EPEPS IBIS Summit
San Jose, California
October 28, 2015

Accurate Statistical Analysis of High-Speed Links with PAM-4 Modulation

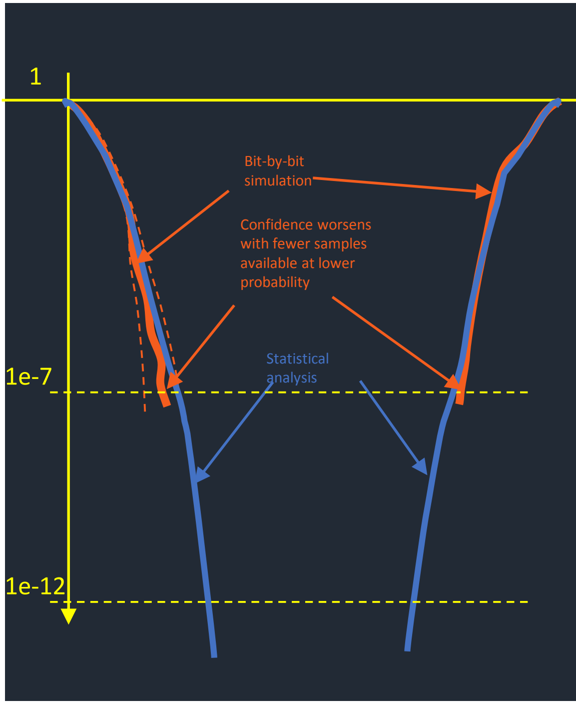
Vladimir Dmitriev-Zdorov

Mentor Graphics Corp.,

vladimir_dmitriev-zdorov@mentor.com

Why statistical analysis of SERDES links?

- Unlike bit-by-bit analysis, statistical simulation can evaluate very low BER (1e-12 to 1e-15 and beyond) in a reasonable time
- First statistical/worst case techniques appeared decades ago in Communication Electronics, however, they do not meet the requirements of the modern technology
- Among other modern approaches, [1,2] described the framework that allows accurate statistical analysis of LTI channels with transmit and receive jitter and edge asymmetry. In [3], it was shown that the method can also consider data-dependent jitter, a set of data-dependent edge transitions, and correlated input patterns. All that was applied to PAM-2 (NRZ) modulation.
- In this work, we generalize these results on PAM-4 modulation



[1] M. Tsuk, D. Dvorscsak, C.S. Ong, and J. White, An electrical-level superimposed-edge approach to statistical serial link simulation, IEEE/ACM International Conf. on Computer-aided Design Digest of Technical Papers, pp. 718–724, 2009.

[2] V. Stojanovic, M. Horiwitz, Modeling and analysis of high-speed links, Proc. IEEE Custom Integrated Circuit Conf., pp. 589–594, Sept 2003

[3] V. Dmitriev-Zdorov, Accurate statistical analysis of SERDES links considering correlated input patterns, data-dependent edge transitions, and transmit jitter, DesignCon 2015.

Major elements of statistical analysis
(general formulation)

The signal at the output of a linear channel can be represented as

$$y(t+\eta_{RX}) = \sum_{k=-\infty}^{n} (b_k - b_{k-1}) S(t - kT + \eta_{TX,k})$$

Where T - symbol interval, b_k, b_{k-1} are symbol values which are {-1,+1} for NRZ and {-1, -1/3, 1/3, 1} for PAM4; *S(t)* is the channels' step response, and η_{TX}, η_{RX} are transmit and receive jitter.

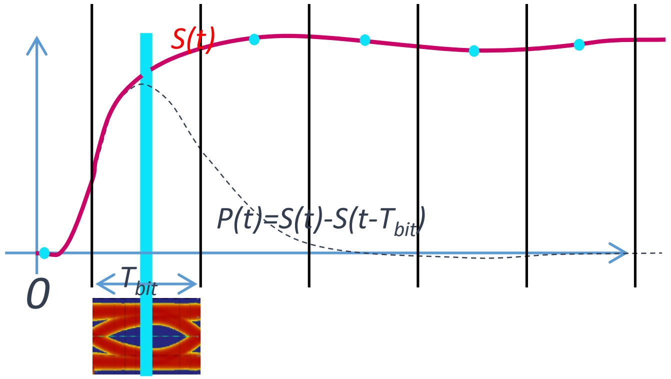
If statistical properties of the input pattern (i.e. distribution of symbol coefficients) and jitter doesn't change in time, *y(t)* is a random variable whose distribution is *T*-periodic in time. Hence, it can be characterized by a collection of *M*>1 PDF or CDF functions. Each one should be computed for $t=nT+\tau_m$, $\tau_m \in [0,T]$. For every τ_m, the problem reduces to finding a single distribution of the variable

$$y_n(\tau_m) = \sum_{k=-\infty}^{n} (b_k - b_{k-1}) S_{n-k}(\eta_{TX,k}),$$

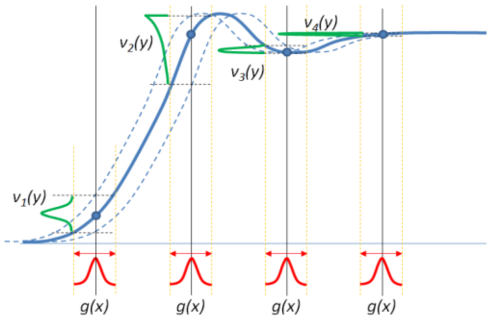
with $$S_{n-k}(x) = S[(n-k)T + \tau_m + x]$$

Major elements of statistical analysis
(samples of the step response)

In absence of transmit jitter, the samples of the step response are discrete values. The set of samples is defined by the timing offset τ_m within unit interval.

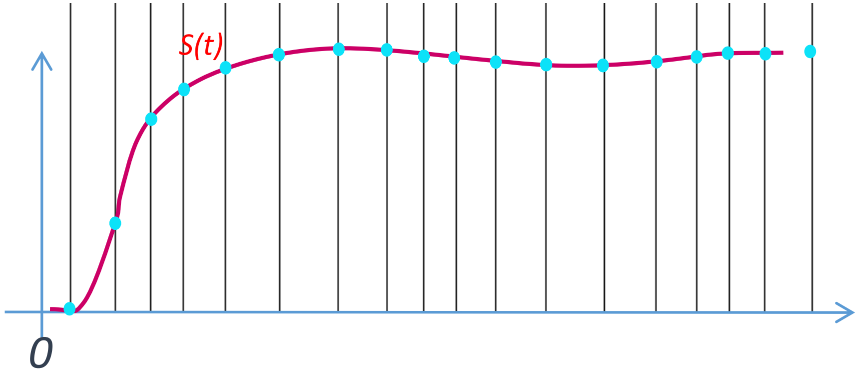


With random transmit jitter, the samples become random values themselves, with their individual distributions (PDFs shown in green).

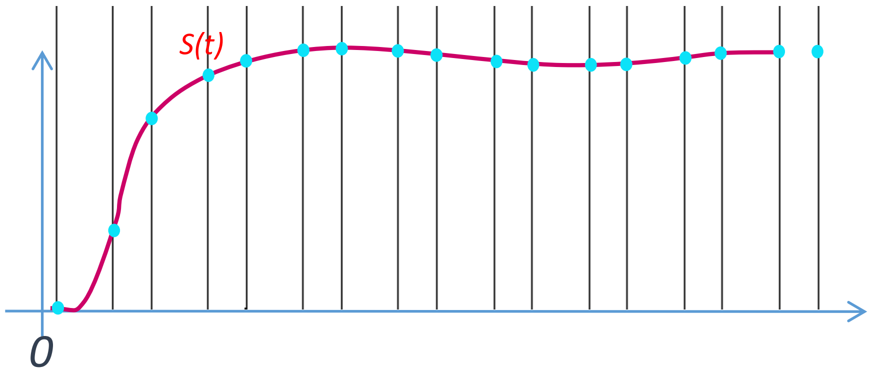


In presence of deterministic Tx jitter, the samples or PDFs should be taken at unequal distances:

Sine jitter:

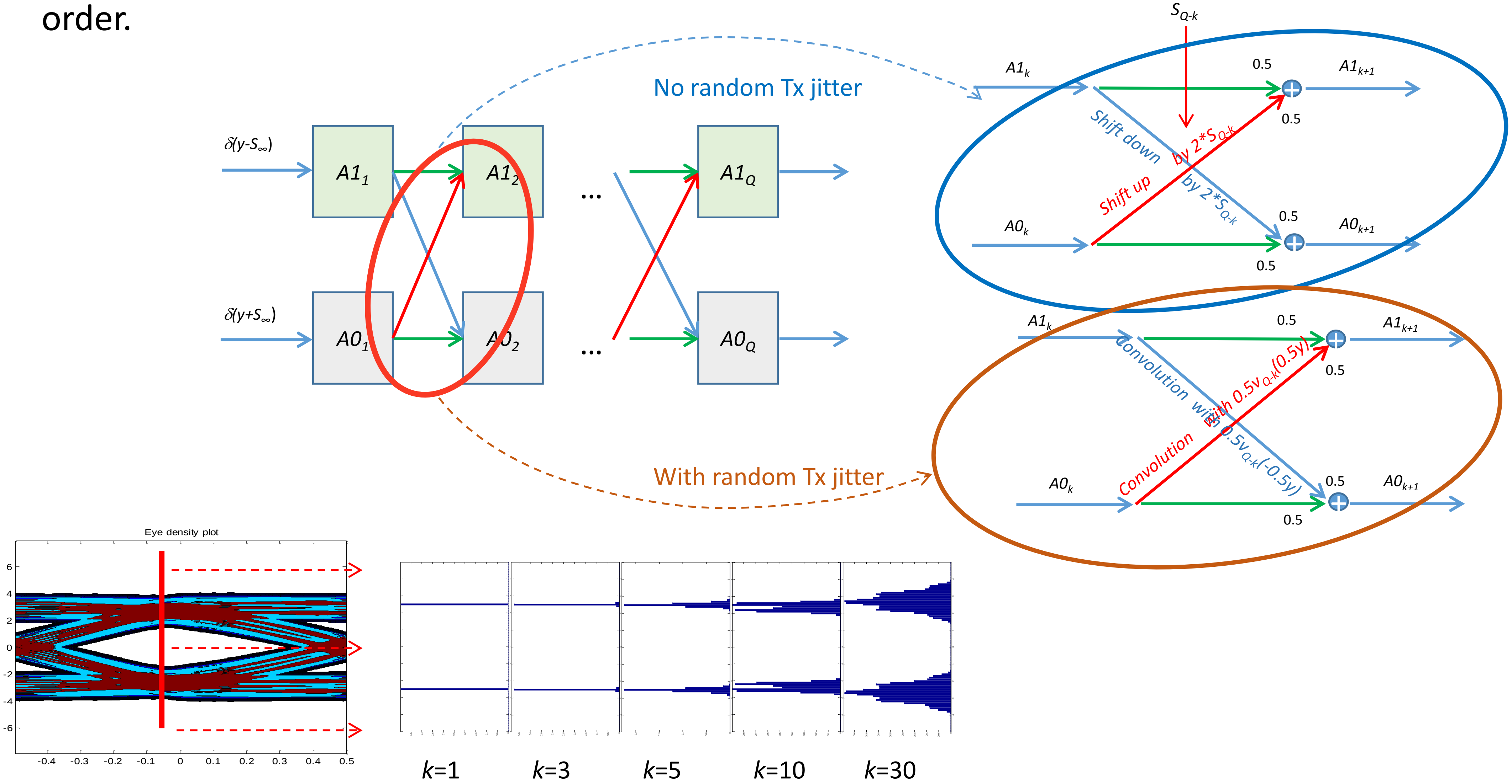


Duty-cycle distortion:



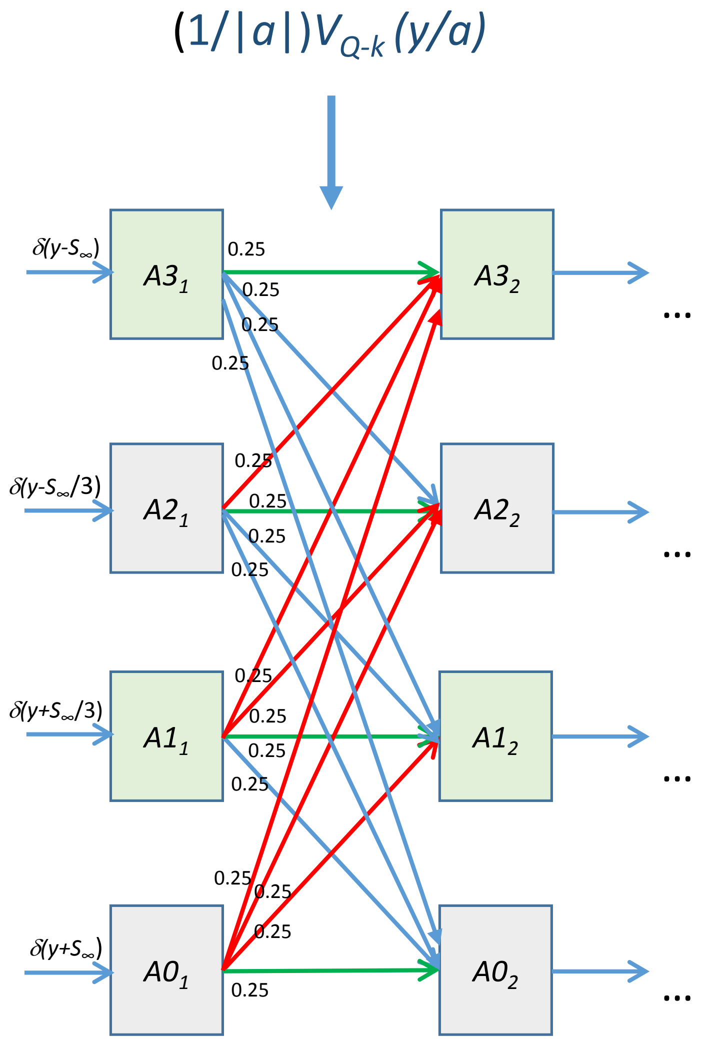
Basic computation flow for PAM-2

For the selected set of samples or vertical PDFs, the cross-section of an eye can be built by a series of shifts (or convolutions) and summations. The samples, *k=1…Q*, of the edge/step response are taken in reverse order.



Basic computation flow for PAM-4

The same samples of the step response or same vertical PDFs are used in a 4-level structure



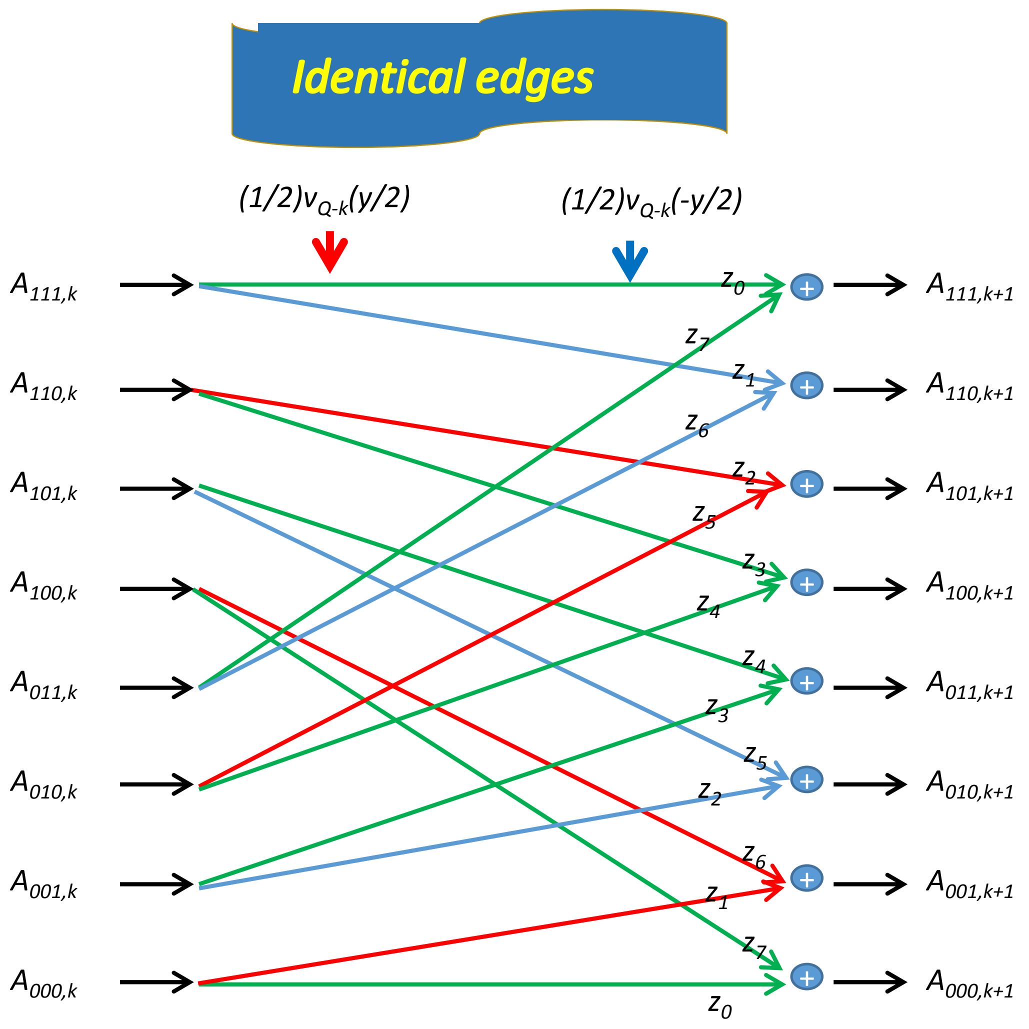
Without random Tx jitter, the shifts of PDFs between the logical states happen in both directions, with increments equal $(2/3)S_{Q-k}$.

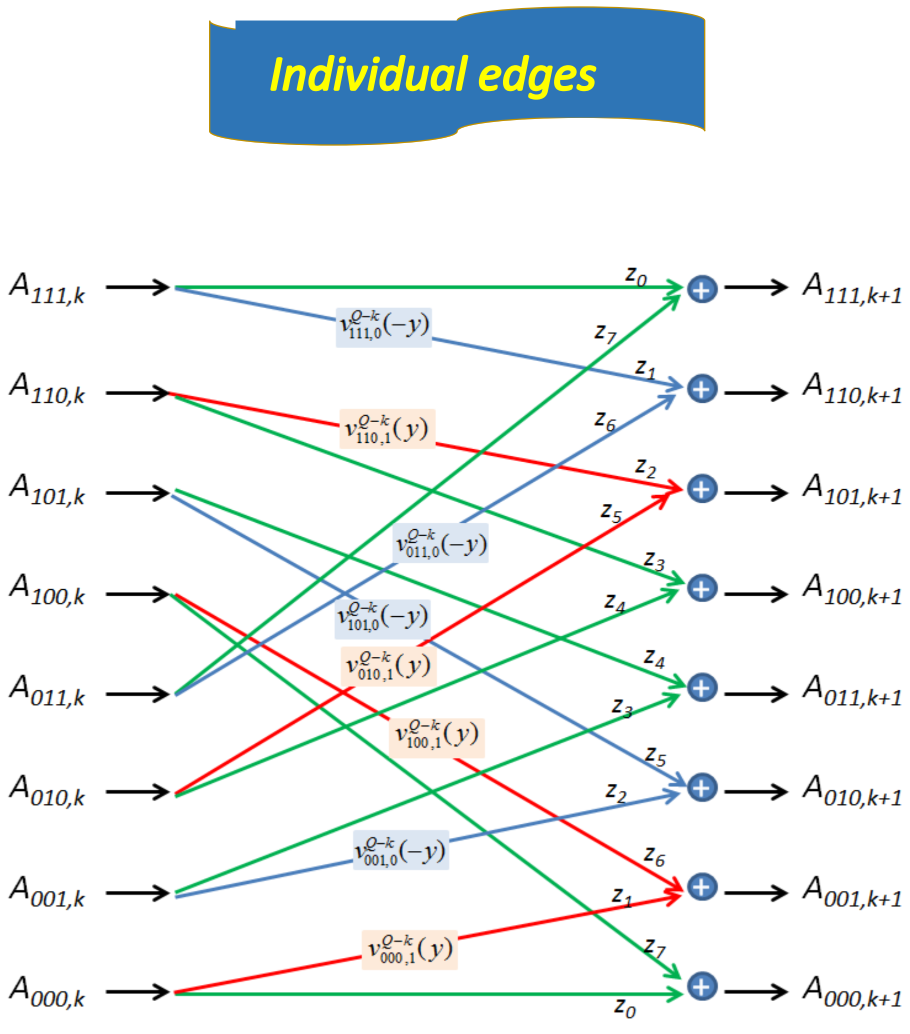
With random Tx jitter, PDFs are modified by way of convolution with scaled/normalized vertical PDFs $V_{Q-k}(y)$. Such scaling/normalization is made by applying the factor a, so that $V_{Q-k}(y)$ becomes $(1/|a|)V_{Q-k}(y/a)$. Negative a means vertically flipped PDF, larger a means expanding along y-axis. As in the discrete case, depending on the direction and the magnitude of the transition, a can be $\pm 2/3$, $\pm 4/3$, and ± 2.

What effects can we simulate statistically?

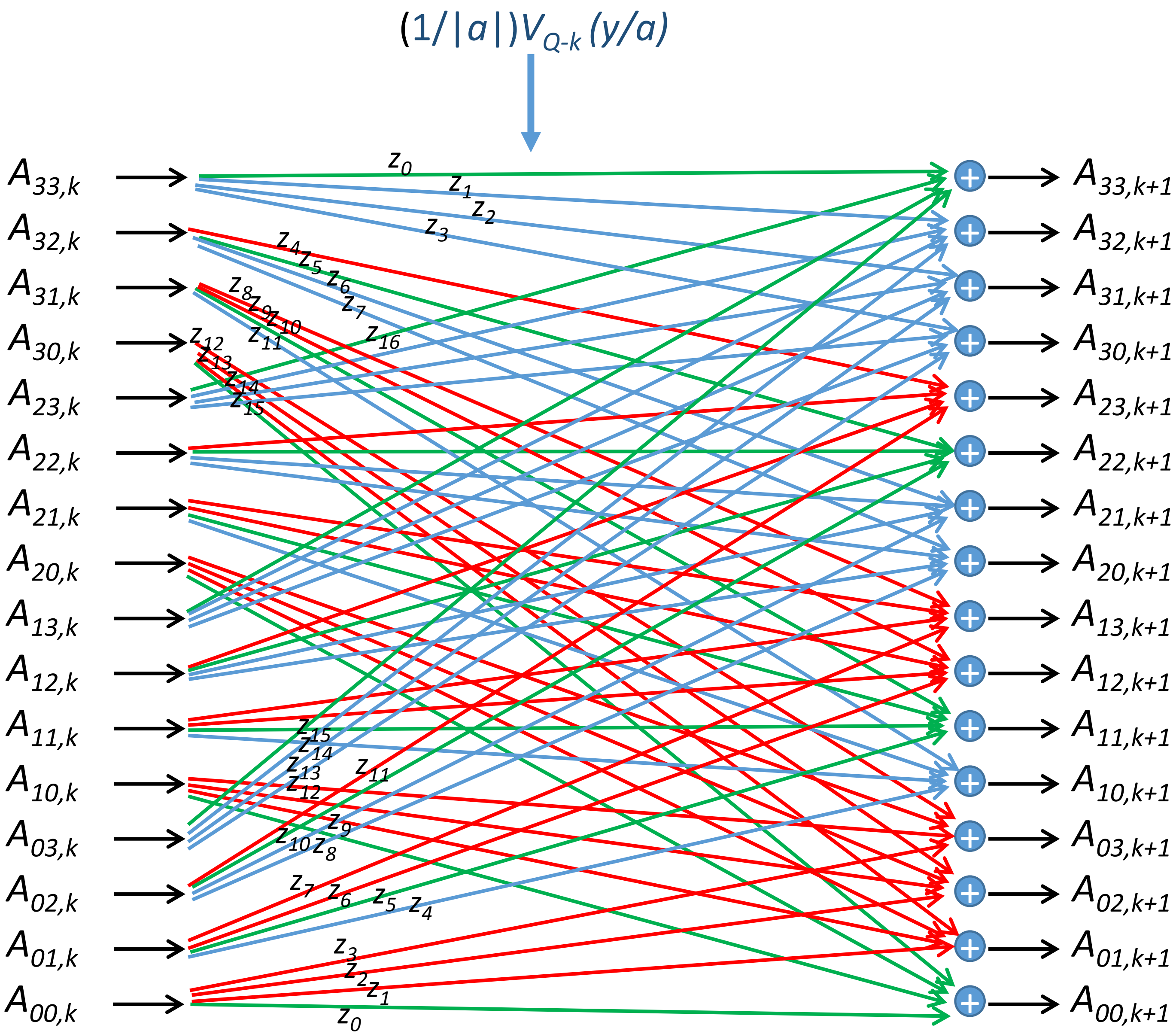
- Random transmit (directly) and receive jitter (added by convolution), with different PDFs
- Deterministic transmit jitter (by uneven sampling of the step response) and receive jitter (by convolution of resulting eye/BER)
- Different shape of the edges (rising/falling asymmetry for PAM-2, or 6 PAM-4 transition edges with different shapes)
- Can we consider the effects that are data-dependent, such as correlation of the input bit values, data-dependent jitter and edge transitions)?

Simulation flow as a multistep Markov chain: consider dependency on more preceding bits, and data-dependent edge transitions. Example of PAM-2 modulation, *N*=3.





Same with PAM-4 modulation, N=2



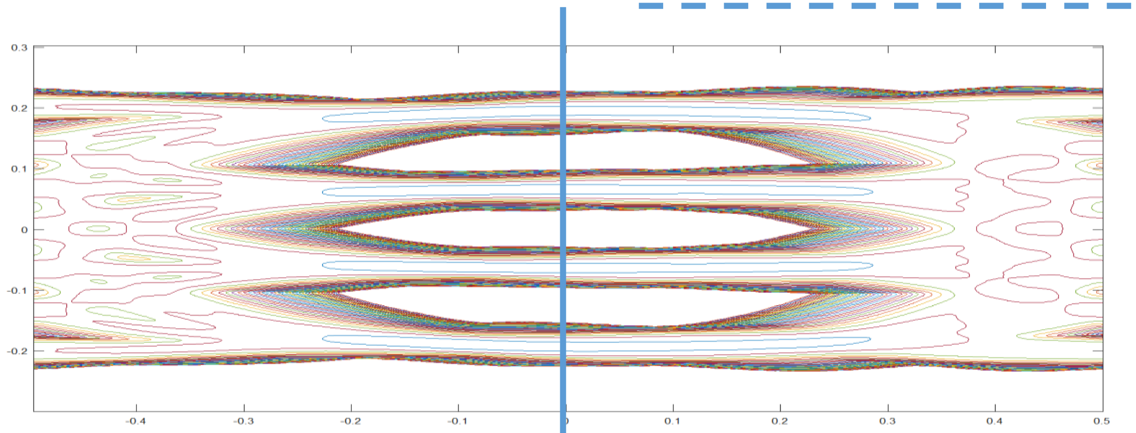
In this structure, we can consider identical or individual transition edges. In the last case, all transitions should be evaluated, assuming dependence on the current bit b_n and a number of previous bits b_{n-2}, b_{n-1}. Convolution should be performed with the vertical PDF

$$\frac{1}{|a|} v^{Q-k}_{b_{n-2} b_{n-1}, b_n}(y/a)$$

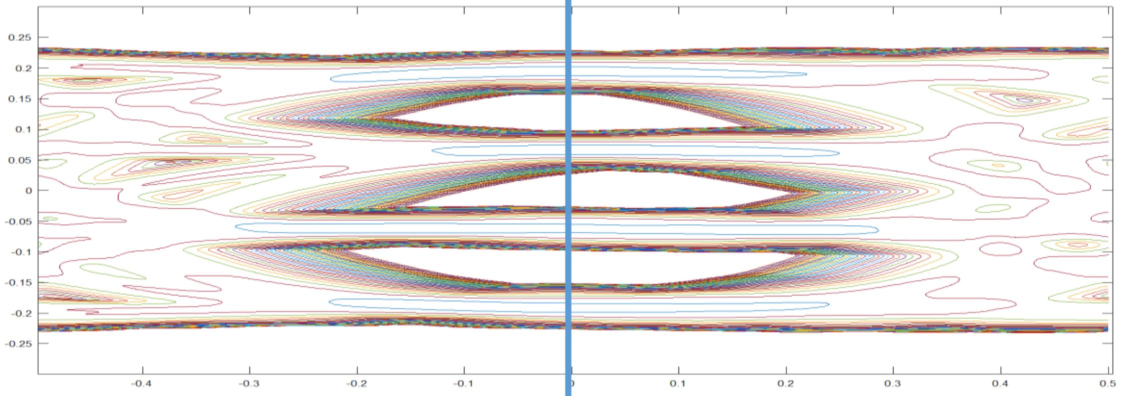
There are 64 transition probability factors z_i total. With statistical symmetry, their number is halved. Among those 32, only 24 are independent (considering 8 normalization conditions). With 24 independent correlation factors measured, we build a system of equations and solve it for unknown transition probabilities.

b_0^2, b_0b_1, b_0b_2, $b_0^2b_1^2$, $b_0^2b_2^2$, $b_0b_1^3$, $b_0b_2^3$, $b_0^3b_1$,
$b_0^3b_2$, $b_0^3b_1^3$, $b_0^3b_2^3$, $b_0^2b_1b_2$, $b_0b_1^2b_2$, $b_0b_1b_2^2$, $b_0^2b_1^2b_2^2$,
$b_0^3b_1^2b_2$, $b_0^3b_1b_2^2$, $b_0^2b_1^3b_2$, $b_0b_1^3b_2^2$, $b_0b_1^2b_2^3$, $b_0^2b_1b_2^3$,
$b_0^3b_1^3b_2^2$, $b_0^3b_1^2b_2^3$ $b_0^2b_1^3b_2^3$

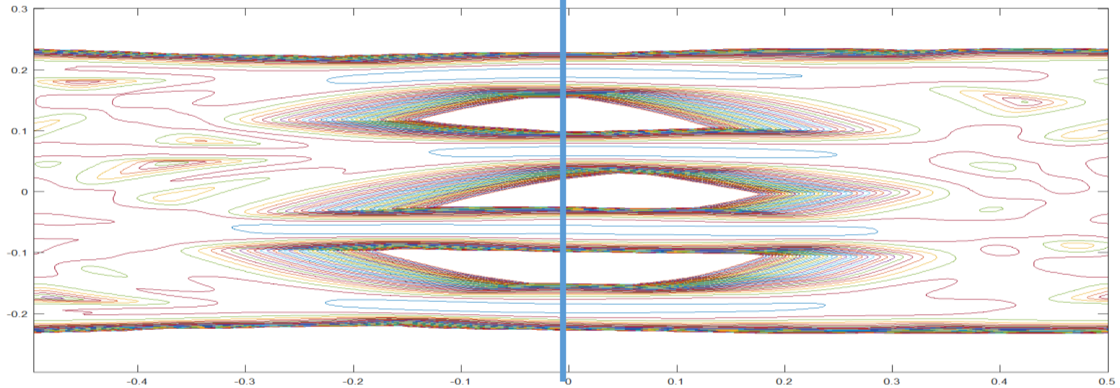
Statistical Eye Diagrams, PAM-4 modulation, *N*=2



PRBS15->Gray->PAM-4,
Identical transitions
N=2

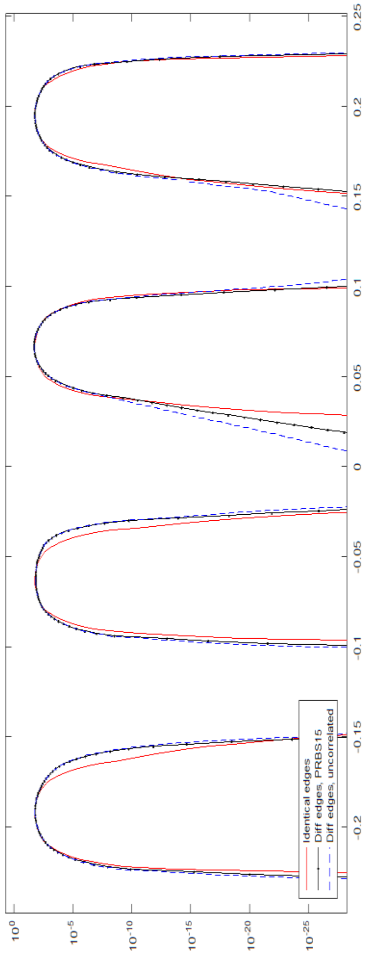


PRBS15->Gray->PAM-4,
Different transition
edges, *N*=2



Uncorrelated binary input
->Gray->PAM-4,
Different edges, *N*=2

Vertical cross-sections
of all three



Sampling thresholds for PAM-4:

we can find them from peak distortion

Assume:

1. Identical transition edges
2. Uncorrelated input pattern (or correlated with statistical symmetry)
3. No jitter, noise, crosstalk and no non-linear equalization.

Then the vertical PDFs at different levels can be described in terms of the pulse response samples:

$$y_3(x) = +p_{K\max}(x) + \sum_{k \neq K\max} b_k p_k(x)$$

$$y_2(x) = \frac{1}{3} p_{K\max}(x) + \sum_{k \neq K\max} b_k p_k(x)$$

$$y_1(x) = -\frac{1}{3} p_{K\max}(x) + \sum_{k \neq K\max} b_k p_k(x)$$

$$y_0(x) = -p_{K\max}(x) + \sum_{k \neq K\max} b_k p_k(x)$$

Here, b_k={±1, ± 1/3} are discrete random variables
x – sampling position within 1UI
$p_k(x)$ – cursors of the pulse response for a given offset

Sampling thresholds for PAM-4:

PAM4_UpperThreshold

First, we find the threshold between levels 3 and 2 (PAM4_UpperThreshold). Let's find the worst margins for the distributions at these levels. These are:

$$y_{3\_wrst}(x) = +p_{K\max}(x) - \sum_{k \neq K\max} |\, p_k(x) \,|$$

$$y_{2\_wrst}(x) = +\frac{1}{3} p_{K\max}(x) + \sum_{k \neq K\max} |\, p_k(x) \,|$$

$$Median_{23}(x) = \frac{1}{2}[y_{3\_wrst}(x) + y_{2\_wrst}(x)] = \frac{2}{3} p_{K\max}(x)$$

$$VertEyeSize_{23}(x) = [y_{3\_wrst}(x) - y_{2\_wrst}(x)] = \frac{2}{3} p_{K\max}(x) - 2 \sum_{k \neq K\max} |\, p_k(x) \,|$$

As we see, the optimal sampling threshold is a function of the horizontal offset. Its dependence is proportional to the shape of the pulse response (near its peak)

Sampling thresholds for PAM-4:

PAM4_CenterThreshold

The mutually-worst margins for levels 2 and 1 are:

$$y_{2\_wrst}(t) = +\frac{1}{3} p_{k\max}(t) - \sum_{k \neq k\max} |p_k(t)|$$

$$y_{1\_wrst}(t) = -\frac{1}{3} p_{k\max}(t) + \sum_{k \neq k\max} |p_k(t)|$$

$$Median_{12}(t) = \frac{1}{2}\left(y_{2\_wrst}(t) + y_{1\_wrst}(t)\right) = 0$$ (As expected)

$$Vec_{12}(t) = y_{2\_wrst}(t) - y_{1\_wrst}(t) = \frac{2}{3} p_{k\max}(t) - 2\sum_{k \neq k\max} |p_k(t)|$$ (Same as before)

Sampling thresholds for PAM-4:

PAM4_LowerThreshold

The mutually-worst margins for levels 1 and 0 are:

$$y_{1\_wrst}(x) = \frac{1}{3} p_{l\max}(x) - \sum_{k \neq k_{\max}} \left| p_{lk}(x) \right|$$

$$y_{0\_wrst}(x) = -p_{l\max}(x) + \sum_{k \neq k_{\max}} \left| p_{lk}(x) \right|$$

$$M_{01}(x) = \frac{1}{2}\left(y_{1\_wrst}(x) + y_{0\_wrst}(x)\right) = -\frac{2}{3} p_{l\max}(x)$$

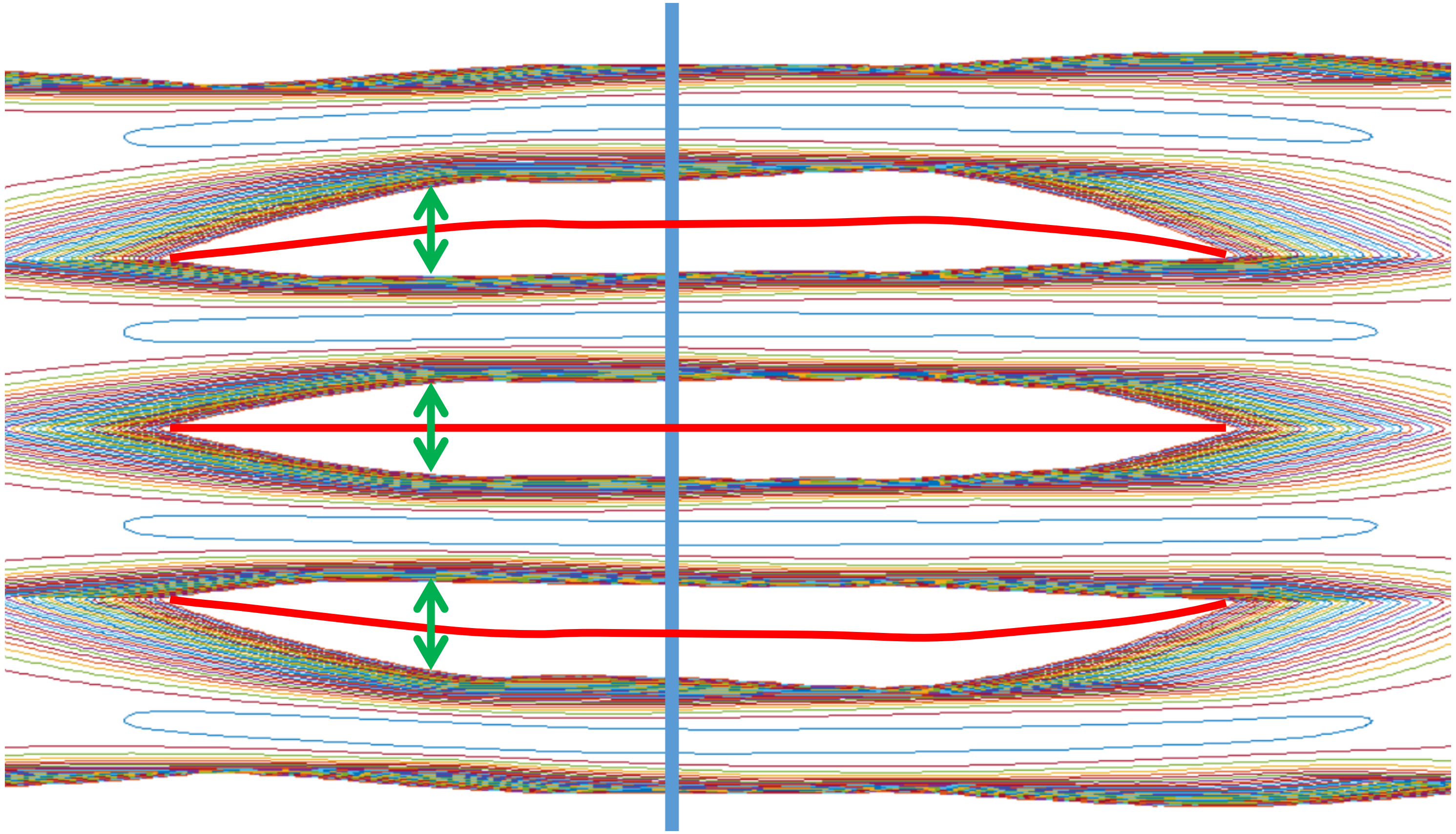
(Opposite to upper threshold)

$$V_{eye01}(x) = y_{1\_wrst}(x) - y_{0\_wrst}(x) = \frac{2}{3} p_{l\max}(x) - 2\sum_{k \neq k_{\max}} \left| p_{lk}(x) \right|$$

(Same as before)

Sampling thresholds for PAM-4:

Can be found from the pulse response of the channel



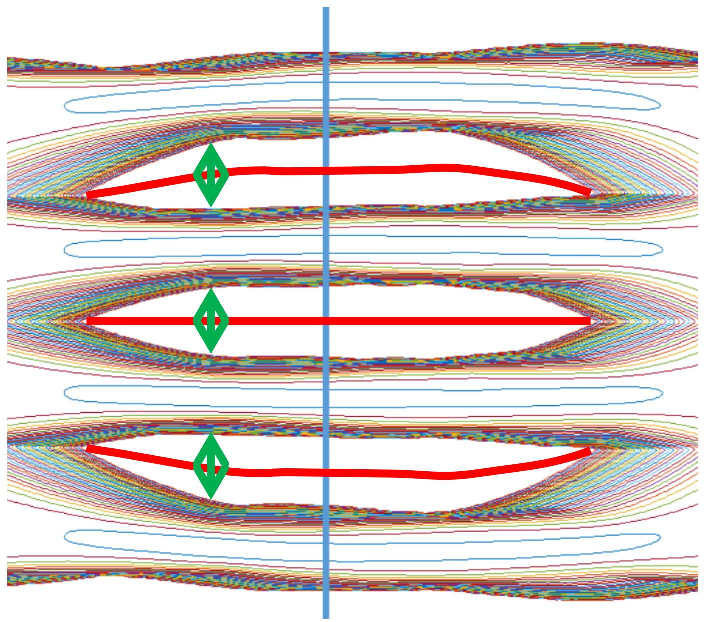
1. Eye median (and the optimal sampling threshold) is a function of the sampling position x_0. It can be found as

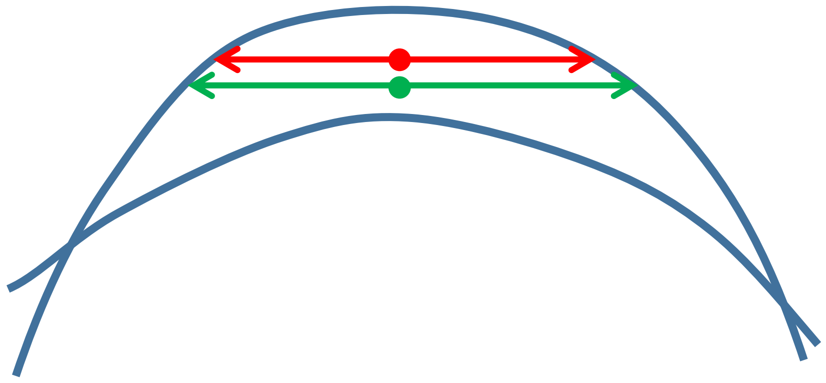
$$\pm\frac{2}{3}p_{\mathrm{Kmax}}(x_0)$$

2. Under assumed conditions, the vertical openings for all 3 eyes are identical.
3. For LTI symmetrical channels and no impairments, the optimal threshold values can be determined from the single bit response (can be found from equalized impulse response of the channel).

Sampling thresholds for PAM-4:

Can be found from the pulse response of the channel, but…

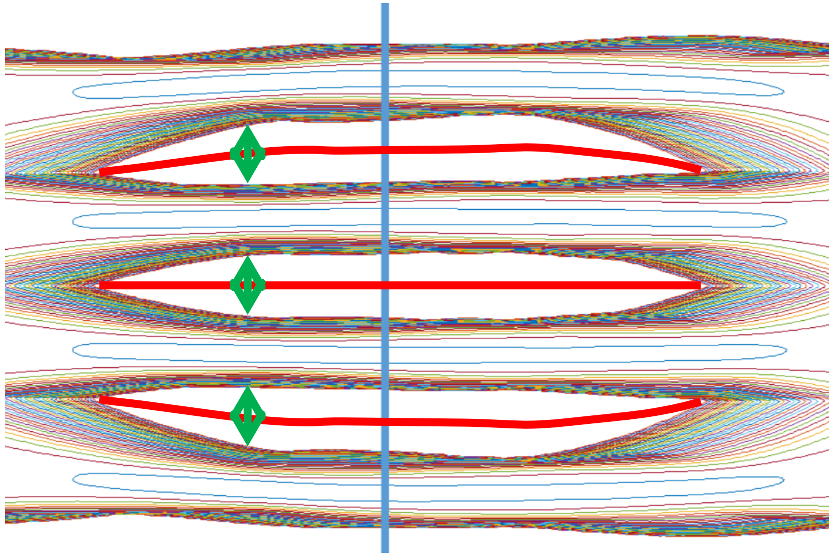


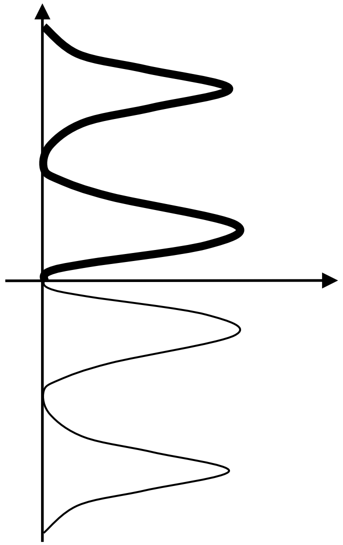


- Uncorrelated crosstalk and noise will not change the relationships found for the eye median and margins (medians still repeat the shape of the pulse response, vertical eye openings are same in the vertical cross-section
- But, Tx and Rx jitter may invalidate them (the top and bottom eyes are most affected)
- Even for the “balanced” eye opening, we may decide to modify upper or lower sampling threshold, if the timing margin is tighter than the voltage allowance. For example, in the case below we trade some of the vertical space for improved horizontal margin (green versus red sampling)

Sampling thresholds for PAM-4:

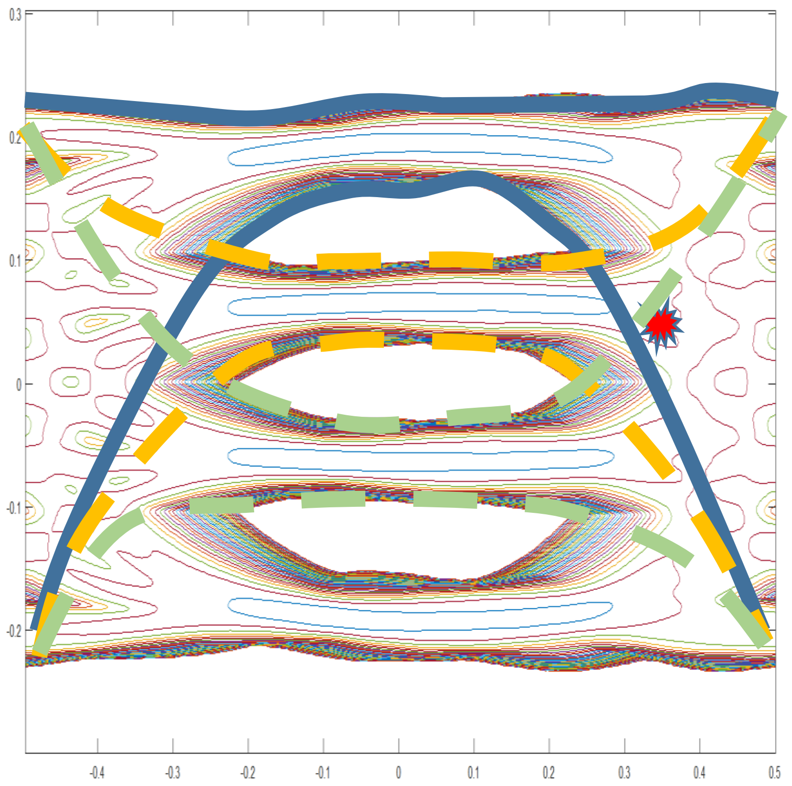
For non-LTI channels, we can build vertical histogram





- For non-LTI channels, we use bit-by-bit analysis and call GetWave function. If each call processes sufficient number of bits (e.g. 2-4 thousand), Rx AMI DLL (or EDA tool) can build a vertical histogram from the waveform sampled at the selected location - e.g. dictated by clock times - and find the best threshold values by analyzing this histogram.
- Or, it can find thresholds and offsets from other considerations, based on specific knowledge about device/slicers
- When simulation advances, the thresholds are updated from the modified waveform etc.

Finding SER and BER from statistical analysis



- In statistical analysis, accurate SER and BER evaluation is possible if we store/keep the "branches" of the eye density plot separately. Then, for a given sample position we can find the source and effect of the error and convert SER into BER
- SER = P(3)[P(2|3)+P(1|3)+P(0|3)] + P(2)[P(3|2)+P(1|2)+P(0|2)] +
  P(1)[P(3|1)+P(2|1)+P(0|1)] + P(0)[P(3|0)+P(2|0)+P(1|0)].
- We can convert SER into BER if we know the mapping (e.g. Gray code):
  '0' <=> '00'
  '1' <=> '01'
  '2' <=> '11'
  '3' <=> '10'
- BER = P(10)[P(11|10)*1+P(01|10)*2+P(00|10)*1] +
  P(11)[P(10|11)*1+P(01|11)*1+P(00|11)*2] +
  P(01)[P(10|01)*2+P(11|01)*1+P(00|01)*2] +
  P(00)[P(10|00)*1+P(11|00)*2+P(01|00)*1]
- Each of the four summands in SER and BER can be found from considering one branch of the eye density

The effects we can consider in statistical simulation with PAM-4 modulation

- Accurate consideration of ISI and crosstalk
- Transmit jitter, random and deterministic, and receive jitter
- Edge asymmetry, or dependence of the edge shapes on a number of preceding bits
- Correlated input pattern
- The effect of data-dependent jitter and noise
- Simulation is fast: typically takes a minute or less
- The branches of eye density are formed separately, this allows accurate BER evaluation
- PAM-4 thresholds are new for IBIS-AMI in IBIS Version 6.1