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Quasi-Analytical Estimation of Very Low Bit Error Rate

Dingqing Lu
Sanjeev Gupta
Mihai Marcu
Xuliang Yuan

Agilent Technologies Inc.

Contact Email: xuliang_yuan@agilent.com

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Background and related works

Very low BER estimation for system with ISI, Jitter and Noise is needed

Monte Carlo (MC) is very slow

- Very few simulation runs contribute to error events in the statistics

Fast BER techniques have been proposed

- Importance sampling – a way to speed up simulation
  - More simulation runs contribute to error events in the statistics
- QA estimation is very efficient
  - 100% simulation runs contribute to error events in the statistics
  - We need to know its statistical properties

System model

Linear system with ISI, Jitter and Noise

The input sample of the decision device can be expressed by

$$y(t_k) = v_s(t_k) + v_n(t_k)$$

- Vn is zero mean white Gaussian noise
- Vs is given by

$$v_s(t_k) = \sum_{i=0}^{M} a_i e^{j\theta_i} h(t_k - iT)$$

Impulse response

i-th sample from the input data set

i-th sample from a random process with zero mean

System model

Ho: $a_k = -A$

H1: $a_k = +A$

Under Ho, BER is given by:

$$P_0 = \int_{\Omega_0} f^0(v_s)\, f_n(v_n)\, dv_s\, dv_n$$

pdf under Ho   Noise pdf and

$$\Omega_0 = [(v_s, v_n) : v_s + v_n \geq T_0]$$

Under H1, BER is given by:

$$P_1 = \int_{\Omega_1} f^1(v_s)\, f_n(v_n)\, dv_s\, dv_n$$

pdf under H1   Noise pdf and

$$\Omega_1 = [(v_s, v_n) : v_s + v_n \leq T_1]$$

The error probability for the system is given by

$$P_e = P(H_0)P_0 + P(H_1)P_1$$

where $P(H_0)$ and $P(H_1)$ are prior probabilities for Ho and H1, respectively

With the assumption of equal prior probabilities for *H0* and *H1*, symmetry of the noise density, and channel symmetry: $P_e = P_0 = P_1$

QA Estimator

Under Ho, the BER expression as:

$$P_0 = \int_{-\infty}^{\infty} \left[\int_{-v_s}^{\infty} f_n(v_n) dv_n \right] f^0(v_s) dv_s$$

A QA estimator can be constructed as

$$\hat{P}_0 = (1/N) \sum_{i=1}^{N} P_i$$

where N is the number of the simulation samples, and for the i-th sample of the signal we define the *Pi* as

complementary distribution function

$$P_i = \int_{-v^i_s}^{\infty} f_n(v_n) dv_n = Q(-v_s^i / \sigma)$$

i-th sample of the signal

noise standard deviation

For the white Gaussian noise with zero mean and variance of σ_{in}^2. We have

$$\sigma^2 = \sigma_{in}^2 \sum_{i=0}^{M} h_i^2$$

Properties of QA Estimation

QA Estimation is unbiased because

$$E[\hat{P}_0] = (1/N)\sum_{i=1}^{N} E[P_i] = 0$$

QA Estimation variance is given by

$$Var[\hat{P}_0] = (1/N)[\int_{-\infty}^{\infty} Q^2(-v_s/\sigma) f^0(v_s) dv_s - P_0^2]$$

QA Estimation variance is upper-bounded by

$$Var[\hat{P}_0] = \sigma_{QA}^2 \le \sigma_{UB}{}^2 = (1/N)[Q_{\max}^2(-v_s/\sigma) - P_0^2]$$

Simulation speed improvement ratio defined by

$$r = \sigma_{MC}^2 / \sigma_{QA}^2$$

Lower-bound improvement ratio is given by

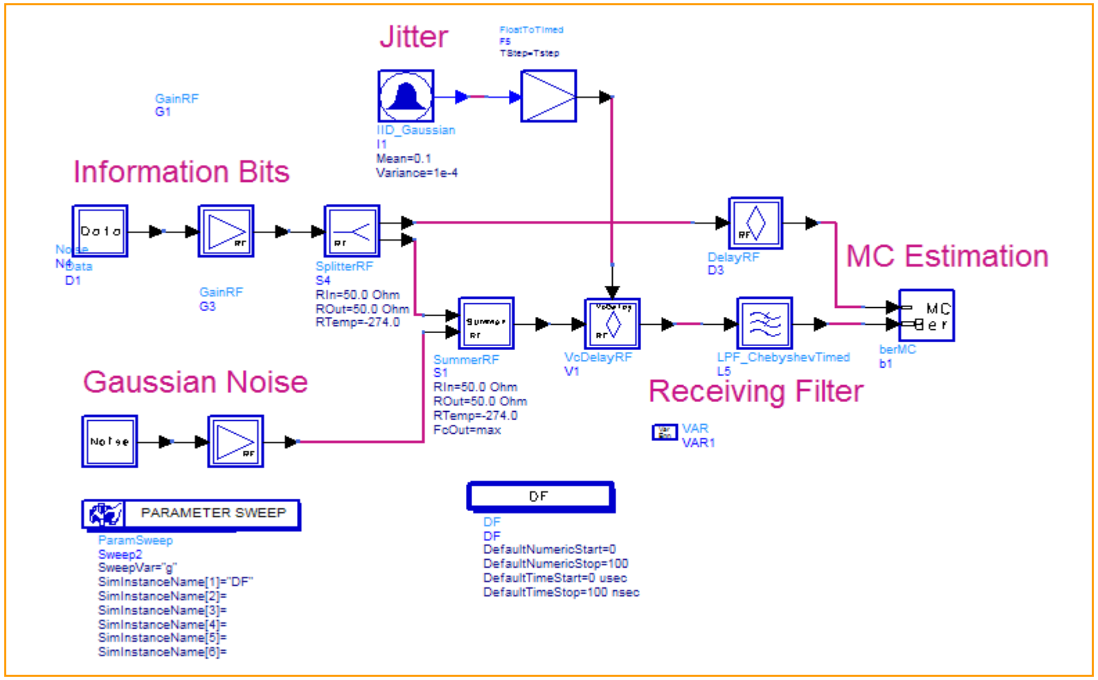
$$r_{LB} \approx P_0 / Q_{\max}^2(-v_s/\sigma)$$

Applications

Let us consider a linear system with ISI, jitter and additive white Gaussian noise.

We start from Monte Carlo simulation to estimate the BER as shown in Figure 1

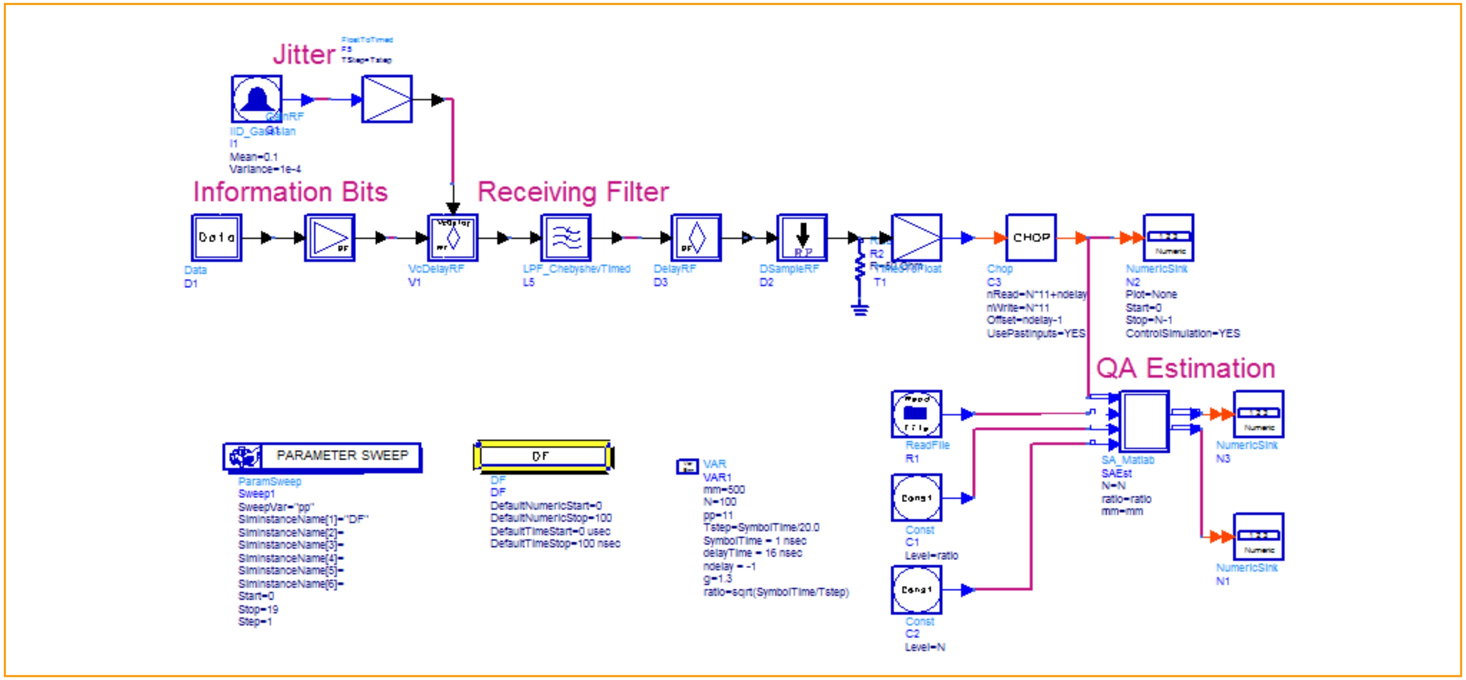
Figure 1. BER Simulation using MC Method



Applications

For the same system as Figure 1, we use the QA Estimation as shown in Figure 2.

Figure 2. BER Simulation using QA Method

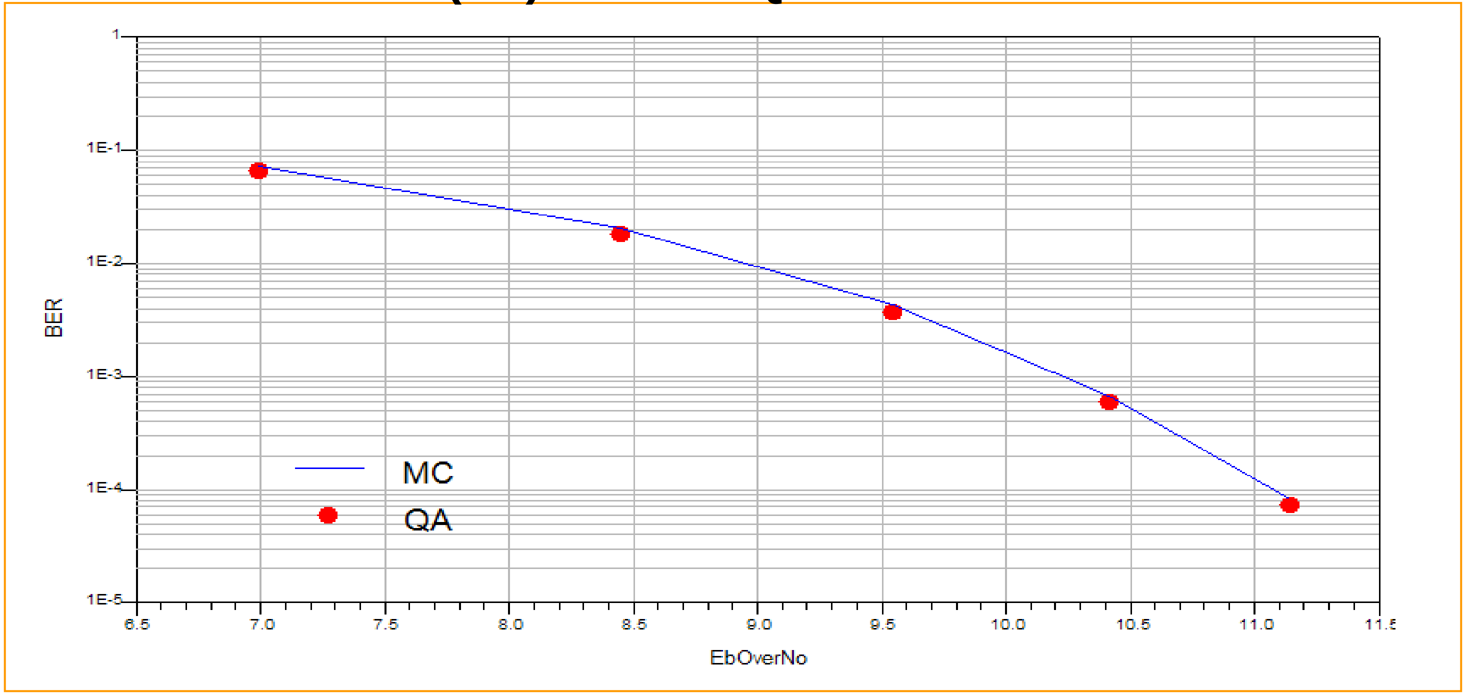


Applications

To show the QA estimation is unbiased, the BER values based on both MC and QA are plotted together in Figure 3.

In the MC and QA, the simulation samples were set to 100000 and 100, respectively. The results from both methods are matched very well.

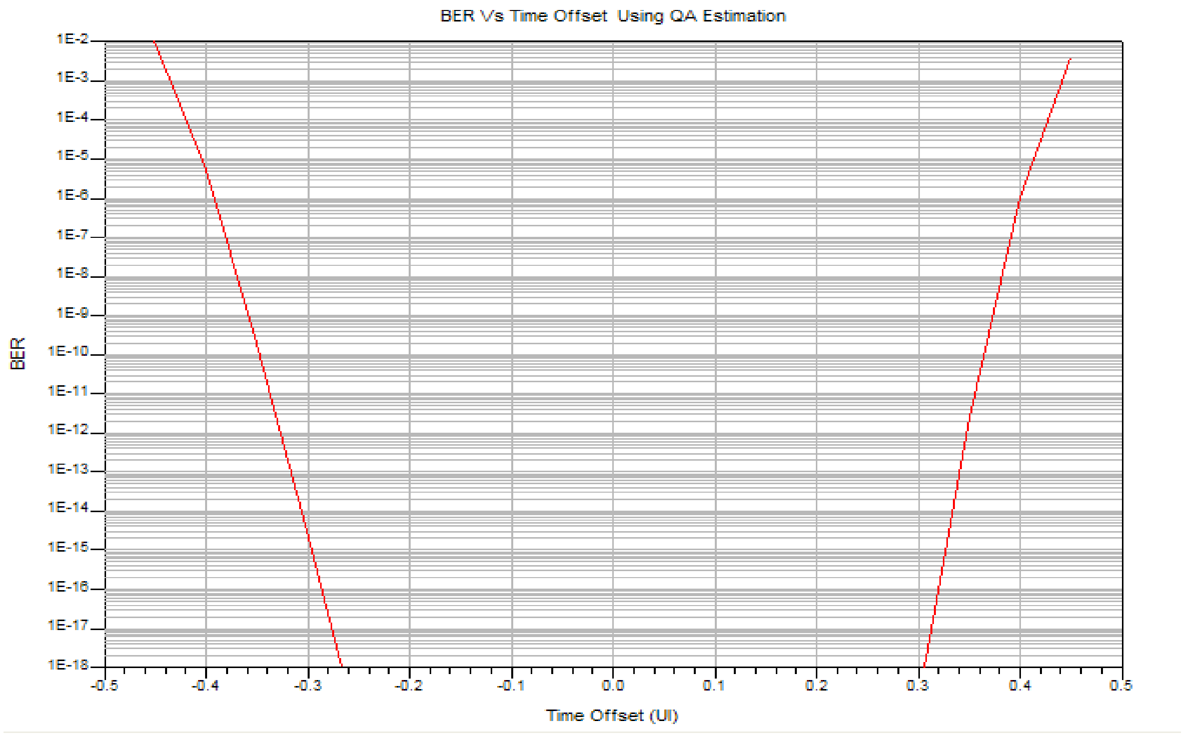
Figure 3. BER Vs Eb/No (dB) for both QA and MC Methods at Time Offset = 0



Applications

In the QA example, a sweep control can be used to plot the BER Vs Time Offset as seen from Figure 4. The simulation time is less than one minutes.

Figure 4. BER Vs Time Offset at input Eb/No=16 dB



Applications

To compare the simulation speed, an improvement ratio for QA vs MC is calculated.

For BER = 3.4e-17, to get a reasonable accuracy the MC samples can be set to 3.4e18 in MC. The estimation variance for MC is 1e-35.

In the QA, we found $Q_{\max}^2(-v_s/\sigma)$ = 5e-33. Using QA the equation of the estimation variance, we know only about 500 samples are required for the same estimation variance.

The improvement ratio QA Vs MC is greater than 6.8e15

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Summary and Further Works

- We have shown that the error probability in system due to ISI, jitter and Gaussian noise can be effectively estimated by using the QA method .
- Further investigations show that the QA method is better than not only the direct MC method, but also many other variance reduction techniques, such as, importance sampling approach and group sampling approach in linear system simulations.
- The QA method can be generalized for M-ary signaling cases. When the noise is non-Gaussian, the QA method also can be used for estimating the error probabilities with a desired accuracy.

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