Enforcement Passivity of S-parameter Sampled Frequency Data

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Agenda

- Causality and passivity fundamentals
- Enforcing passivity for sampled frequency data
- S-parameter rational function approximation
- A new format of S-parameter rational function
- Summary

S-parameter Model

Touchstone file description

Hz S RI R 50

Frequency	S-parameter raw data							
1.000e+7	1.028e-4	-5.022e-4	3.666e-5	2.586e-4	9.998e-1	-2.709e-3	4.533e-5	-4.858e-5
	1.747e-5	2.512e-4	9.197e-5	-5.294e-4	-6.600e-5	-5.758e-5	9.998e-1	-2.660e-3
	9.998e-1	-2.710e-3	-4.670e-5	-4.861e-5	1.155e-4	-5.020e-4	3.726e-5	2.604e-4
	2.603e-5	-5.806e-5	9.998e-1	-2.659e-3	1.804e-5	2.493e-4	7.785e-5	-5.294e-4
2.000e+7	1.268e-4	-1.096e-3	5.455e-5	4.873e-4	9.997e-1	-5.339e-3	3.647e-5	-1.011e-4
	3.735e-5	4.941e-4	1.116e-4	-1.132e-3	-5.493e-5	-1.077e-4	9.997e-1	-5.231e-3
	9.997e-1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	1.962e-5	-1.074e-4	9.997e-1	-5.229e-3	3.722e-5	4.813e-4	9.901e-5	-1.132e-3
3.000e+7	1.322e-4	-1.705e-3	6.496e-5	7.263e-4	9.996e-1	-7.954e-3	-1.425e-6	-1.547e-4
	6.417e-5	7.199e-4	1.198e-4	-1.751e-3	-1.140e-5	-1.558e-4	9.996e-1	-7.787e-3
	9.996e-1	-7.953e-3	-9.924e-6	-1.548e-4	1.249e-4	-1.704e-3	6.277e-5	7.226e-4
	-3.206e-6	-1.545e-4	9.996e-1	-7.785e-3	6.165e-5	7.242e-4	1.141e-4	-1.750e-3
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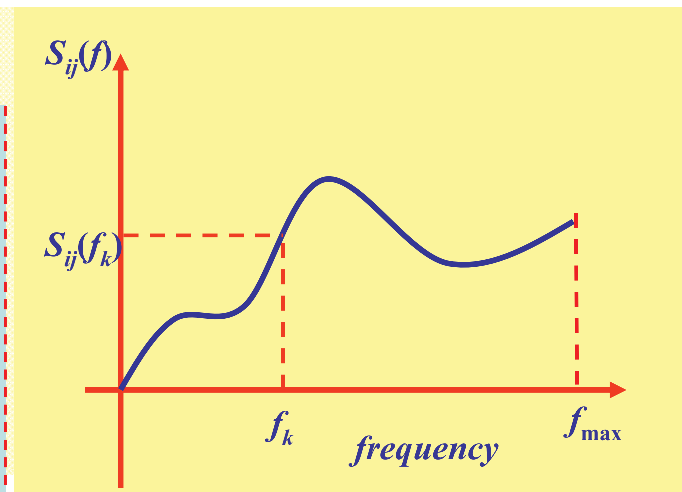
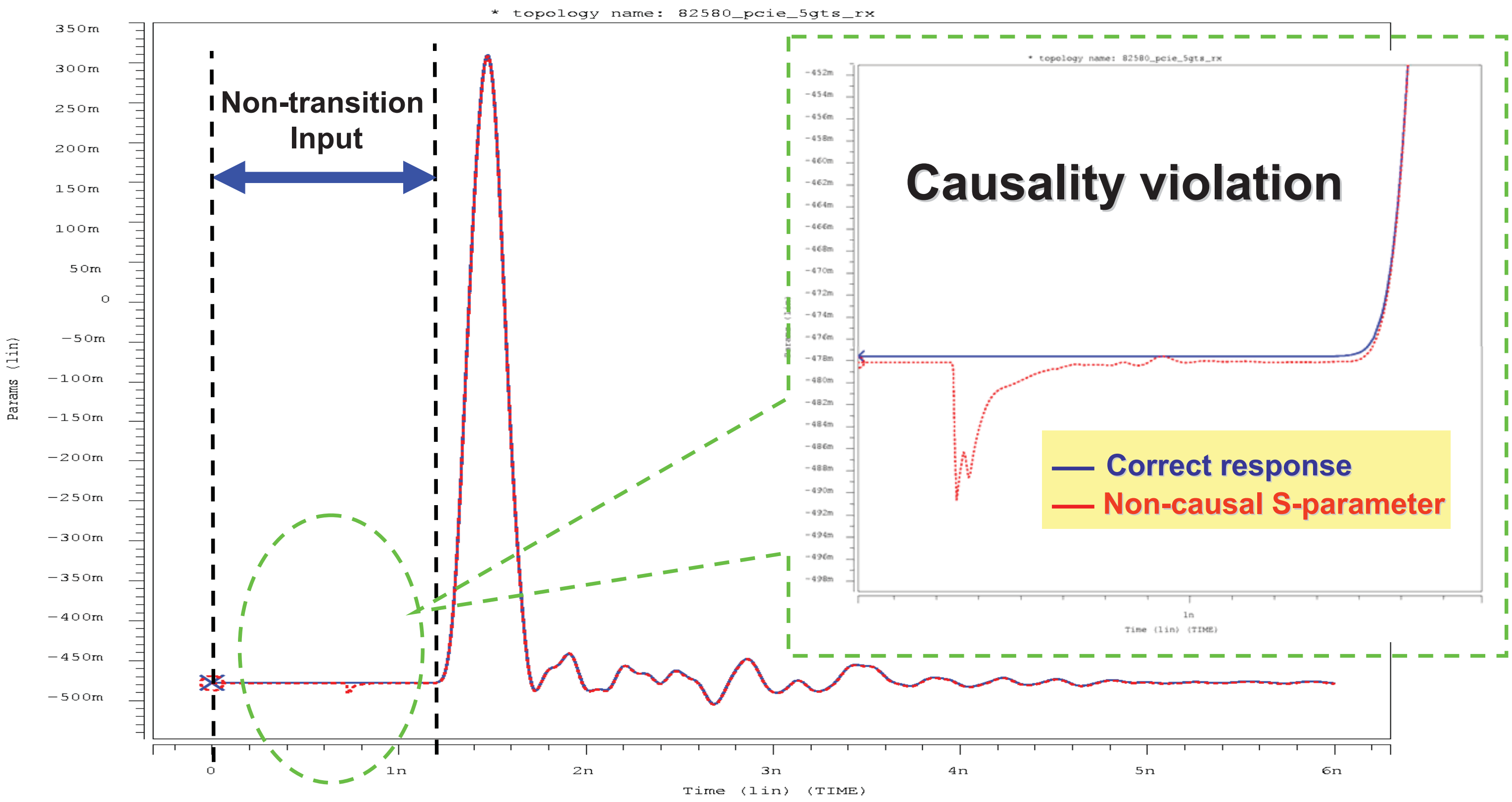
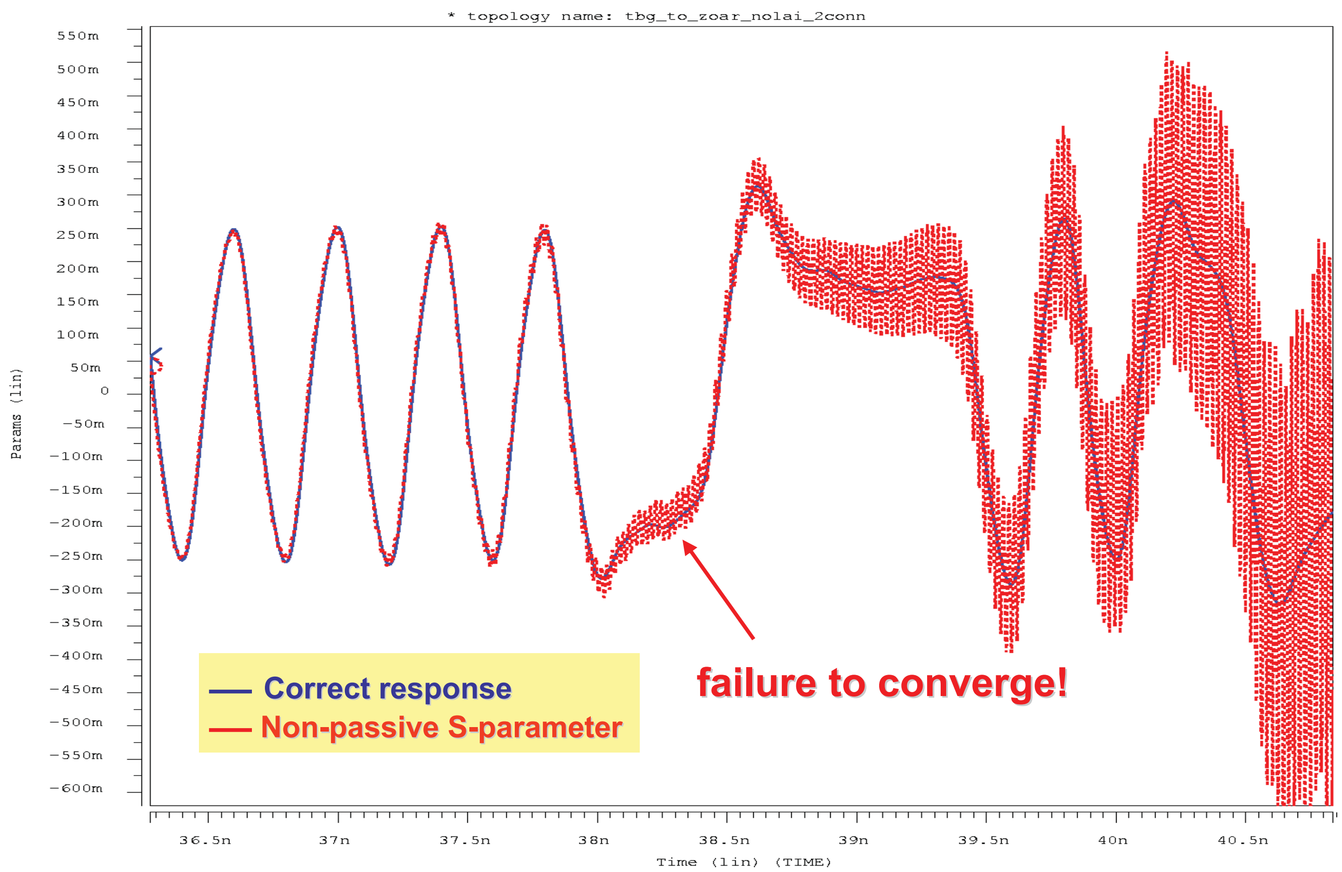


Impact of Non-Causality



Impact of Non-Passivity



Causality & Passivity Conditions

Causality Theorem

Theorem: An LTI system is causal if and only if all the elements $h_{ij}(t)$ of its impulse response matrix $h(t)$ are vanishing for $t<0$, i.e.,

$$h(t) = 0, \quad t < 0.$$

Passivity Theorem

Theorem: An scattering matrix $S(s)$ represents a passive linear system iff

1. $S(s^*) = S^*(s)$ where "*" is the complex conjugate operator
2. each element of $S(s)$ is analytic in $\mathrm{Re}\{s\} > 0$
3. $[I - S^H(s)S(s)] \geq 0.$ for all ω .

Causality & Passivity Conditions

Theorem: If $h(t)$ admits a Fourier transform, the following facts are equivalent.

1. $h(t) = 0, \quad t < 0.$
2. $H(j\omega)$ is the limit, as $\sigma \to 0$, of a function $H(s)$ defined in $\mathrm{Re}\{s\} > 0$ and here analytic and of polynomial growth.
3. $H(j\omega) = \mathrm{F}\{h(t)\}$ satisfies Kramers-Kronig relations.

Theorem: If an LTI system is passive, then it is also causal.

Passivity Condition 1 & 2

Passivity Condition 1: $S(-\omega) = S^*(\omega)$ $\quad S(\omega) = S_1(\omega) + jS_2(\omega)$

$\boxed{S_1(-\omega) = S_1(\omega)}$ **even function**

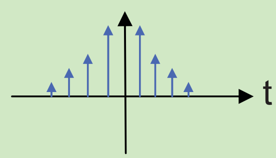
$\boxed{S_2(-\omega) = -S_2(\omega)}$ **odd function**

Passivity Condition 2: $S(\omega)$ satisfies Kramers-Kronig relations.

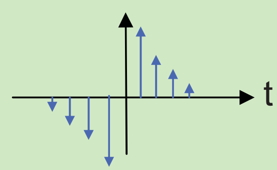
$$S(\omega) = S_1(\omega) + jS_2(\omega) \qquad \boxed{S_1(\omega) = \frac{1}{\pi}\,\mathrm{pv}\int_{-\infty}^{\infty} \frac{S_2(\omega')}{\omega' - \omega} d\omega'} \qquad \boxed{S_2(\omega) = -\frac{1}{\pi}\,\mathrm{pv}\int_{-\infty}^{\infty} \frac{S_1(\omega')}{\omega' - \omega} d\omega'}$$

equivalent to

$S_1(\omega) \xrightarrow{\text{IFT}} s_1(t)$ **even function**



$S_2(\omega) \xleftarrow{\text{FT}}$ **odd function**

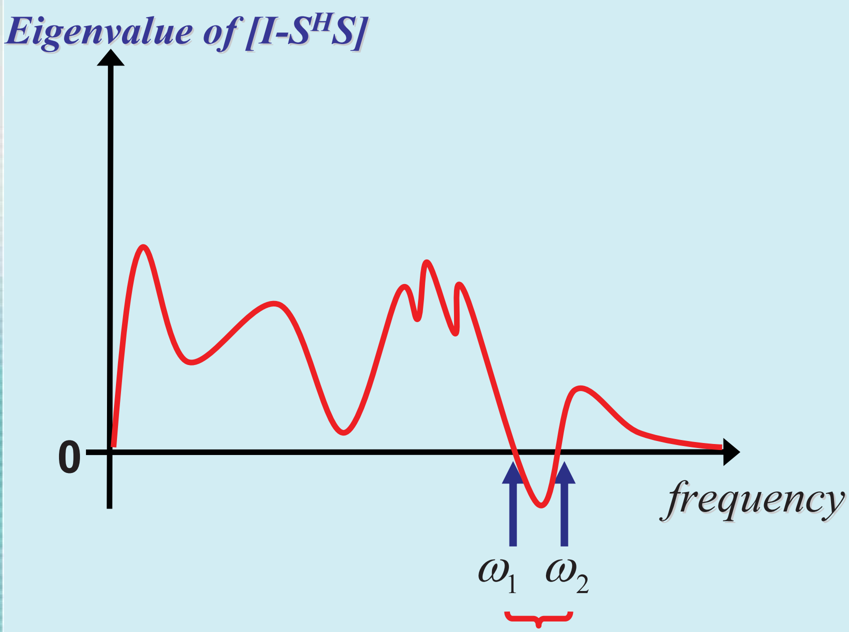


$s_2(t) = \mathrm{sgn}(t)s_1(t)$

Passivity Condition 3

Check the eigenvalue of

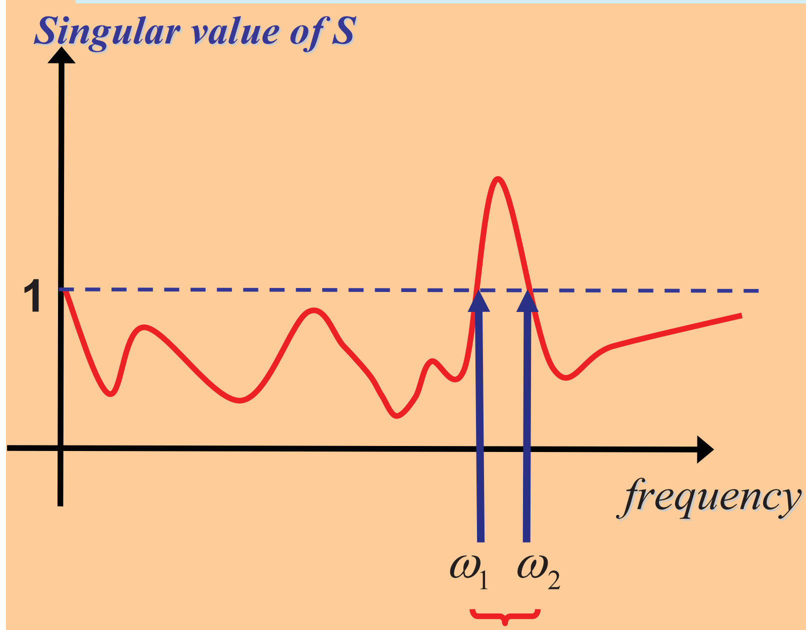
$I-S^HS$



Leads to Passivity violation

Check the singular value of S

$$[I-S^HS]\geq 0 \qquad S=U\sum V^H \qquad U^{-1}SV=\sum$$

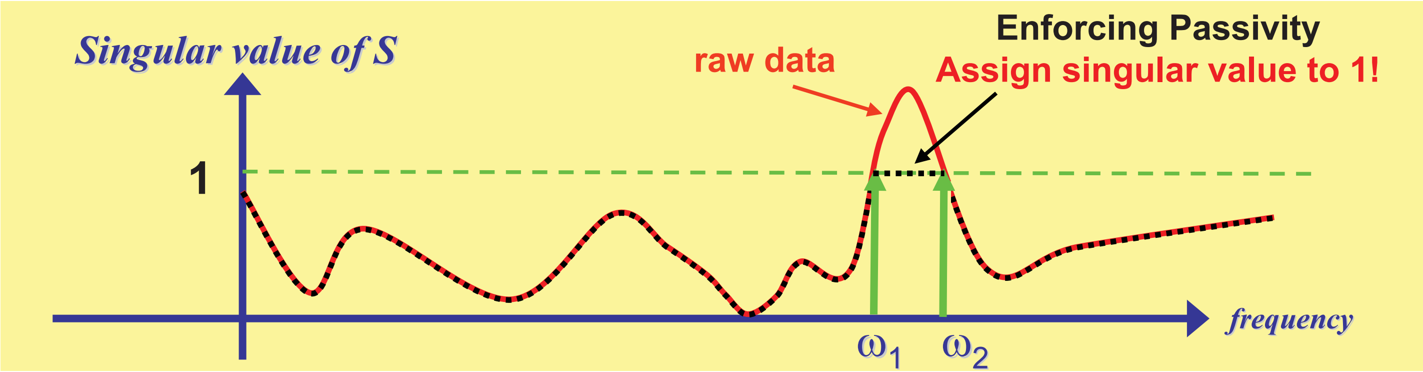


Leads to Passivity violation

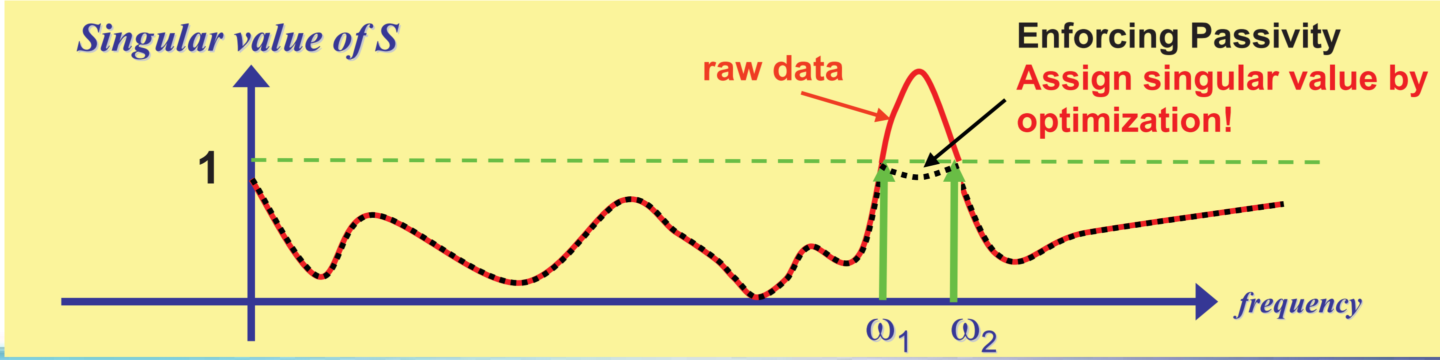
Enforcing Passivity for S-Parameter Raw Data

Currently the methodologies of enforcing passivity focused on Passivity Condition 3 only.

- Simple method (E.D. Campbell, IEEE ICCE 2010)

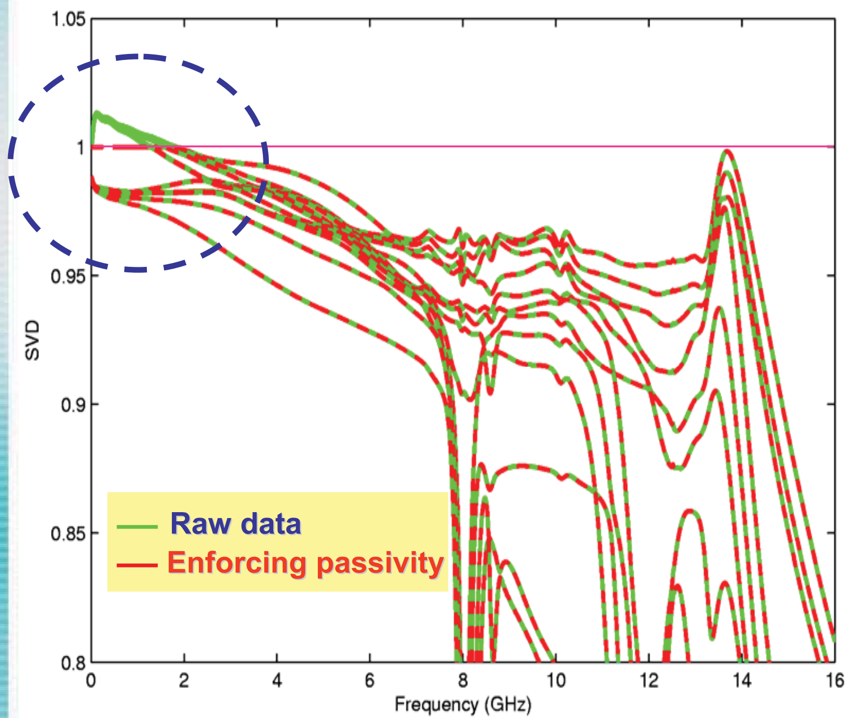


- Perturbation method (M.S. Nakhla, IEEE MTT-S 2005)

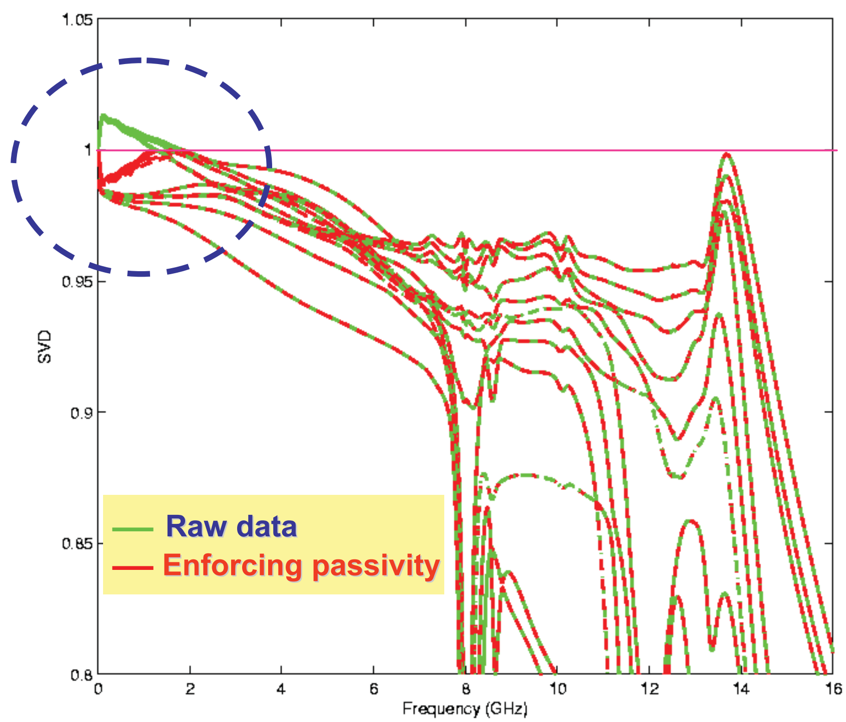


Enforcing Passivity
Singular Value Results

Simple Method

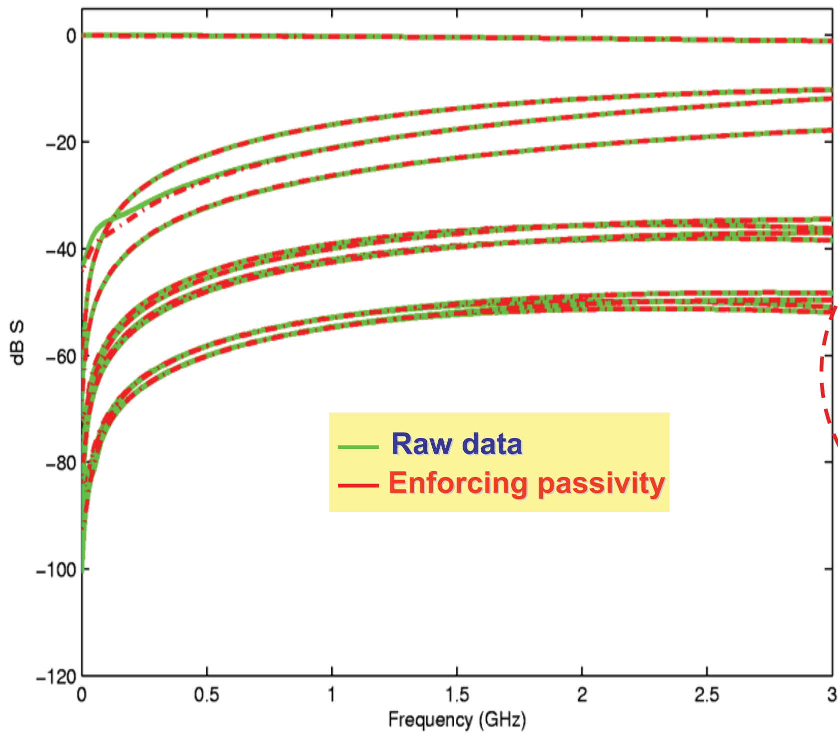


Perturbation Method

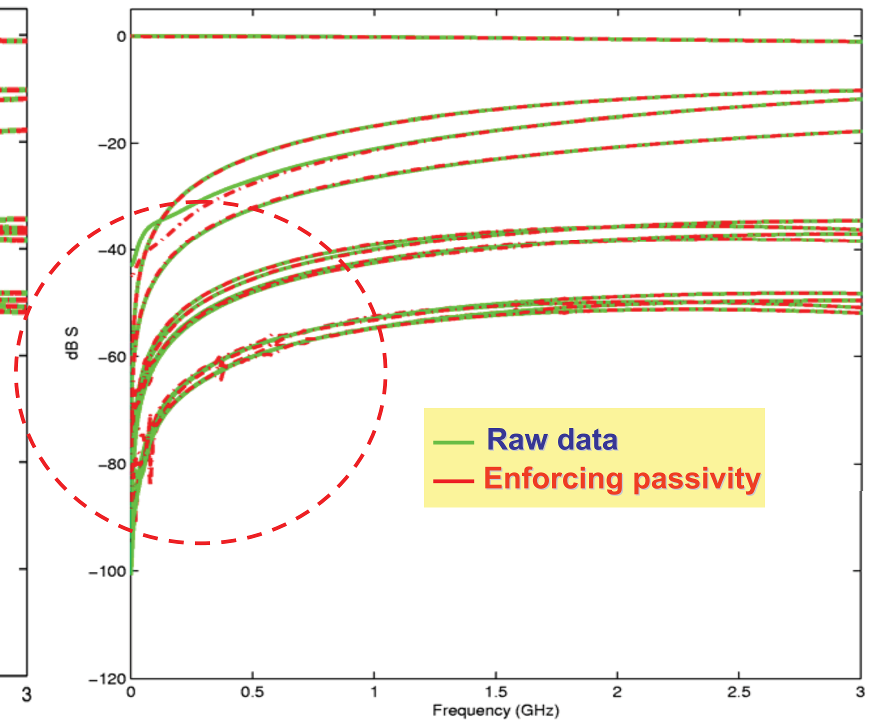


Enforcing Passivity S-Parameter Results

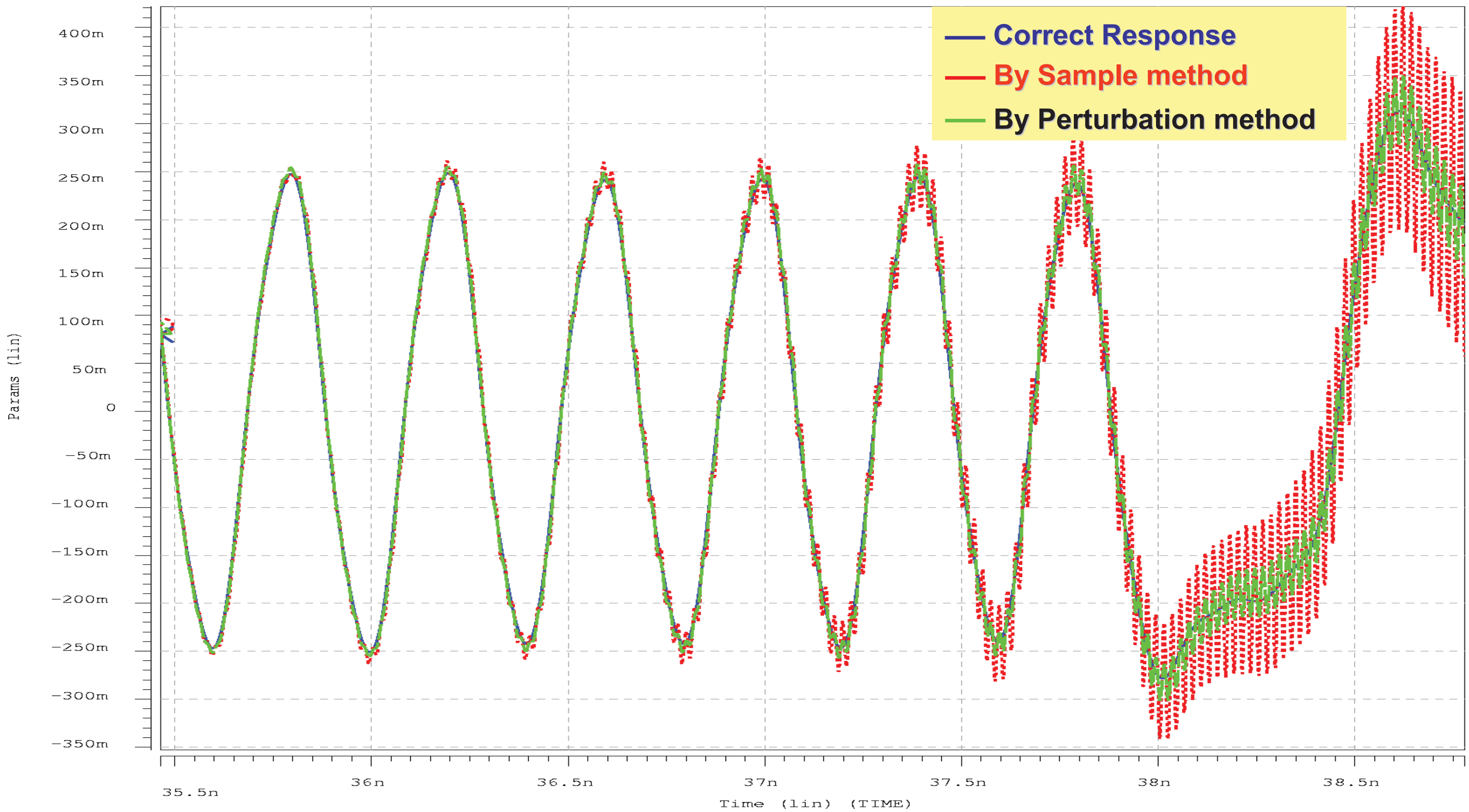
Simple Method



Perturbation Method



Transient Response



Rational Function Approximation

Touchstone file description

Hz S RI R 50

Frequency

S-parameter raw data

1.000e+7	1.028e-4	-5.022e-4	3.666e-5	2.586e-4	9.998e-1	-2.709e-3	4.533e-5	-4.858e-5
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2.0[illegible]e+7	1.268e-4	-1.096e-3	5.455e-5	4.873e-4	9.997e-1	-5.339e-3	3.647e-5	-1.011e-4
	3.735e-5	4.941e-4	1.116e-4	-1.132e-3	-5.493e-5	-1.077e-4	9.997e-1	-5.231e-3
	9.997e-[illegible]	-5.3[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]995e-4
	1.962e-[illegible]	-1.0[illegible]4e-4	9.997e-1	-5.229e-3	3.722e-5	4.813e-4	[illegible]	-1.132e-3
3.0[illegible]e+7	1.322e-4	-1.705e-3	6.496e-5	7.263e-4	9.996e-1	-7.954e-3	-1.425e-6	-1.547e-4
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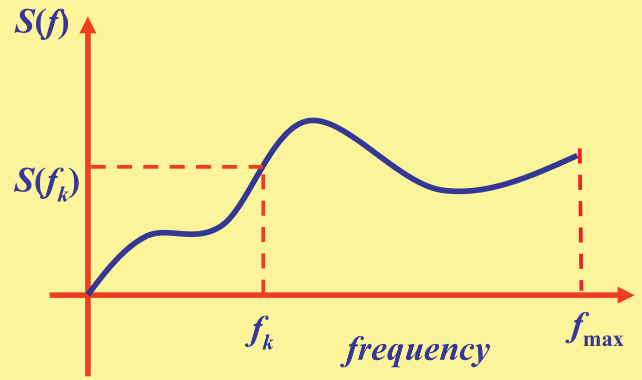


Rational Function Approximation by Vector Fitting Algorithm

$$f(s) \approx \sum_{n=1}^{N} \frac{r_n}{s - p_n} + d + se$$

Rational Function Passivity Conditions

- If $S(s)$ is a rational function, the passivity condition 1 is satisfied.
- If $S(s)$ is a rational function of s, the passivity condition 3 implies the analyticity of $S(s)$ for $\mathrm{Re}\{s\} > 0.$

Passivity Theorem for rational function

Theorem: A rational function of scattering matrix $S(s)$ represents a passive linear system iff

$$[I - S^H(s)S(s)] \geq 0. \quad \text{for all } \omega .$$

Perturbation Method with Rational Function

Enforcing passivity: $S + \Delta S$

$$S = \sum_{m=1}^{M} \frac{R_m}{s + p_m} + D$$

Perturbation of eigenvalue →

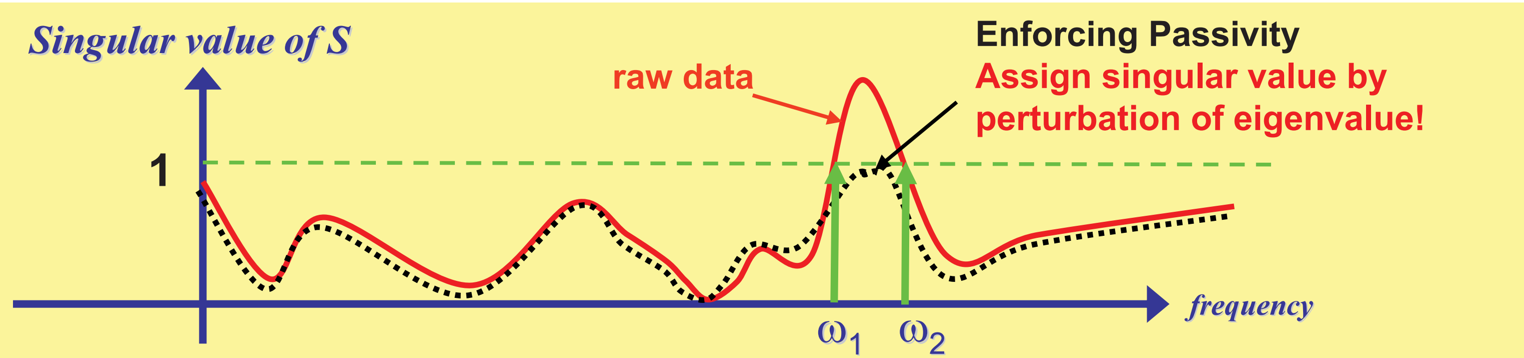
$$\frac{R_m + \Delta R_m}{s + p_m} \cong \frac{P_m(\Lambda_{R_m} + \Delta\Lambda_{R_m})P_m^T}{s + p_m}$$

(by B. Gustavsen, 2010 IEEE Trans. on Advanced Packaging)

Target →

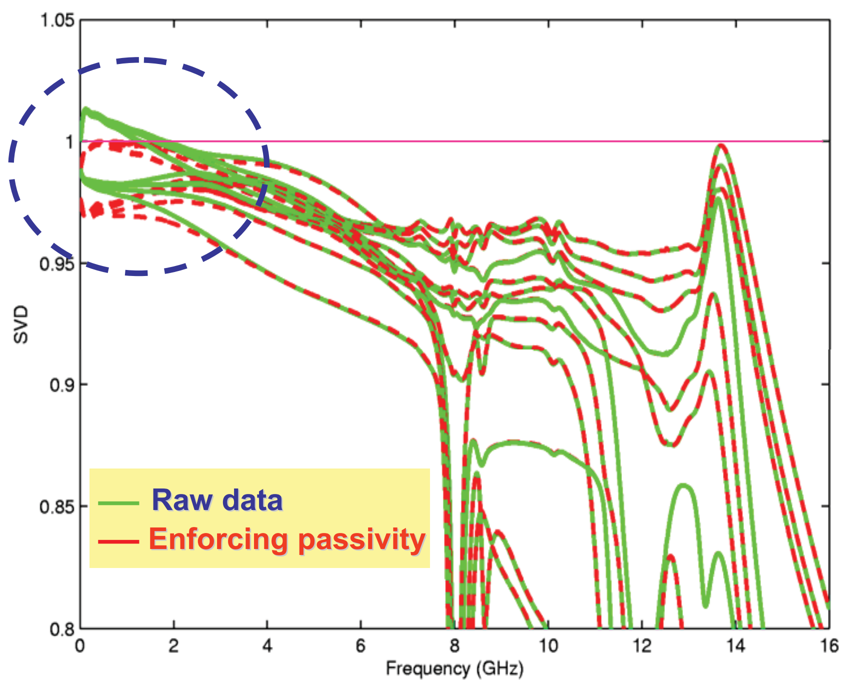
$$\Delta S \cong 0$$

$$\sigma_i + \Delta\sigma_i < 1 \quad \text{for} \quad i = 1 \ldots n$$

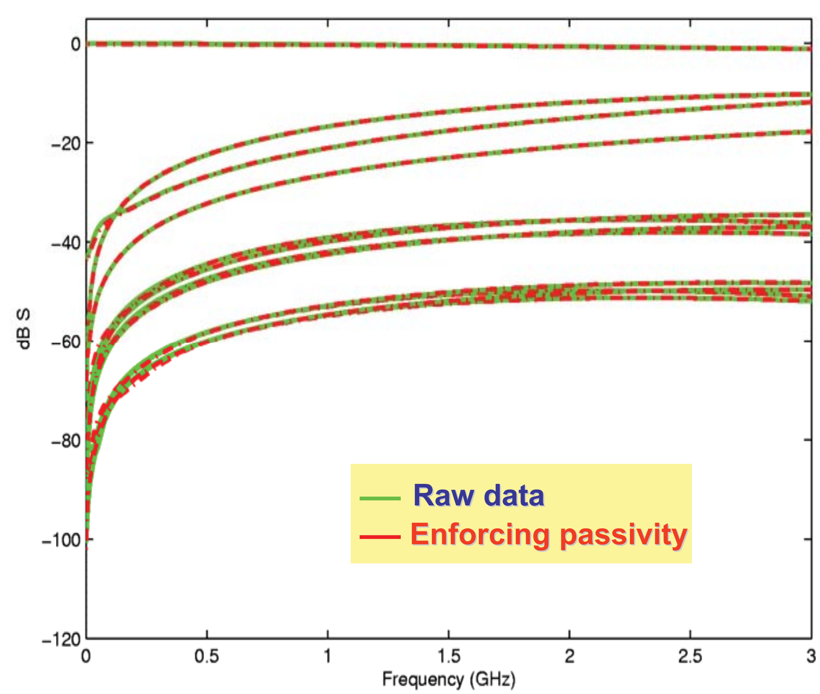


Enforcing Passivity for S-Parameter Rational Function

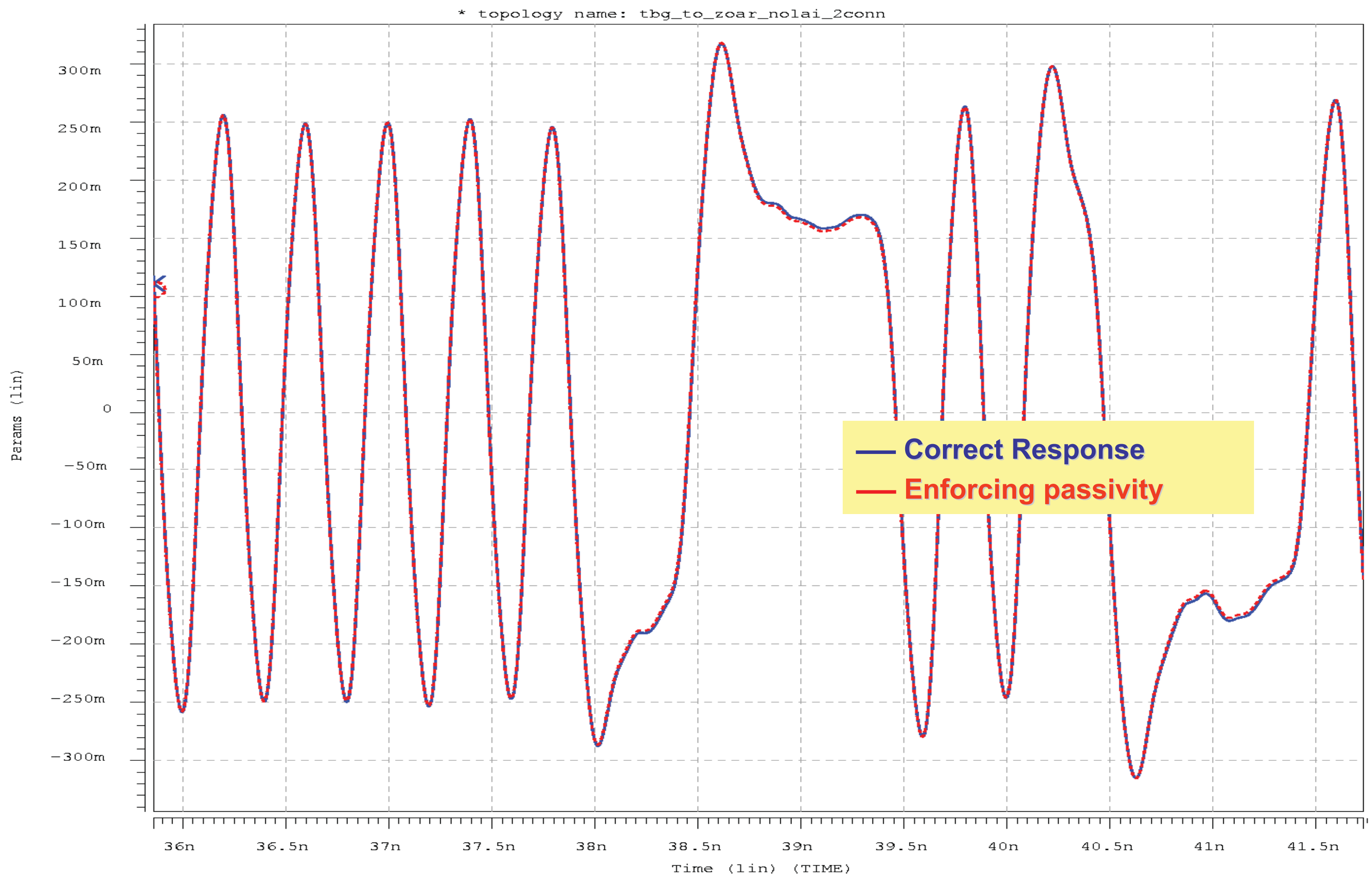
Singular values



S-parameter



Transient Response



Proposed Common-Pole Rational Function Standard File

$$S_{i,j} = D_{i,j} + j\omega E_{i,j} + \sum_{n=1}^{N} \frac{Rr_{i,j}^{n}}{s + pr_n} + \sum_{m=1}^{M} \left(\frac{Rc_m}{s + pc_m} + \frac{Rc_m^{*}}{s + pc_m^{*}} \right)$$

R Zo1 Zo2 Zo3 Zo4	! reference resistor of ports
Common pole	! Common pole definition
Real *N*	! Real pole definition
$pr_1 \; pr_2 \; \cdots \; pr_N$	! Real pole data
Complex *M*	! Complex pole definition
$\mathrm{Re}(pc_1) \; \mathrm{Re}(pc_2) \; \cdots \; \mathrm{Re}(pc_M)$	! Complex pole real part data
$\mathrm{Im}(pc_1) \; \mathrm{Im}(pc_2) \; \cdots \; \mathrm{Im}(pc_M)$	! Complex pole imagery part data
i j	! Element definition, i is row and j is column
DC D_{ij}	! DC element definition
E E_{ij}	! E element definition
Real residue *N*	! Real residue definition
$Rr_{ij}^1 \; Rr_{ij}^2 \; \cdots \; Rr_{ij}^N$	! Real residue data
Complex residue *M*	! Complex residue definition
$\mathrm{Re}(Rc_{ij}^1) \; \mathrm{Re}(Rc_{ij}^2) \; \cdots \; \mathrm{Re}(Rc_{ij}^M)$	! Complex residue real part data
$\mathrm{Im}(Rc_{ij}^1) \; \mathrm{Im}(Rc_{ij}^2) \; \cdots \; \mathrm{Im}(Rc_{ij}^M)$	! Complex residue imagery part data

Advantage of Rational Function Standard File

- Present S-parameter on full spectrum.
- Causality and passivity guarantee after enforcing passivity.
- Size reduction of S-parameter raw data.

Type	No. of Sampled data	No. of port	No. of pole-residue	Size of Touchstone file	Size of proposed file	Size reduced to
Via	2000	4	12	1.66M	6.88K	0.41%
Connector	2000	12	78	9.21M	334.38K	3.63%
Trace	2000	42	240	105.16M	1.35M	1.28%

Summary

- The traditional enforcing passivity for S-parameter raw date are not sufficient to satisfy the passivity conditions.

- Using enforcing passivity rational function to fit S-parameter raw data could be causality and passivity guarantee on full spectrum.

- The proposed rational function standard file has the advantage in size reduction of S-parameter sampled frequency data.

Reference

[1] Piero Triverio, S. Grivet-Talocia, M.S. Nakhla, F.G. Canavero and R. Achar, "Stability, Causality, and Passivity in Electrical Interconnect Models," IEEE TRANS. on ADVANCED PACKAGING, VOL. 30, NO. 4, NOVEMBER 2007.

[2] B. Gustavsen and A. Semlyen, ”Rational approximation of frequency domain responses by vector fitting,” IEEE Trans. Power Delivery, vol. 14, no. 3, July 1999, pp. 1052-1061.

[3] Altan Odabasioglu, Mustafa Celik and Lawrence T. Pileggi,"PRIMA: Passive Reduced-Order Interconnect Macromodeling Algorithm," IEEE TRANS. on COMPUTER-AIDED DESIGN OF INTEGRATED CIRCUITS AND SYSTEMS,, VOL. 17, NO.8, Aug. 1998.

Thank you for your attention.