

Background



- S-parameter is becoming more and more popular in high speed simulation
- In Howard Johnson's book, we believe the F_{knee} is the upper limit of T-Line model

$$F_{knee} = \frac{0.5}{Tr}$$

 But is it accurate enough for transient analysis in GHz analysis?



Fourier transform

 Fourier transform provide mathematical mechanism for transforming frequency data to timing domain

$$f(t) = \int_{-\infty}^{\infty} S(f)E(f)e^{2\pi ift}df$$

- There are two approximations in real application
 - Finite band width
 - Discrete data point





• Finite bandwidth S parameter like a "window" function $W\left(f\right)$

$$W(f) = \begin{cases} 1 & -b \le f \le b \\ 0 & \text{ot her} \end{cases}$$

$$f(t) = \int_{-\infty}^{\infty} W(f)S(f)E(f)e^{2\pi jft}df$$
$$= \int_{-b}^{b} S(f)E(f)e^{2\pi jft}df$$

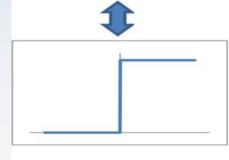
Finite bandwidth Problems

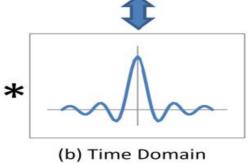


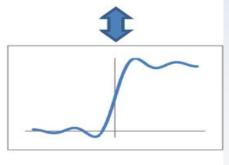
"Window" function will cause Oscillation

$$W(f) \leftrightarrow 2bSa(\pi bt) = \frac{\sin \pi bt}{\pi t}$$

$$\times \boxed{ } = \boxed{ }$$
(a) Frequency Domain



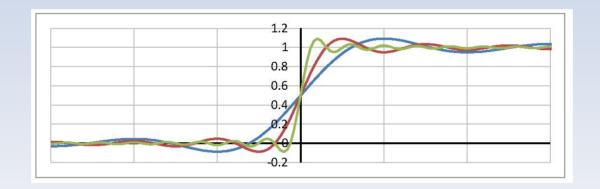




Finite bandwidth Problems



 Increase bandwidth cause sharper oscillation, but never disappear



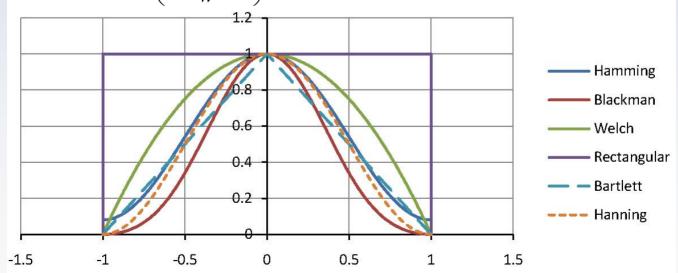
If expand bandwidth to infinite

$$\lim_{b\to\infty}\frac{\sin(bt)}{\pi t}=\delta(t)$$

Using other "windows"



- **Hamming** $0.54 + 0.46\cos(\frac{2\pi f}{2\pi})$
- Hanning $0.5 \left[1 + \cos(\frac{2\pi f}{w})\right]^{W}$ Blackman $0.42 + 0.5\cos(\frac{2\pi f}{w}) + 0.08\cos(\frac{4\pi f}{w})$ Welch $1 \left(\frac{2 f}{w}\right)^{2}$
- Bartlett $1 \left(\frac{2 f}{1 \frac{1}{2}}\right)$



Results of other "windows"



- Attenuate the spectrum near upper limit
- Trade-off between edge and oscillation

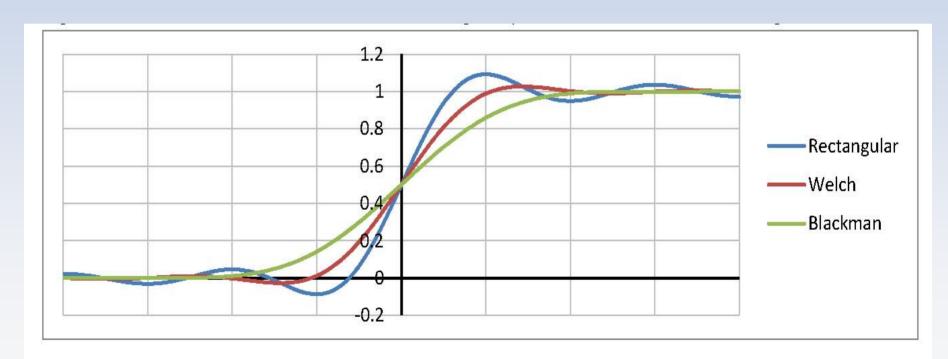


Figure 7. Step response for three different windows, each with the same bandwidth.

Idea step response of different "windows"



Table 1. Characteristics of selected window functions for continuous time

Window	Step Response $f_s(t)$	10-90 Edge Rate	Max Overshoot
Rectangular	$\frac{1}{2} + \frac{\operatorname{Si}(2\pi bt)}{\pi}$	0.45 b	8.95%
Welch	$\frac{1}{2} + \frac{\cos(2\pi bt)}{2\pi^2 bt} - \frac{\sin(2\pi bt)}{4\pi^3 b^2 t^2} + \frac{\sin(2\pi bt)}{\pi}$	0.70 b	2.70%
Hanning	$\frac{2\pi + 2\mathrm{Si}(2\pi bt) - \mathrm{Si}(\pi - 2\pi bt) + \mathrm{Si}(\pi + 2\pi bt)}{4\pi}$	0.97 b	0.64%
Blackman		1.19 b	0.02%

$$Si(x) = \int_0^x \frac{\sin(a)}{a} da$$

Finite edge response of Rectangular "window"



Finite edge response of Rectangular

"window" $f(t) = \frac{1}{2} + \frac{Si(2\pi b(r-t))}{\pi} + \frac{\cos(2\pi bt) - \cos(2\pi b(t-r)) + 2\pi bt(Si(2\pi b(r-t)) + Si(2\pi bt))}{2\pi^2 br}$

$$f(0) = \frac{1}{2} + \frac{1 - \cos(2\pi br)}{2\pi^2 br} - \frac{Si(2\pi br)}{\pi}$$

Rectangular "window" is Non-Causal system

Finite edge response of Rectangular "window"



ullet If increase br , f(0) will closer to ullet

Table 2. Finite edge response for rectangular windows for continuous time

Bandwidth*Risetime (br)	Edge Response at t = 0	Max Overshoot
0.5	0.113	5.34%
0.75	0.056	2.41%
1	0.049	1.19%
2	0.025	0.72%
3	0.017	0.51%
5	0.010	0.32%
10	0.005	0.17%

Discrete data point



- Simulation is discrete point data both in Timing domain and Frequency domain:
- Relationships of step

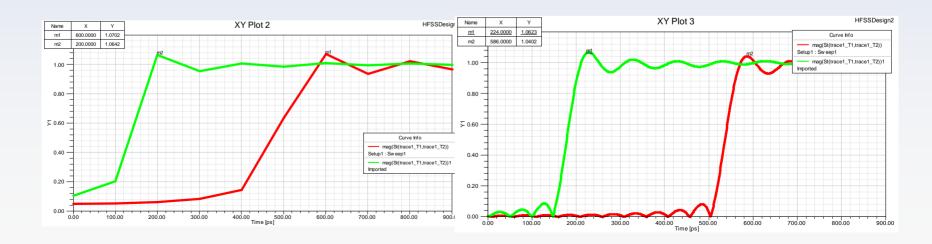
$$T_{step} = \frac{1}{2F_{\text{max}}} \qquad T_{\text{max}} = \frac{1}{2F_{step}}$$

 Minimum timing step is decided by the bandwidth which has been discussed

Timing step influence



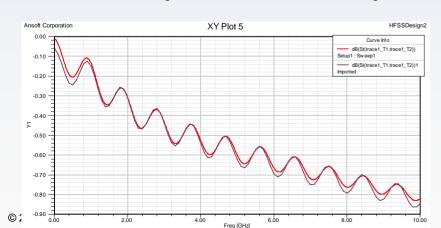
- 1 inch and 3 inch matched T_line's step response using different timing step:
- Only different delay are expected, but different oscillation when bigger timing step
- Reduce timing step get correct results



Frequency step influence



- Discrete frequency point data convert to timing domain look like repeating series of long pulse.(Discrete Fourier transform)
- If frequency step is bigger, the repeating series pulse will interrupt each other
 - Same T_line S21 using different frequency step and corresponding transient results





Different requirements of parallel and Serial signal simulation



- Parallel signal is repeating of one bit.
 - One rising and falling edge
 - Frequency step should be denser than 1/T
- Serial signal is series of bit.
 - Random bits for testing ISI
 - Frequency step should be denser than 1/n*UI

conclusion



- Mathematical based simulation or testing have native limitation
- Serdes GHz simulation need broadband channel characteristic
 - F_{knee} is limited, 5/Tr or 10/Tr
- Discrete frequency point data is also critical for analysis

$$T_{step} = \frac{1}{2F_{\text{max}}} \qquad T_{\text{max}} = \frac{1}{2F_{step}}$$



Thank you

谢谢