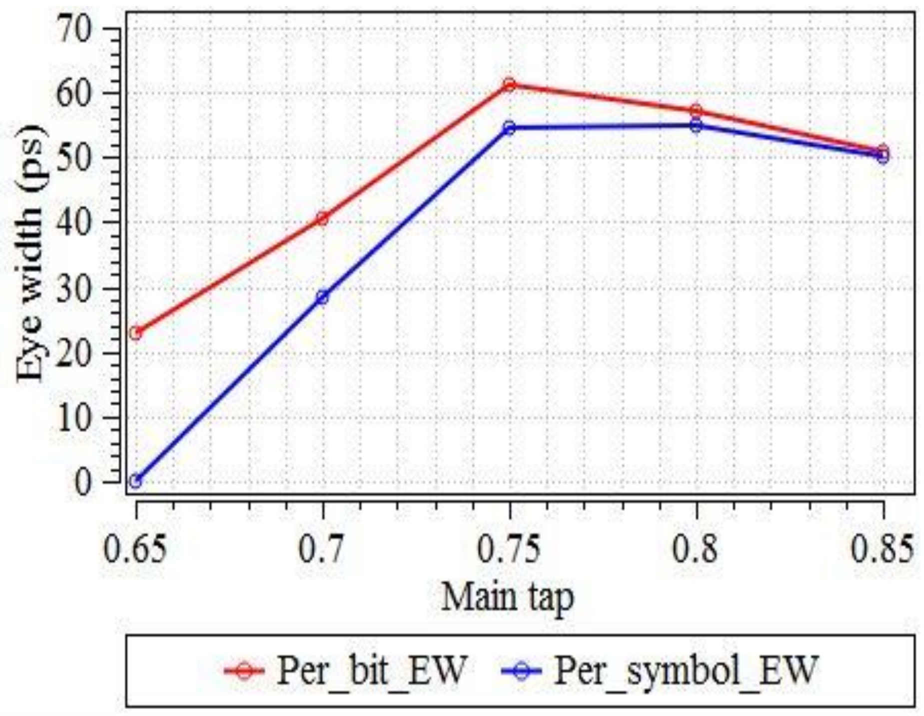
Per symbol method



Eye diagrams for taps (0.75, -0.25) at 25Gbps

Comparison of test results



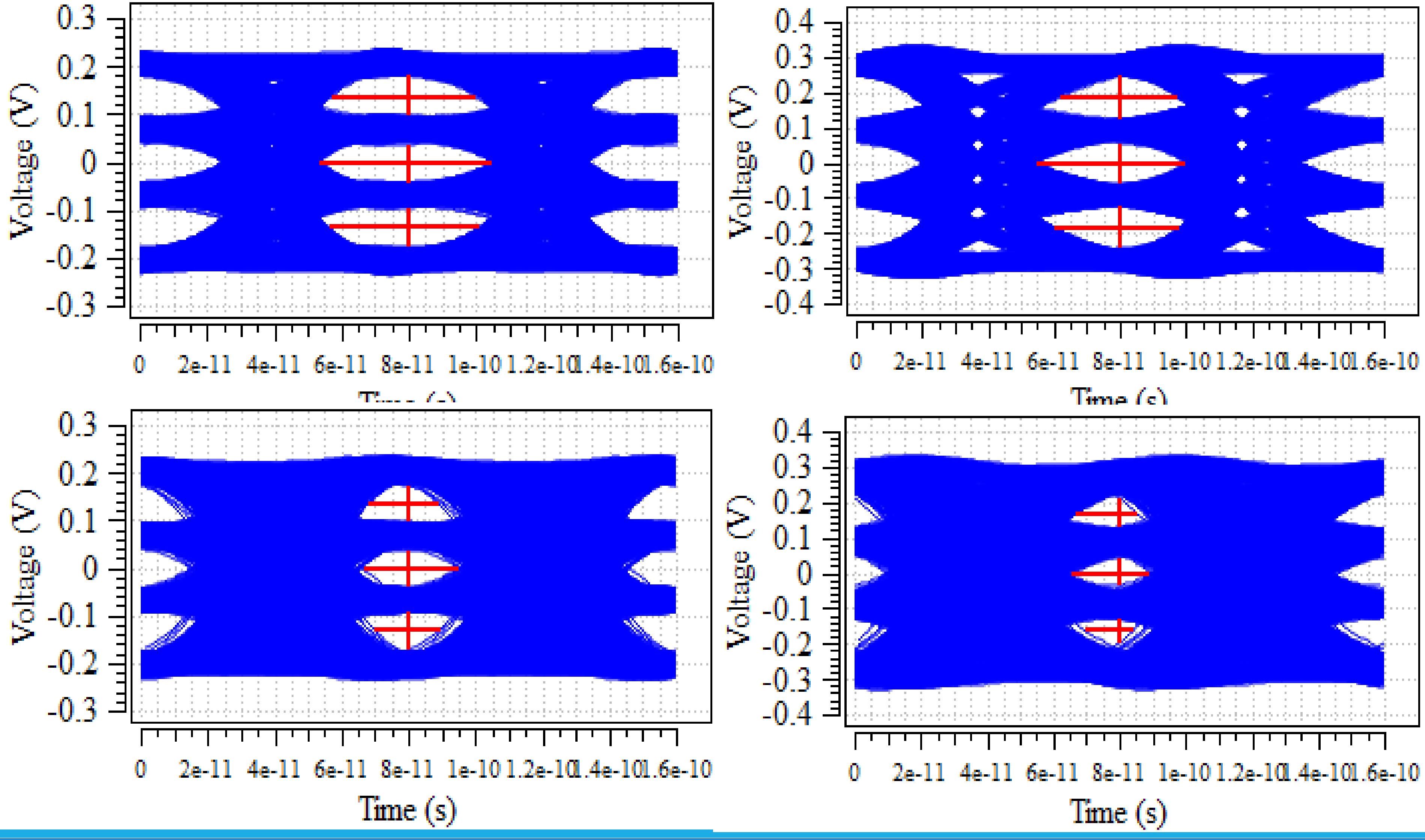


For a two-tap de-emphasis, the eye height and eye width results are plotted when the main tap changes from 0.65 to 0.85. It can be shown that per-bit approach gives larger eye width results. When the de-emphasis is optimal or weak, per-symbol equalization has its advantage in eye height results but it's also more sensitive to the tap coefficient than per-bit equalization.

Eye diagrams for optimized taps at 25Gbps, with 5 ps jitter

Per bit method

Per symbol method



When the tap coefficients are chosen from optimizations, equalizing by symbol approach gives larger eye height results while equalizing by bit approach gives larger eye width results. Therefore, per-bit equalization is less susceptible to jitter.

When 5ps random jitter is injected, a larger eye is obtained by per-bit equalization.

Compensating the level in de-emphasis

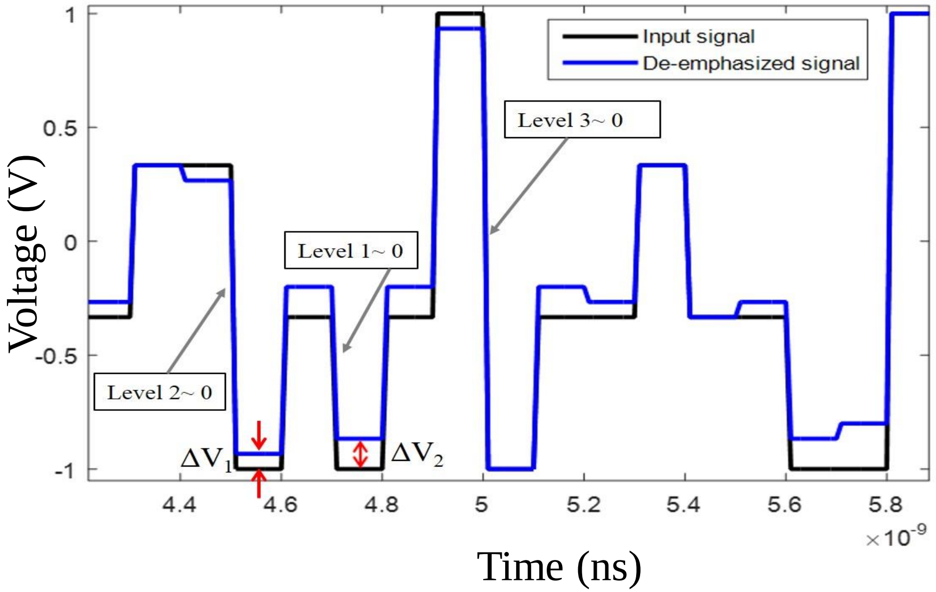
In NRZ, de-emphasis will have the same strength for signal transitions. In PAM4, since there are four levels (level 0, 1, 2 and 3) and sixteen transitions, the de-emphasis or pre-emphasis strength will not be the same for each level. One level will be emphasized to different levels according to previous and following symbol levels in the de-emphasis or pre-emphasis process.

Level 3~level 0: $C_0\times(-a)+C_1\times a = -a$

Level 2~level 0: $C_0\times(-a)+C_1\times a/3 = -a - 2aC_1/3$

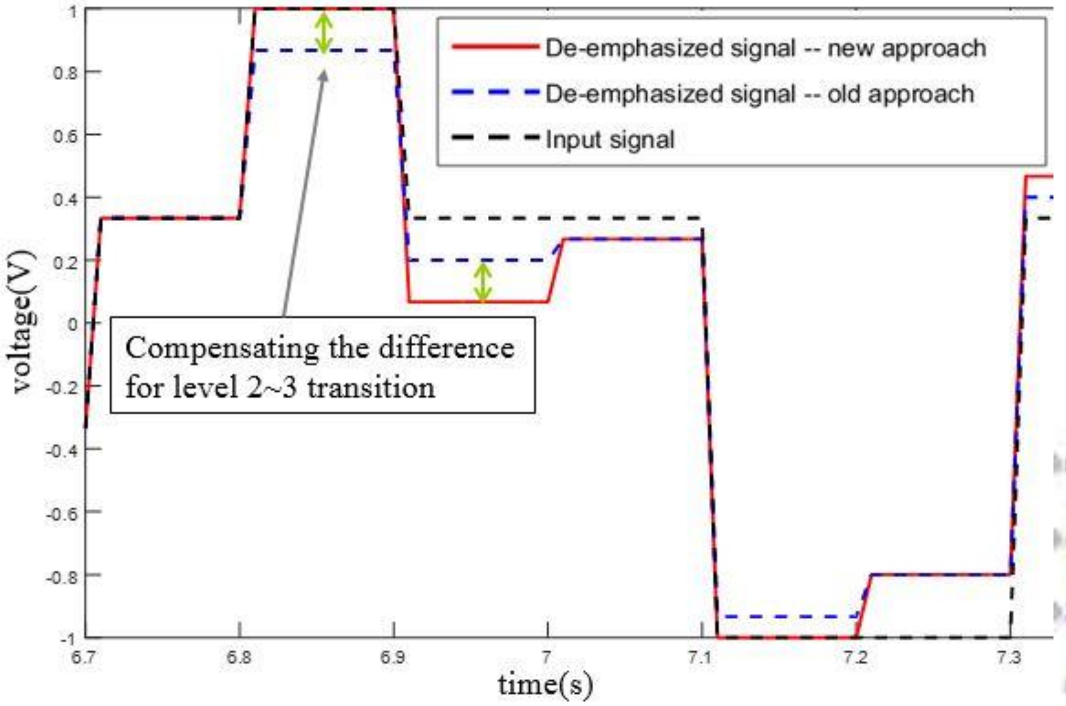
Level 1~level 0: $C_0\times(-a)+C_1\times(-a/3) = -a - 4aC_1/3$

$\Delta V_1=2aC_1/3$, $\Delta V_2=4aC_1/3$



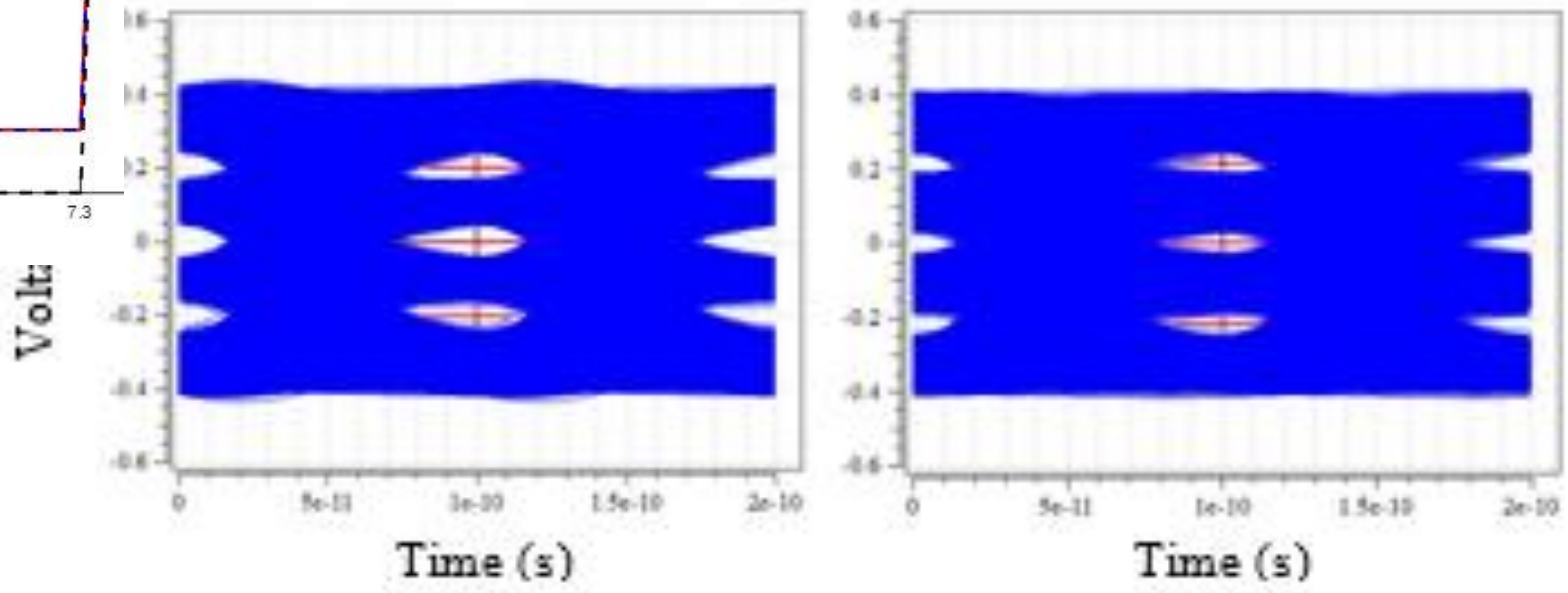
Another enhancement method is to compensate the voltage level when two adjacent symbols in transition don't have equal strength in de-emphasis process.

Comparison of test results



Since the compensation value is proportional to the post-tap, C_1, the signal tends to shoot more when C_1 gets larger, which will cause levels mixing with each other and close the eye. So the over-shooting effect is the main limitation of this approach.

The improvement in eye height results is significant especially when the signal is not heavily emphasized.



Thanks for your attention