24th European IBIS Summit, 26 May 2022, Siegen, Germany

Fast Simulation of Analog Circuit Blocks under Nonstationary Operating Conditions via Reduced Order Equivalent Circuits

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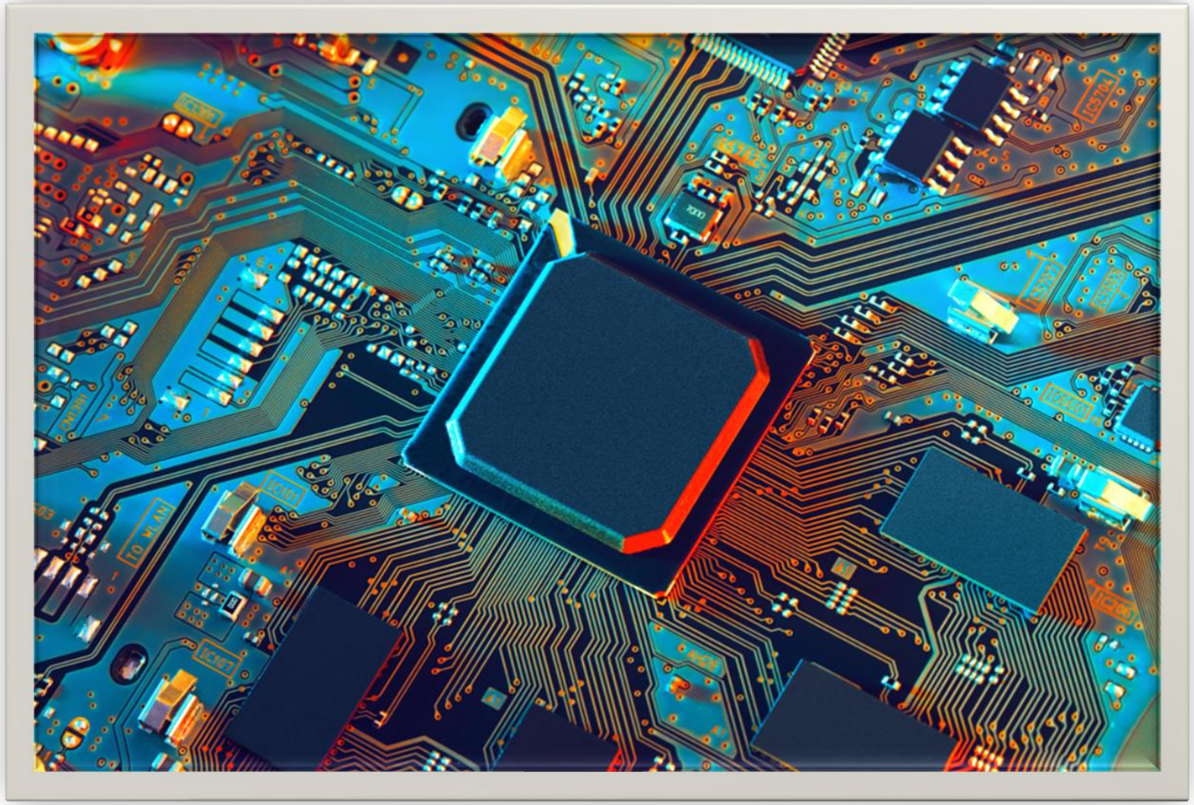
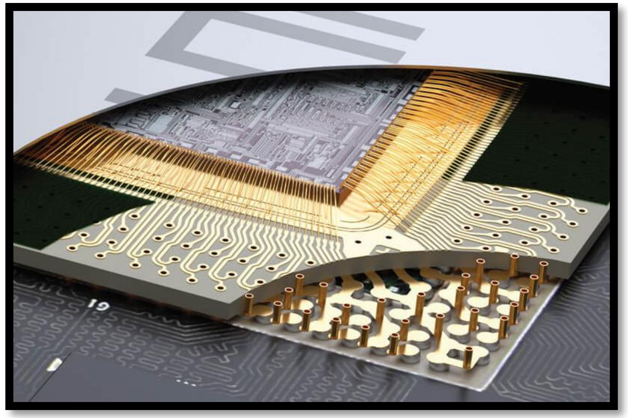
POLITECNICO DI TORINO

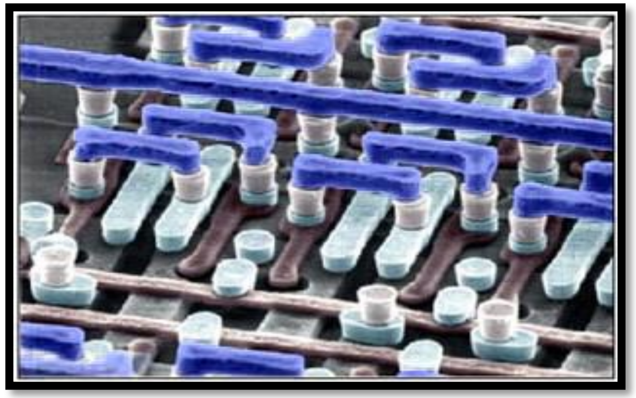
EMC Group

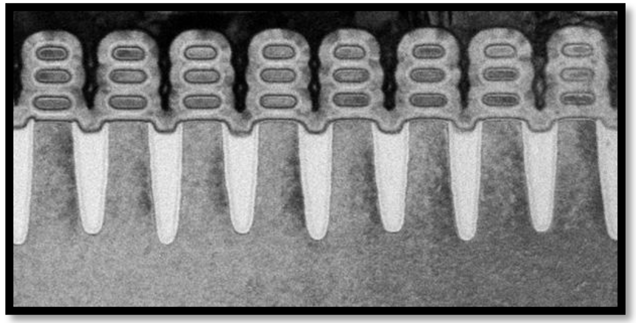
Intro

SOC/SIP



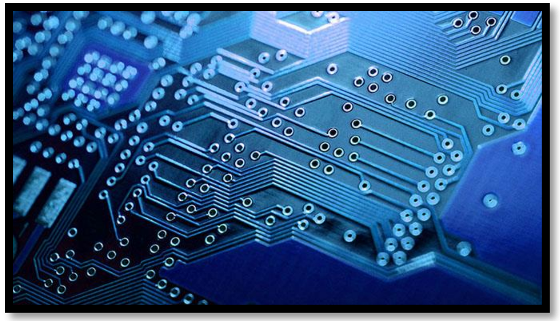


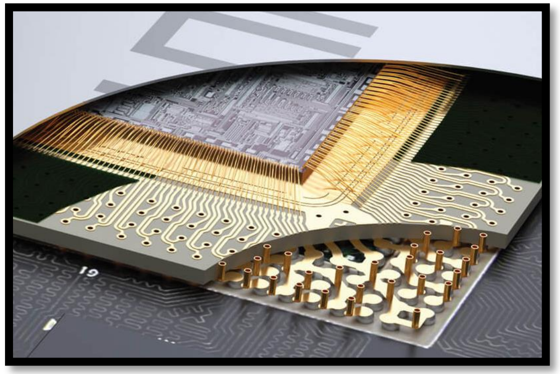




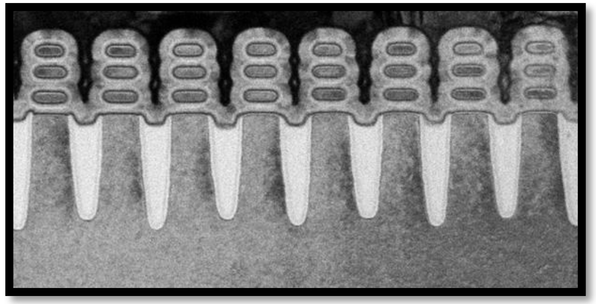
Simulating components behaviors

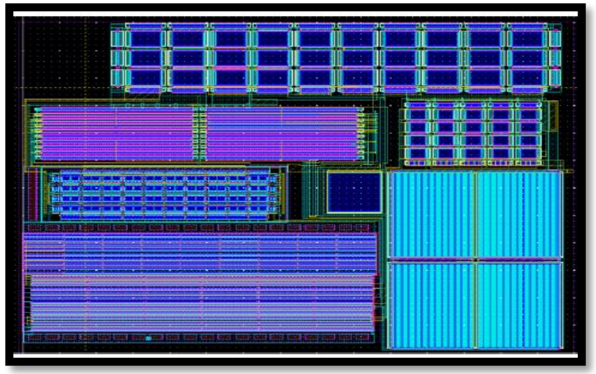
Raw subdivision of the components





Passive - Linear





Active devices - Nonlinear

Simulating components behaviors

Different first-principle models

e.g. Maxwell

$$\nabla \cdot D = \rho$$
$$\nabla \cdot B = 0$$
$$\nabla \times E = -\frac{\partial B}{\partial t}$$
$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

Passive - Linear

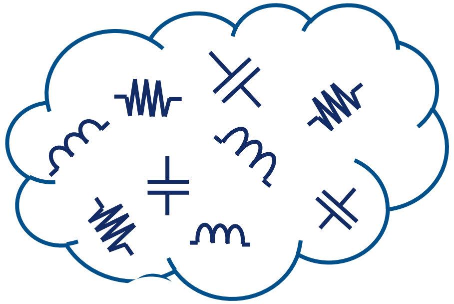
e.g. Transistor Models

BSIM 3,4
BSIM-CMG
EKV
…

Active devices - Nonlinear

Simulating components behaviors

Reduced Order Equivalent circuits



$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Small equivalent linear circuit

well-established

$$\dot{w} = F(w,u)$$
$$y_{NL} = G(w,u)$$

?

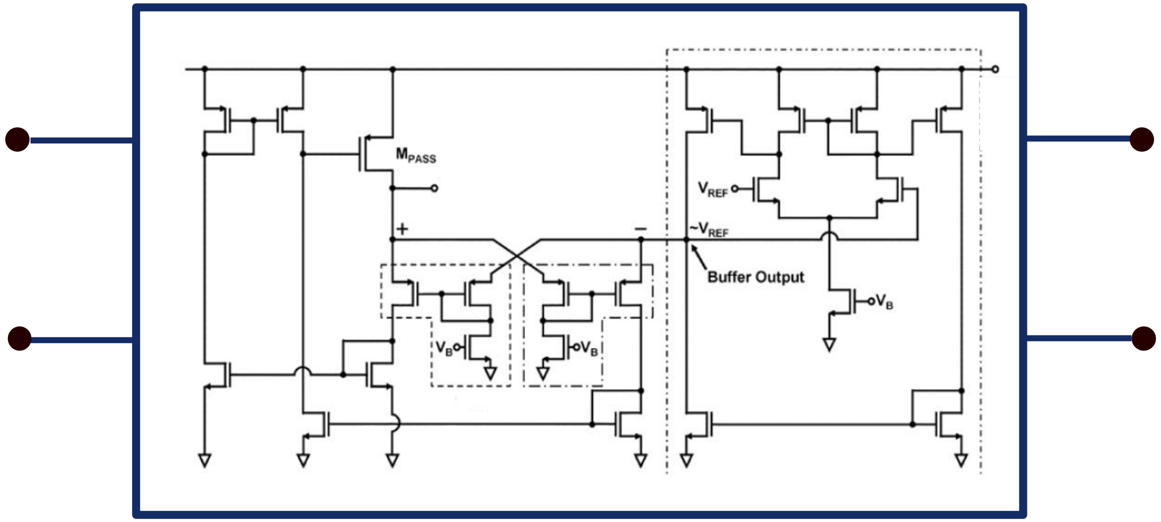
Application dependent

Semiconductors-NL

Analog Circuit Blocks

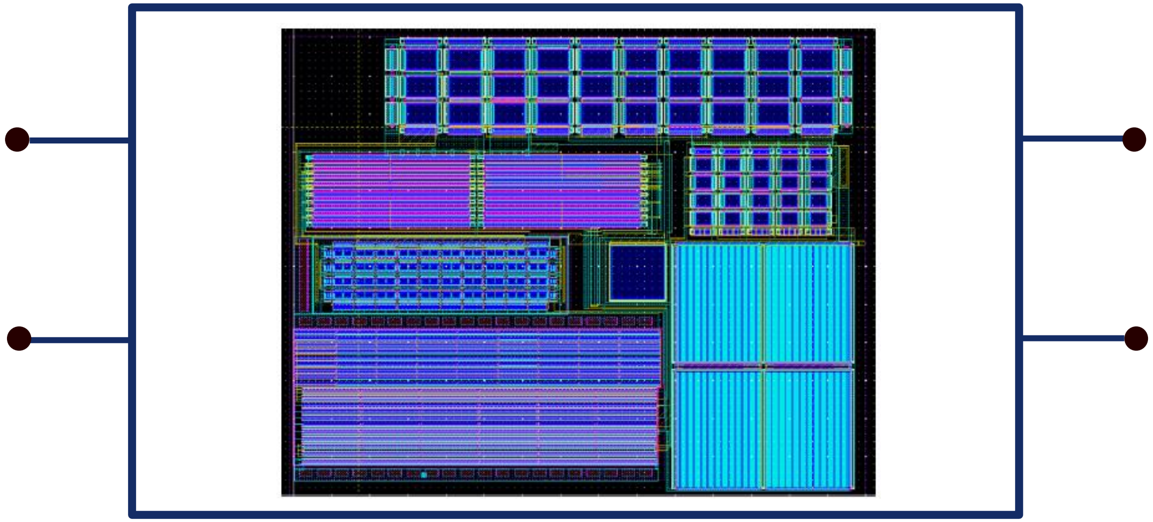
Amplifiers, LDOs, Filters, Oscillators…

Schematic. Preliminary design



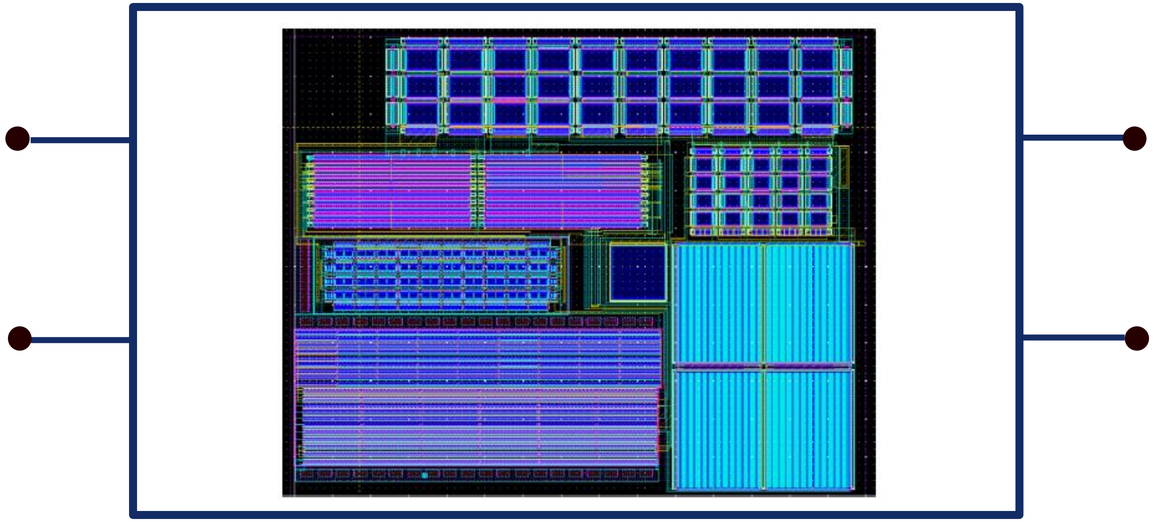
Transistor models only

Including layout (possibly packaging)



Transistor models + RLC elements

LDO test-case



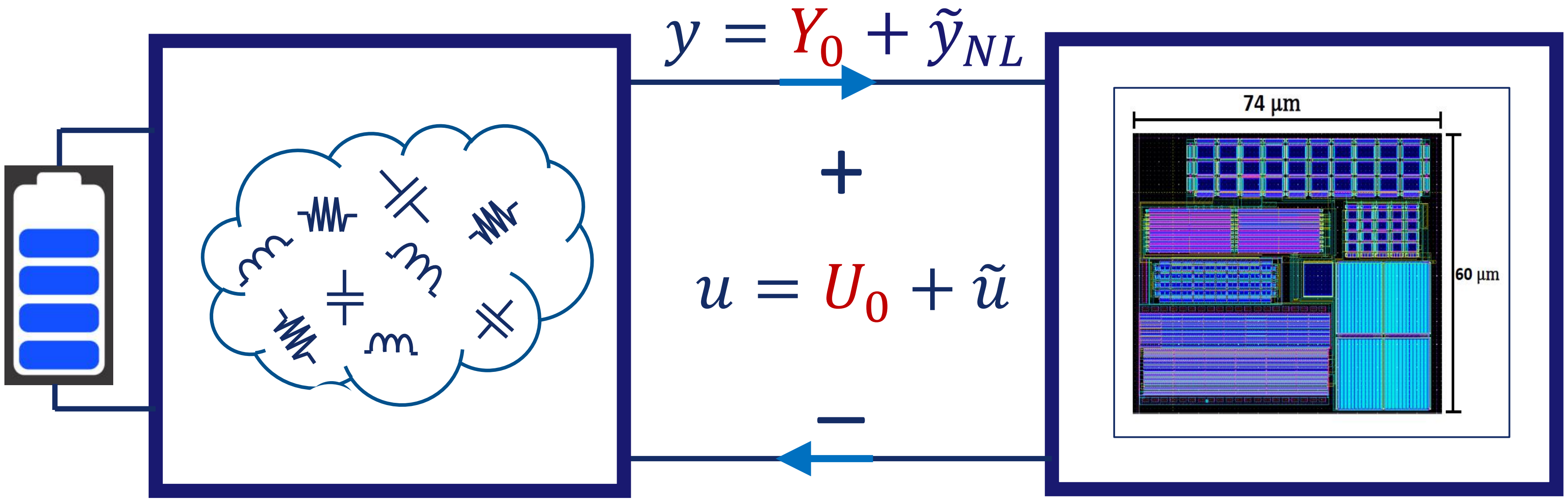
30 MB netlist !

A Transient simulation requires minutes on our hardware (72-core 2.3GHz CPUs, 128Gb RAM)

T. Y. Man, P. K. T. Mok and M. Chan, "A High Slew-Rate Push-Pull Output Amplifier for Low-Quiescent Current Low-Dropout Regulators With Transient-Response Improvement," in IEEE Transactions on Circuits and Systems II: Express Briefs, vol. 54, no. 9, pp. 755-759, Sept. 2007.

Linearized Models for Small Signal Analyses

The CB Interacts with the external system

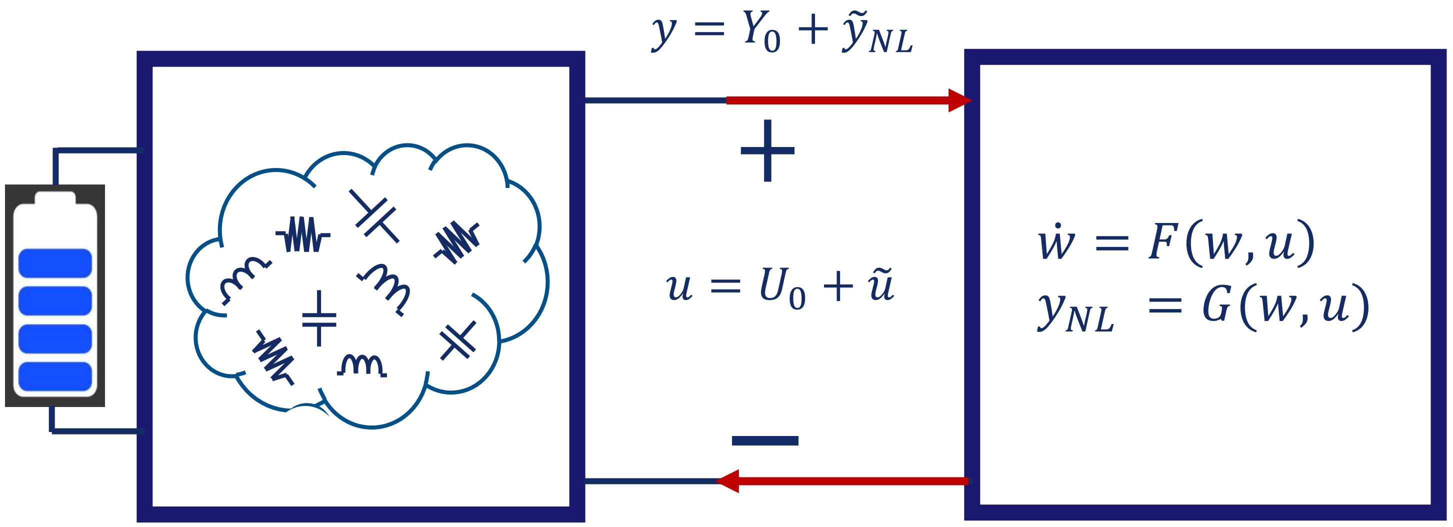


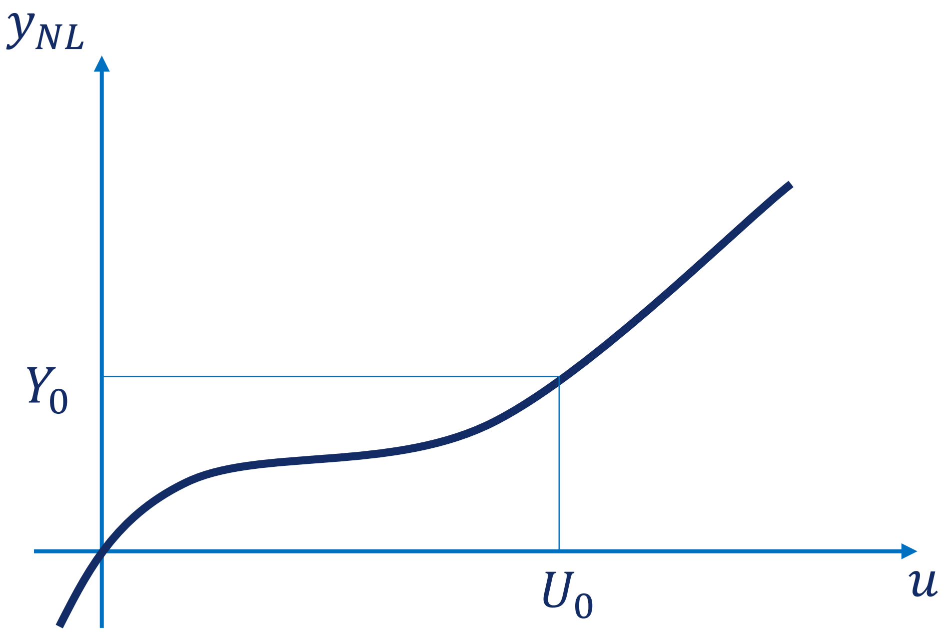
$u \equiv$ Port Voltage
$y \equiv$ Port Current

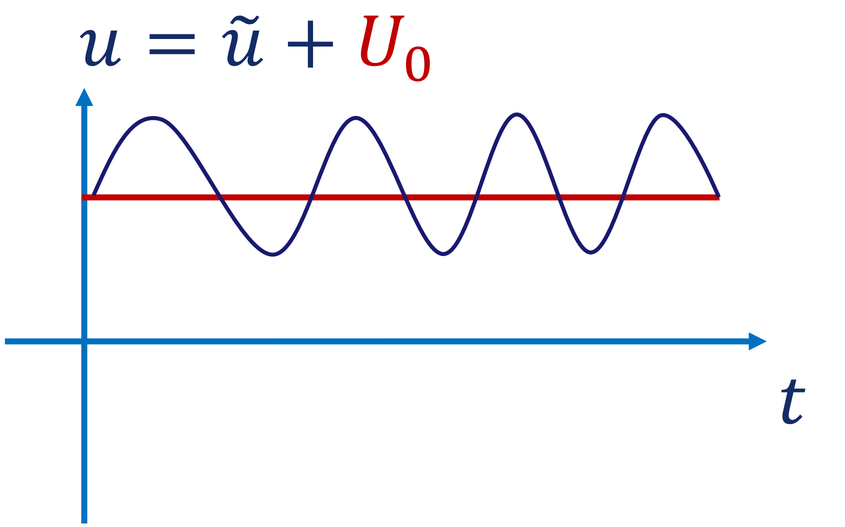
U_0: Bias voltage
Y_0: Bias current
} DC

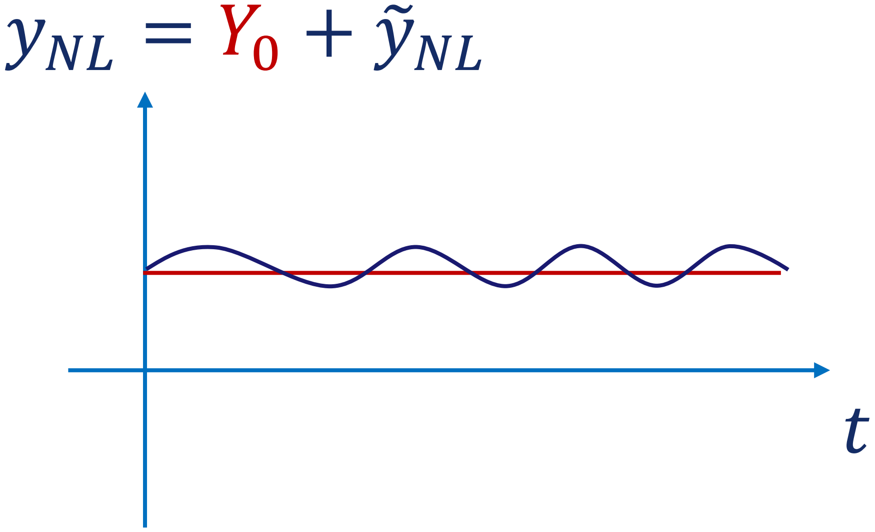
$\tilde{u}$: voltage small signal
$\tilde{y}$: current small signal
} AC

Linearized Models for Small Signal Analyses

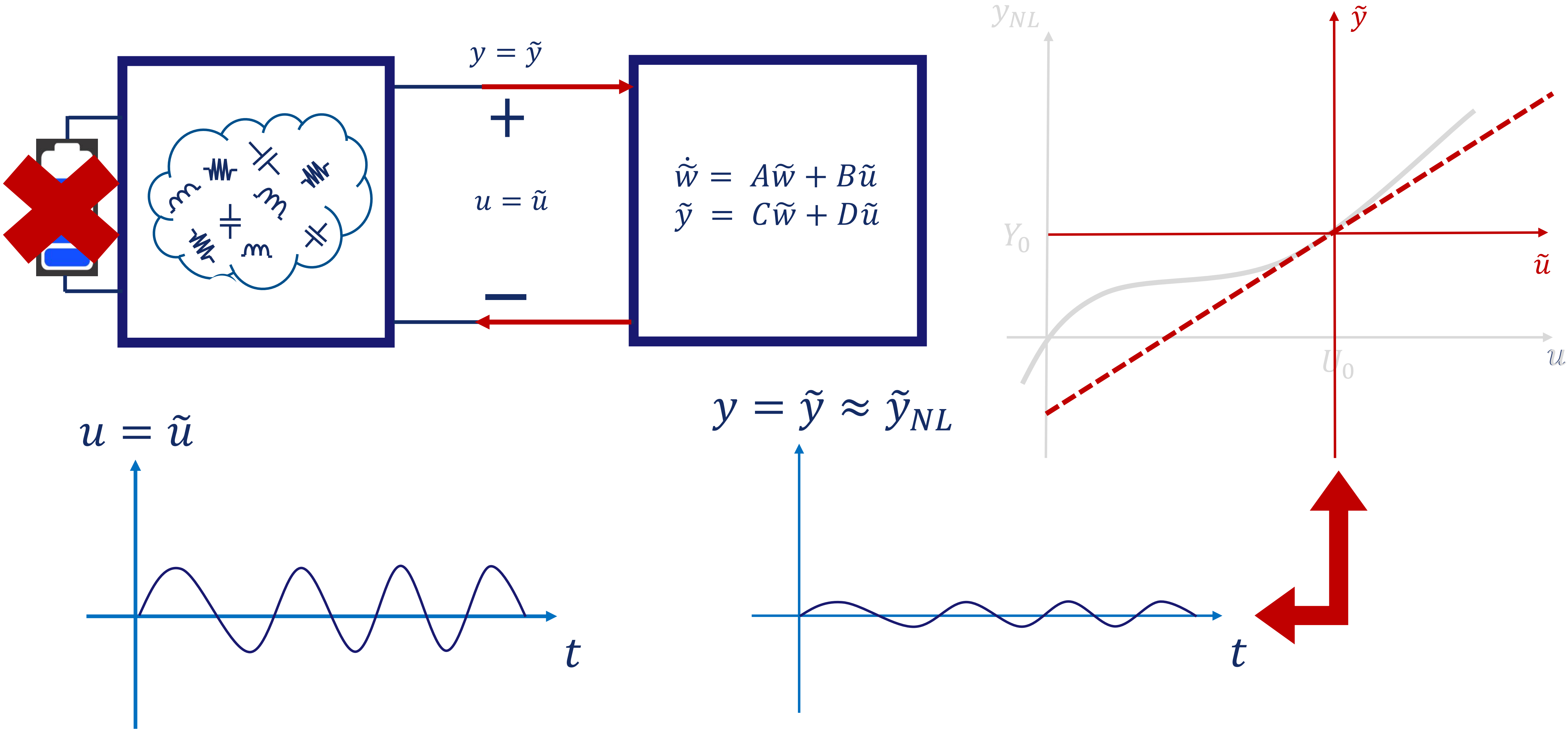




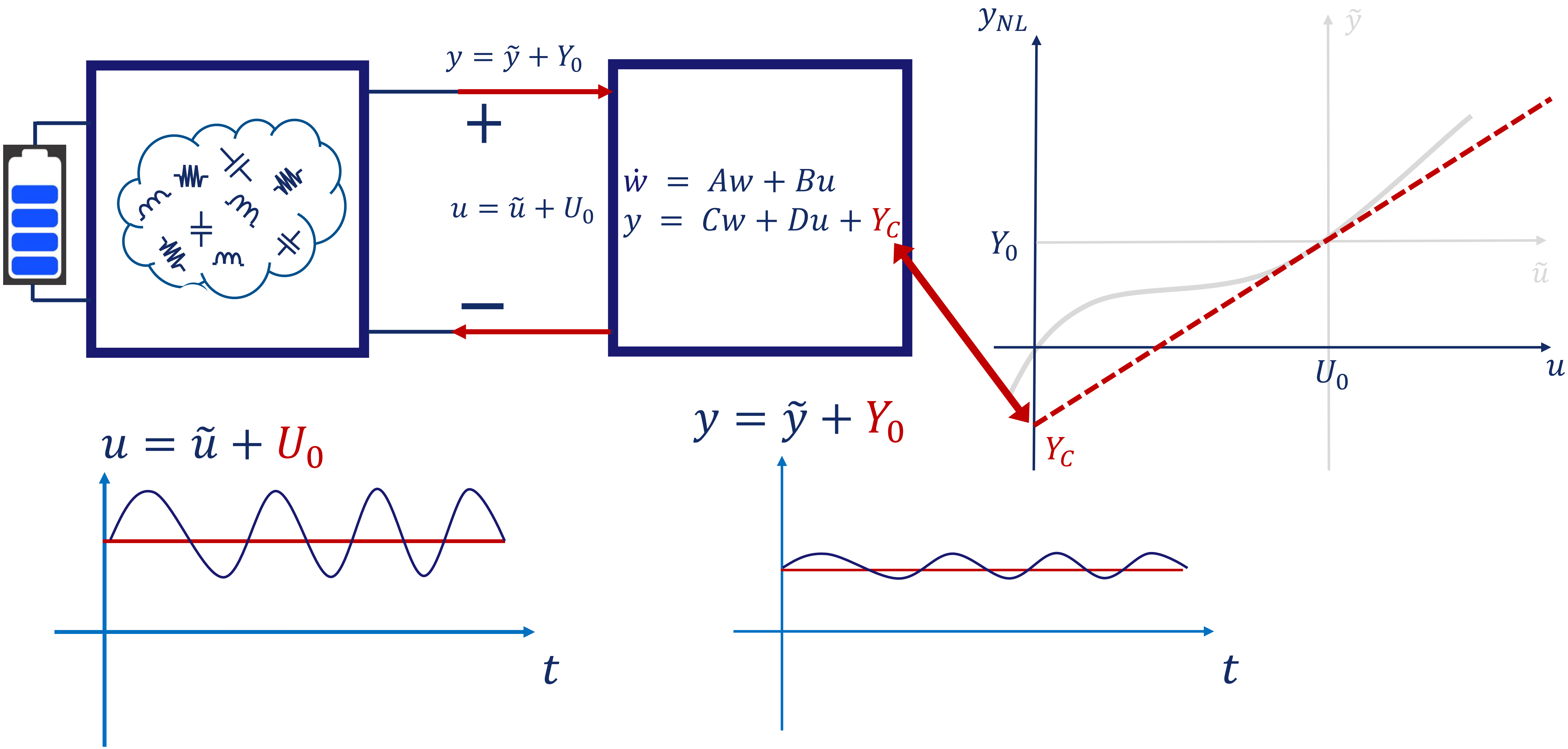




Small signal componenents approximation

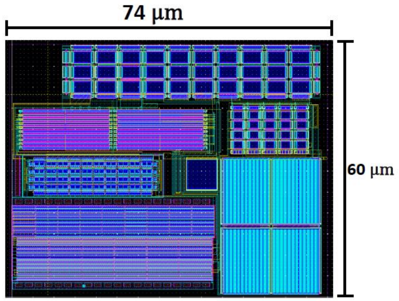


Restoring the appropriate bias level



Modeling workflow

Post layout circuit equations are encrypted in the netlist.
We collect AC data and perform rational approximation.



FIXED
BIAS

AC DATA

Rational Approximation
(Vector Fitting)

Small signal model

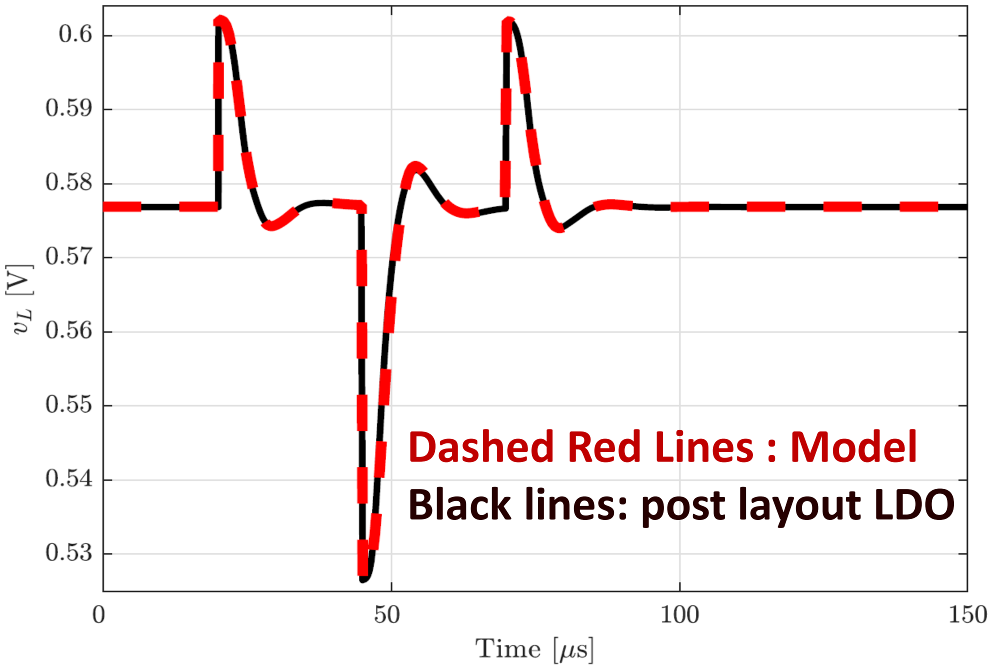
$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Once the small signal model is available, compute the required Y_C from a DC analysis. Equivalent netlist synthesis is possible

Accurate and fast results

Bias point: $V_{DD} = 1V,\ I_L = 5mA$
Small-signal: Sequential square pulses of amplitude $\pm 25mV$ over V_{DD}
Computed time span: $100\mu s$

Regulated Voltage



Model time requirements: 93 ms

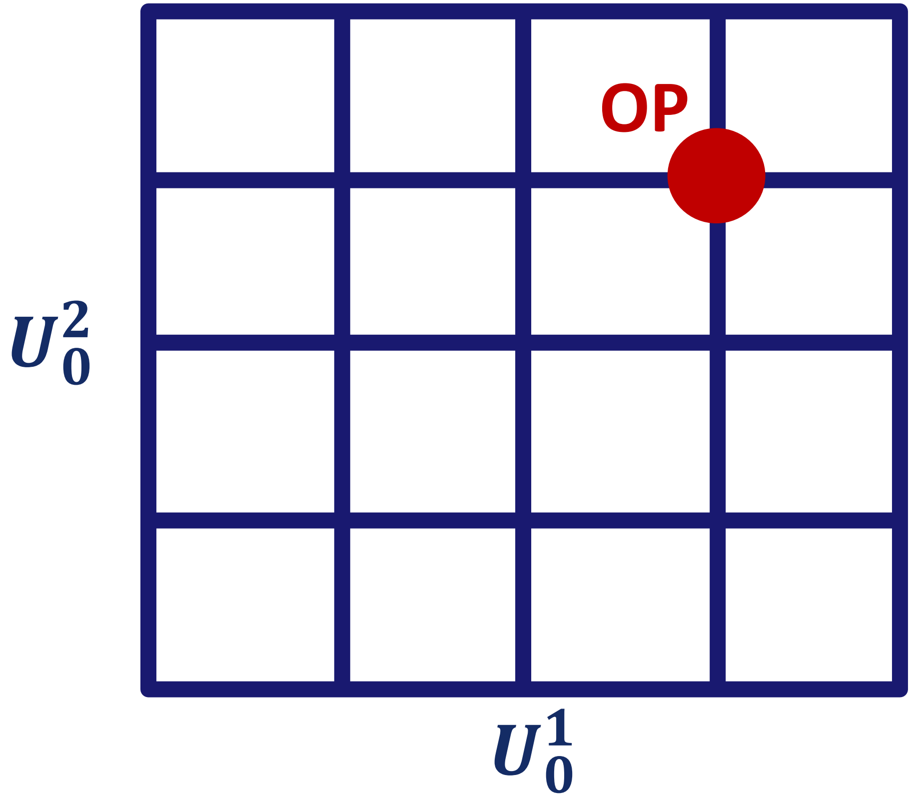
Transistor level post Layout: 63 s

SPEED UP FACTOR: 675X

Handling uncertain operating points

Affine linearized models are valid only for a single operating point

Admissible bias conditions for a 2-P circuit

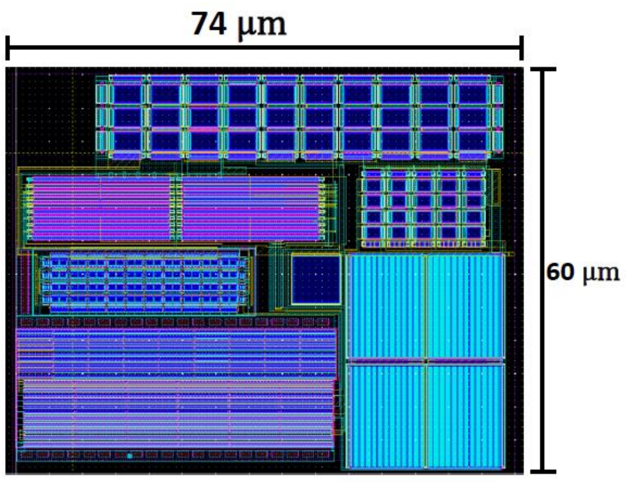


DC components combinations at each of the P ports define different bias conditions

↓

The bias point parameterizes the small signal transfer function (and the output correction level)

Equivalent Circuit via Parameterized Rational Fitting



Parametric sweep

For finite U_0 configurations

IN

Frequency response data

$$\breve{\mathrm{H}}_{k,m} = \breve{\mathrm{H}}(j\omega_k, U_{0m})$$
$$k = 1, \dots, K;\ m = 1, \dots, M$$

Multivariate Rational Fitting

$$H(s, U_o) = \frac{\mathrm{N}(s, U_0)}{\mathrm{D}(s, U_0)}$$

State-space realization

Circuit synthesis

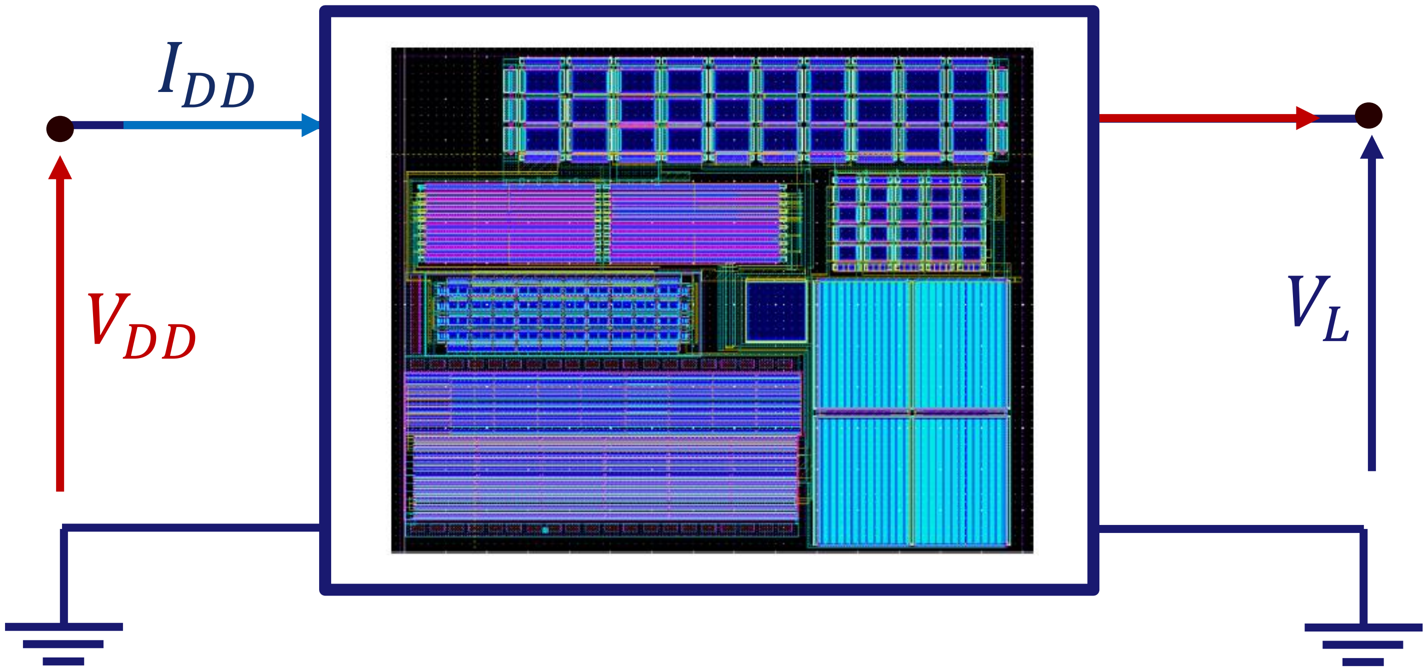
OUT

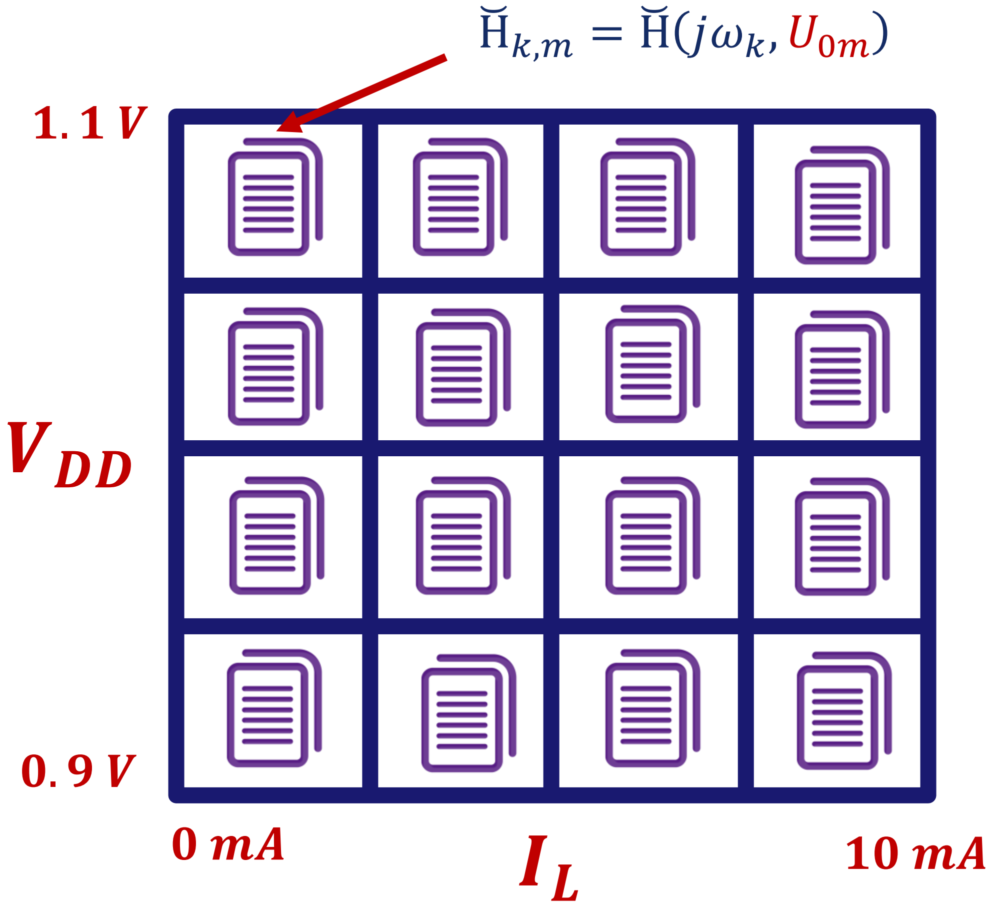
SPICE netlist

$$\dot{x} = A(U_0)x + B(U_0)u$$
$$y = C(U_0)x + D(U_0)u$$

For our LDO test case

$U_0 = \begin{bmatrix} V_{DD} \\ I_L \end{bmatrix}$ Induce the parameterization





$V_{DD} \in [0.9, 1.1]\ V \quad I_L \in [0, 10] mA$

Model structure and generation

Rational model structure

$$H(s,U_0) = \frac{N(s,U_0)}{D(s,U_0)} = \frac{\sum_n \sum_\ell R_{n,\ell} \times \zeta_\ell(U_0)\varphi_n(s)}{\sum_n \sum_\ell r_{n,\ell} \times \zeta_\ell(U_0)\varphi_n(s)}$$

To be estimated

$$\frac{N(j\omega_k, U_{0m})}{D(j\omega_k, U_{0m})} \approx \breve{H}(j\omega_k, U_{0m})$$

Partial Fractions $\varphi_n(s) : \frac{1}{s-q_n}$

Multiv. polynomials $\zeta_\ell(U_0)$

$D_0 = 1;$

for $\mu = 1,2,\ldots$

$$min \left\| \frac{N_\mu(s_k, U_{0m}) - D_\mu(s_k, U_{0m})\breve{H}(s_k, U_{0m})}{D_{\mu-1}(s_k, U_{0m})} \right\|$$

Re-weighted Least Squares

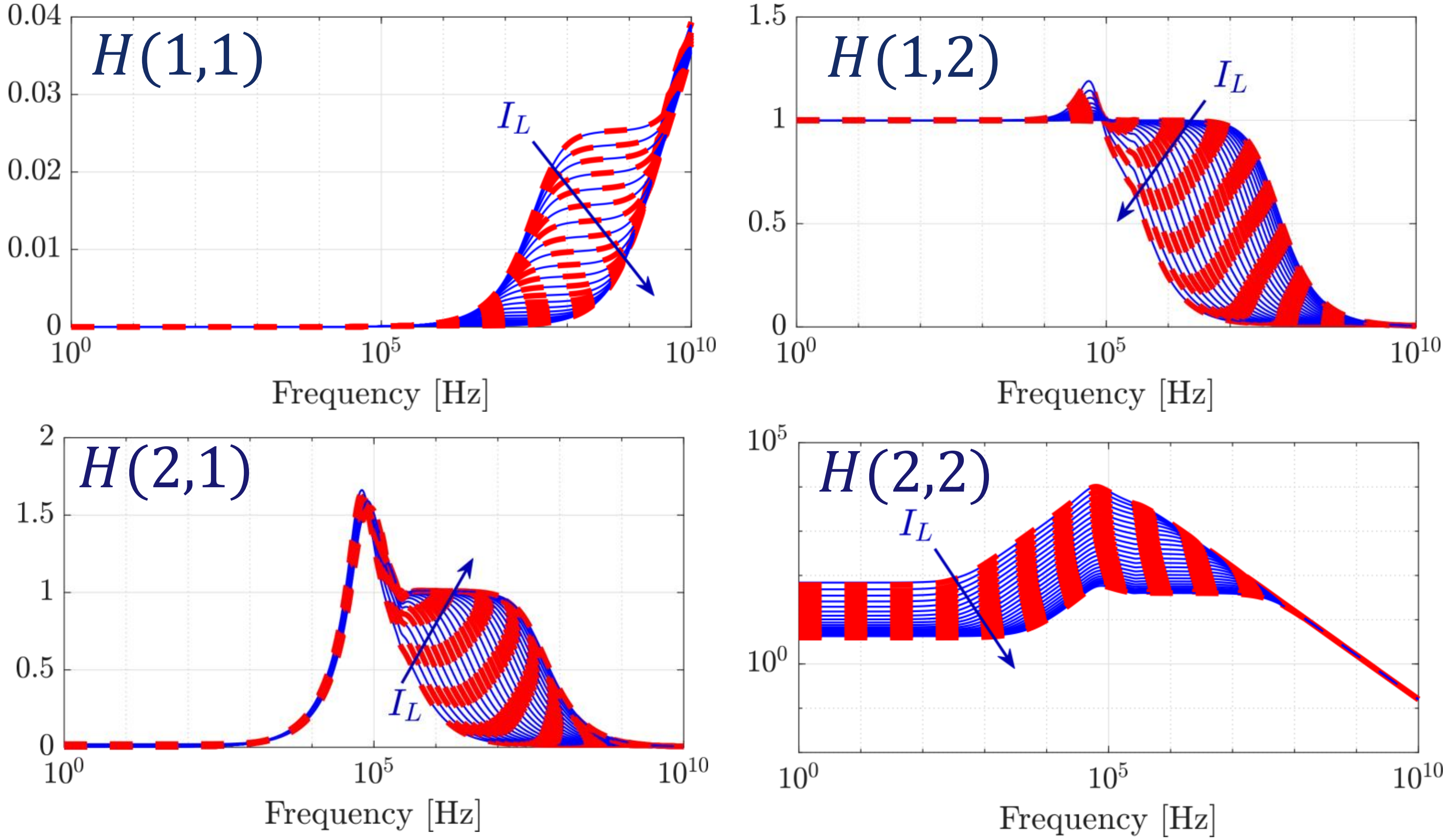
Known!

Small-Signal Model ($\mathbf{V_{DD}} = \mathbf{1}\boldsymbol{V}$)

Dynamic order: 9

Parameter Dependent Basis: Chebyshev Polynomials of order 4 and 3 for numerator and denominator respectively

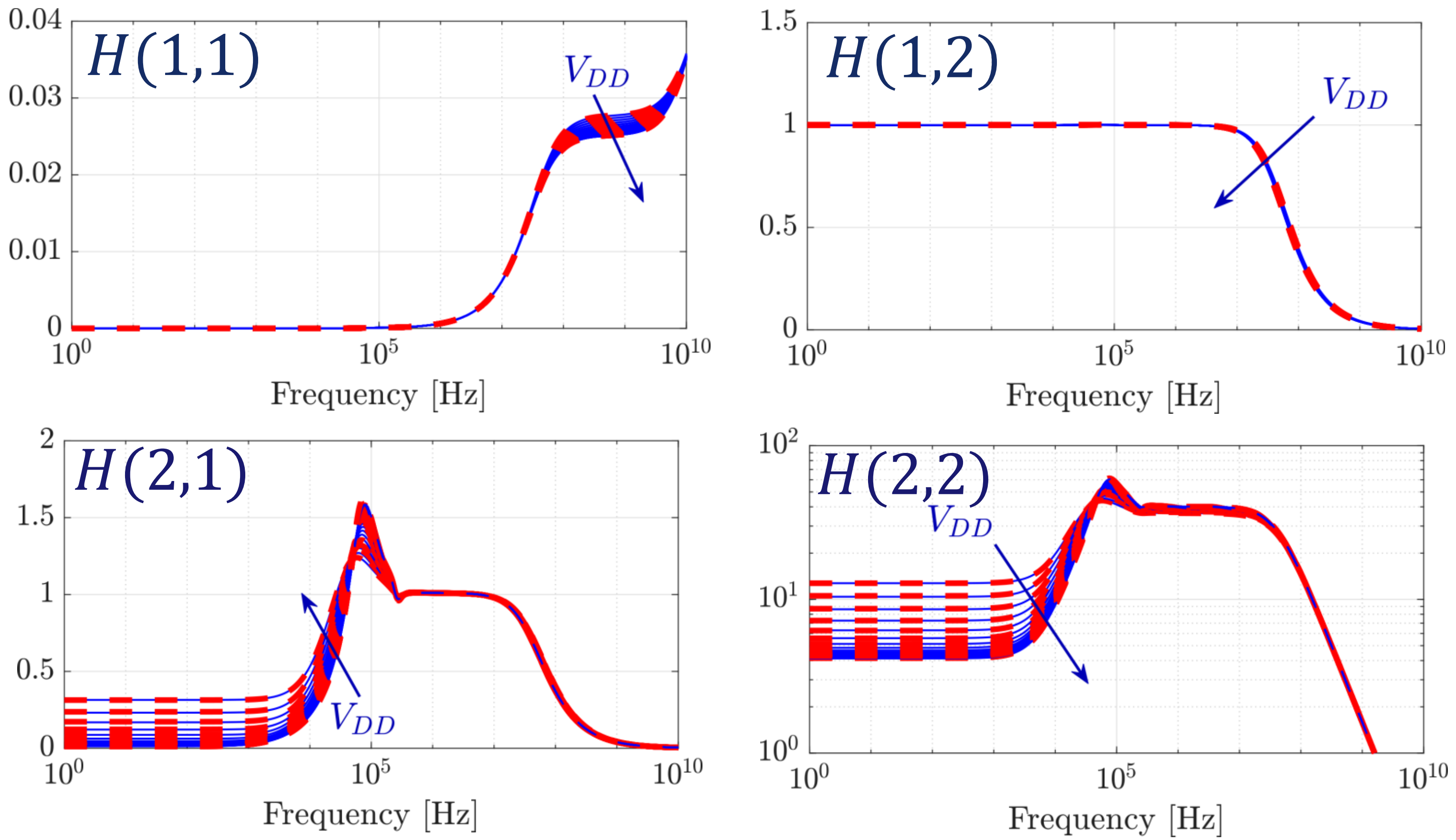
208 AC responses were exploited for fitting



Dashed Red Lines : Model   Blue Lines: AC sweep data

Parameterized Small-Signal Model ($I_L = 10mA$)

Stability is preserved for all bias conditions using standard parameterized macromodeling techniques (more on this later)

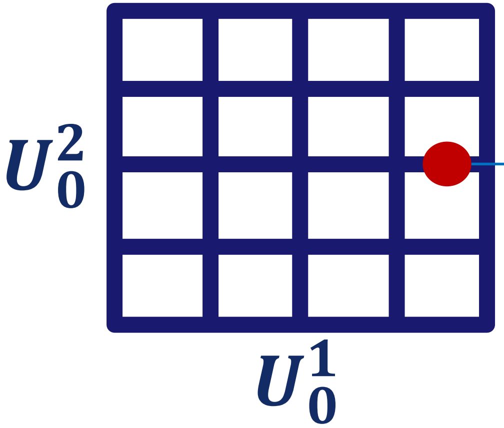


Dashed Red Lines : Model

Blue Lines: AC sweep data

A more involved simulation scenario

Uncertain bias ⟷ Parameterized models



Once OP is found

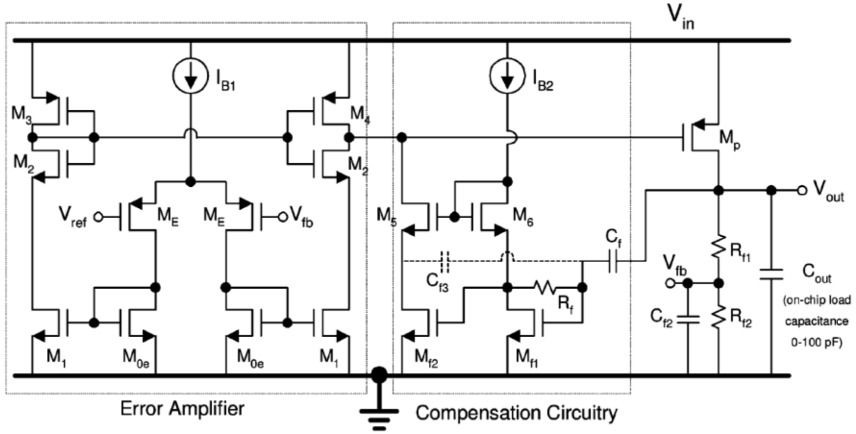
U_0

$$\dot{x} = A(U_0)x + B(U_0)u$$
$$y = C(U_0)x + D(U_0)u + Y_c(U_0)$$

A Linear Time Invariant (LTI) equivalent circuit is instantiated depending on the SPICE DC point analysis. U_0 is uncertain but constant

What if the operating point changes during the simulation?

Nonstationary bias conditions



$V_{in} \in [2.85, 3]$ V

$$\dot{x} = A(U_0)x + B(U_0)u$$
$$y = C(U_0)x + D(U_0)u + Y_c(U_0)$$

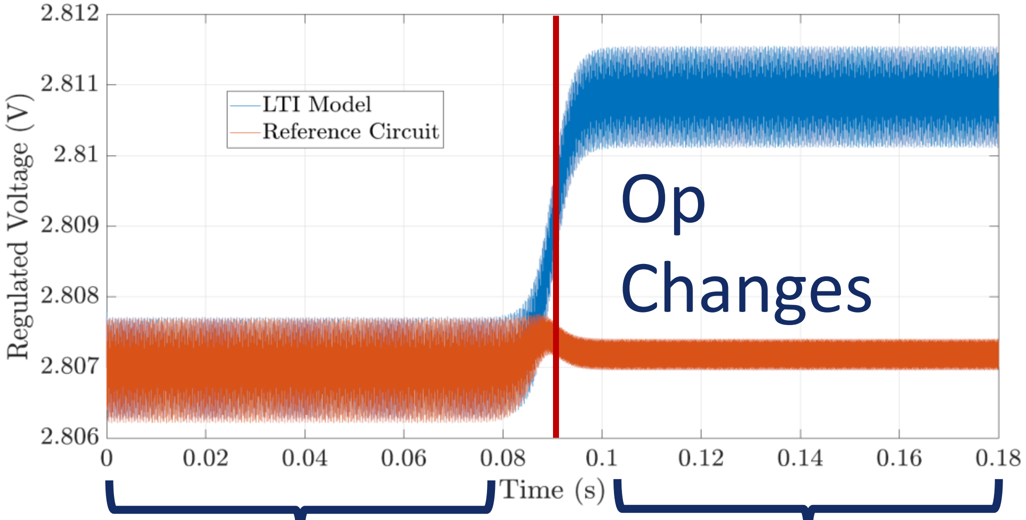
Model in transient simulation

$V_{in}(0) = 2.85$ V $\quad V_{in}(0.1) \approx 3$ V

Netlist instantiation

$$\dot{x} = A(2.85)x + B(2.85)u$$
$$y = C(2.85)x + D(2.85)u + Y_c(2.85)$$

BAD!



$U_0^1 \equiv V_{in} = 2.85$ V $\quad U_0^1 \equiv V_{in} = 3$ V

A Linear Parameter Varying Model

We would like our model to track the operating point variation

Standard small sig. analysis

$$u(t) = U_0 + \tilde{u}(t), \qquad \tilde{u}(0) = 0$$

- The bias is U_0 is constant and determined via OP analysis ($U_0 = u(0)$)
- The linearized model is instantiated accordingly
- The equivalent circuit is linear time invariant

$$\dot{x} = A(U_0)x + B(U_0)u$$
$$y = C(U_0)x + D(U_0)u + Y_c(U_0)$$

Dynamic small sig. analysis

$$u(t) = U_0(t) + \tilde{u}(t), \qquad \tilde{u}(0) = 0$$

- The bias is determined by a large time varying signal $U_0(t)$
- The linearized model should depend on the instantaneous value of $U_0(t)$
- The equivalent circuit should be time varying

$$\dot{x} = A(U_0(t))x + B(U_0(t))u$$
$$y = C(U_0(t))x + D(U_0(t))u + Y_c(U_0(t))$$

A Linear Parameter Varying Model

The same modeling framework can be adapted to the LPV case

ASSUMPTIONS

$$u(t) = U_0(t) + \tilde{u}(t), \qquad \tilde{u}(0) = 0$$

If it holds that

- $\tilde{u}(t)$ is still a small signal component
- $U_0(t)$ is slow wrt the circuit response

→

The circuit instantaneously works around the OP induced by $U_0(t)$ *as if it was constant*

Modeling workflow

Under the previous assumptions, the modeling workflow is as follows

1. Define the range of admissible values for $U_0(t) \in \mathcal{U}_0$ (a hypercube)
2. Repeatedly sample the circuit SS transfer function over $\mathcal{U}_0$, via AC analyses with different STATIC bias configurations
3. Build the multivariate rational approximation $H(s, U_o) = \frac{N(s,U_0)}{D(s,U_0)}$, and the parameterized affine term $Y_C(U_0)$
4. Cast the rational transfer function into a parameterized time-varying equivalent circuit with components depending on $U_0(t)$ or in SS

$$\dot{x} = A(U_0(t))x + B(U_0(t))u$$
$$y = C(U_0(t))x + D(U_0(t))u + Y_c(U_0(t))$$

Technical issues

Two main problems must be solved to generate reliable models

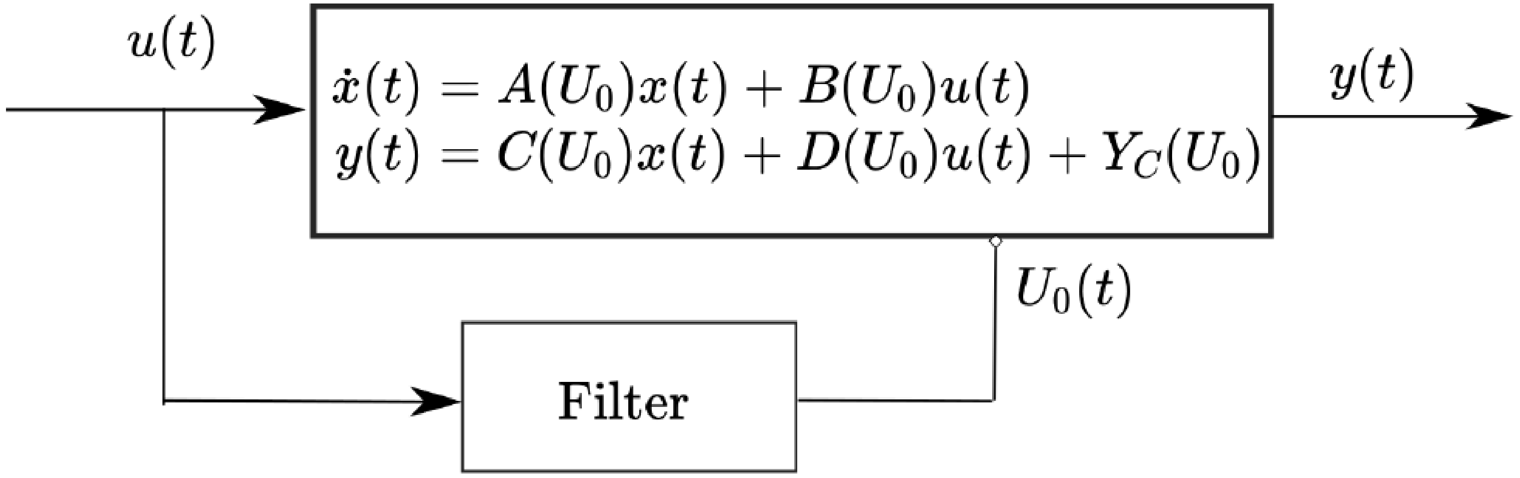
1. During online operation the bias component $U_0(t)$ is merged with the small signal.
2. If the circuit block is known to be stable under the action of $u(t)$ so must be the model.

To tackle the above we propose to

1. Embed in the model a low pass filter that extracts $U_0(t)$ from $u(t)$
2. Constraint the model generation to guarantee stability in a time varying setting

Bias Component Extraction

The total input signal $u(t) = U_0(t) + \tilde{u}(t)$ is filtered to extract $U_0(t)$



$$H(s, U_o) = \frac{N(s, U_0)}{D(s, U_0)}$$

→ Sample poles for different U_0 →

Determine the slowest pole angular frequency ω_p

→

Design 2nd order Butterworth, $\omega_c \leq 0.1\ \omega_p$

Stability enforcement

Robust LTI stability for standard parameterized macromodels

$$H(s, U_0) = \frac{\mathrm{N}(s, U_0)}{\mathrm{D}(s, U_0)} = \frac{\sum_n \sum_\ell \mathrm{R}_{n,\ell}\, \zeta_\ell(U_0)\varphi_n(s)}{\sum_n \sum_\ell r_{n,\ell}\, \zeta_\ell(U_0)\varphi_n(s)}$$

with

Partial Fractions $\varphi_n(s) : \dfrac{1}{s - q_n}$

$\mathrm{N}(s, U_0)$ and $\mathrm{D}(s, U_0)$ share the same set of common poles → Poles of $H(s, U_0)$ are the zeros of $\mathrm{D}(s, U_0)$

If $\mathrm{D}(s, U_0)$ is a passive immittance function, then its zeros have strictly negative real part.

Stability enforcement

The same criterion is not applicable for the proposed LPV model

The dynamic equations
$$\dot{x} = A(U_0(t))x$$
may be unstable
even if $A(U_0)$ is Hurwitz
for constant U_0

$\forall U_0(t)$ →

$$\dot{x} = A(U_0(t))x$$
is asymptotically stable if
$$\exists\, P \succ 0:\ A^T(U_0)P + P\,A(U_0) \prec 0$$
$$\forall U_0 \in \mathcal{U}_0$$

$$H(s, U_0) = \frac{\sum_n \sum_\ell \mathrm{R}_{n,\ell}\, \zeta_\ell(U_0)\varphi_n(s)}{\sum_n \sum_\ell r_{n,\ell}\, \zeta_\ell(U_0)\varphi_n(s)}$$

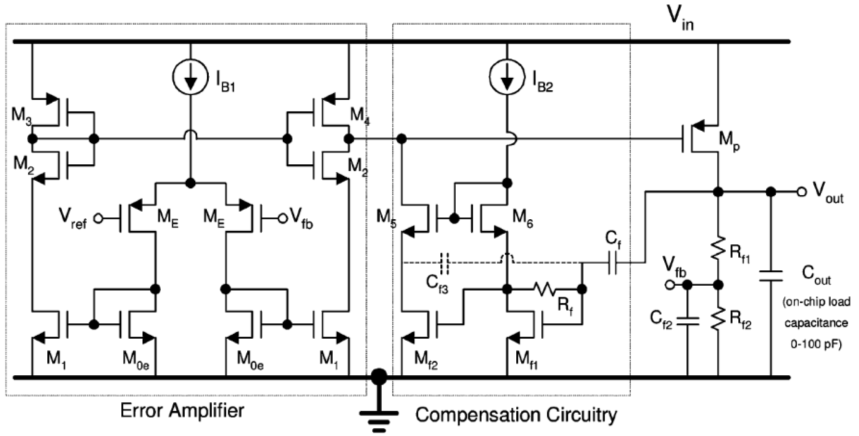
⟷

$$\dot{x} = A(U_0)x + B(U_0)u$$
$$y = C(U_0)x + D(U_0)u$$

With appropriate model structure, the unknowns $r_{n,\ell}$ can be found via convex programming to ensure asymptotic stability

Bradde, Tommaso, et al. "Fast Simulation of Analog Circuit Blocks Under Nonstationary Operating Conditions." *IEEE Transactions on Components, Packaging and Manufacturing Technology* 11.9 (2021): 1355-1368.

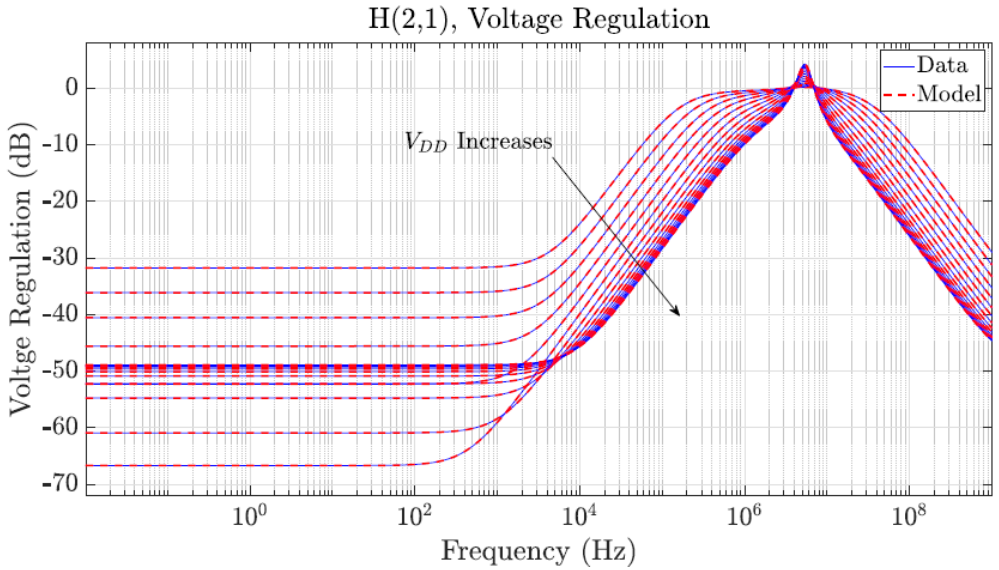
Nonstationary bias conditions

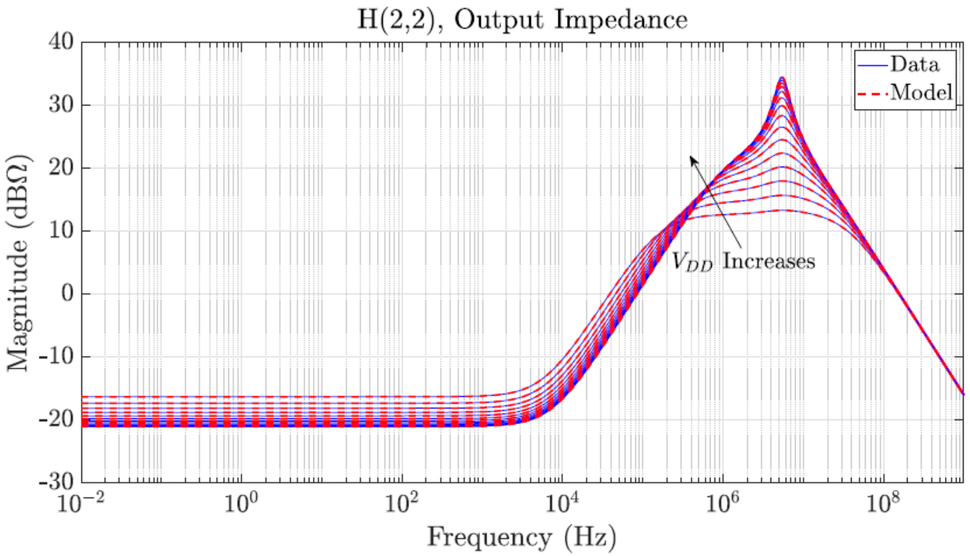


$V_{DD} \in [2.85, 3]$ V

$$\dot{x} = A(U_0(t))x + B(U_0(t))u$$
$$y = C(U_0(t))x + D(U_0(t))u + Y_c(U_0(t))$$

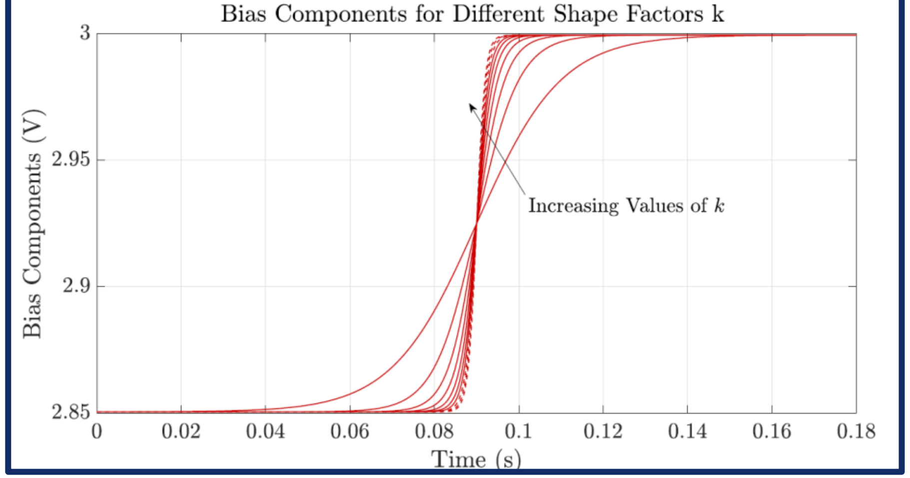
Excellent AC accuracy



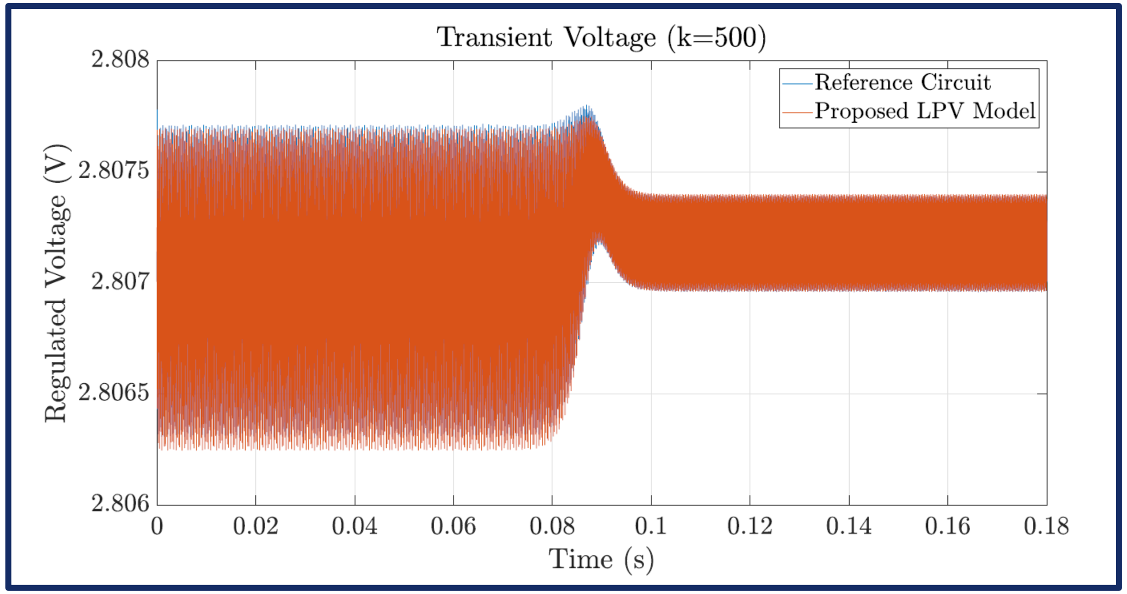


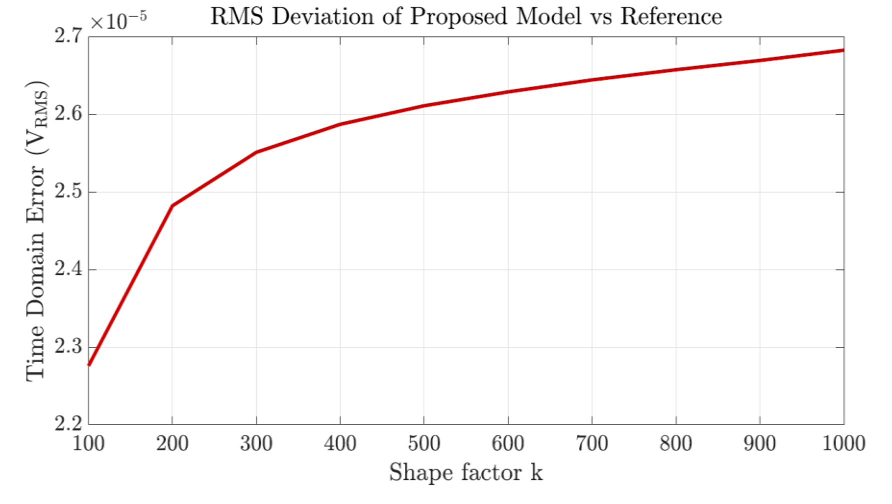
Transient simulation

$V_{DD}(t)$ with different slopes



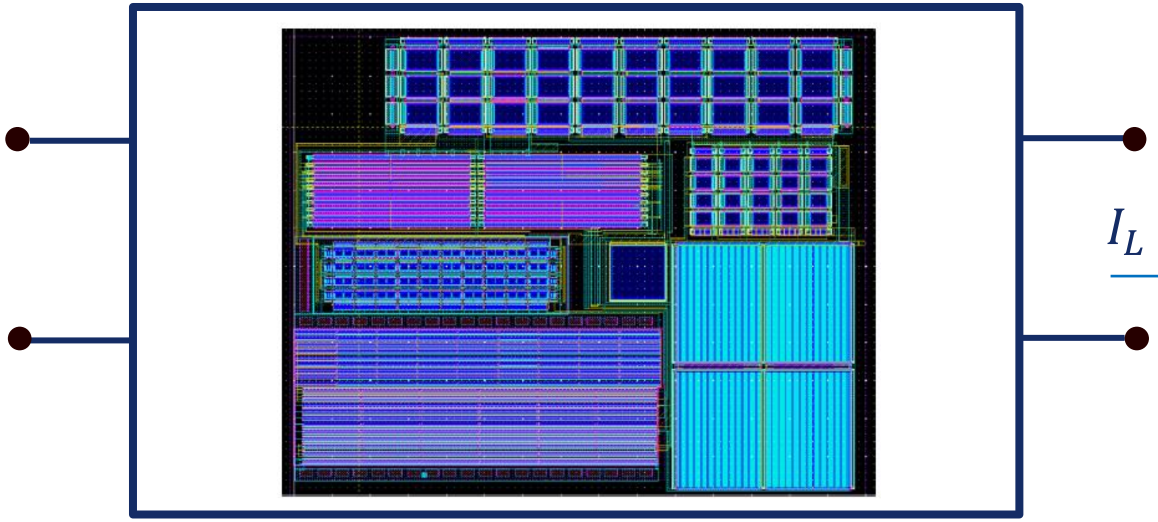
Plus $\tilde{u}(t)$





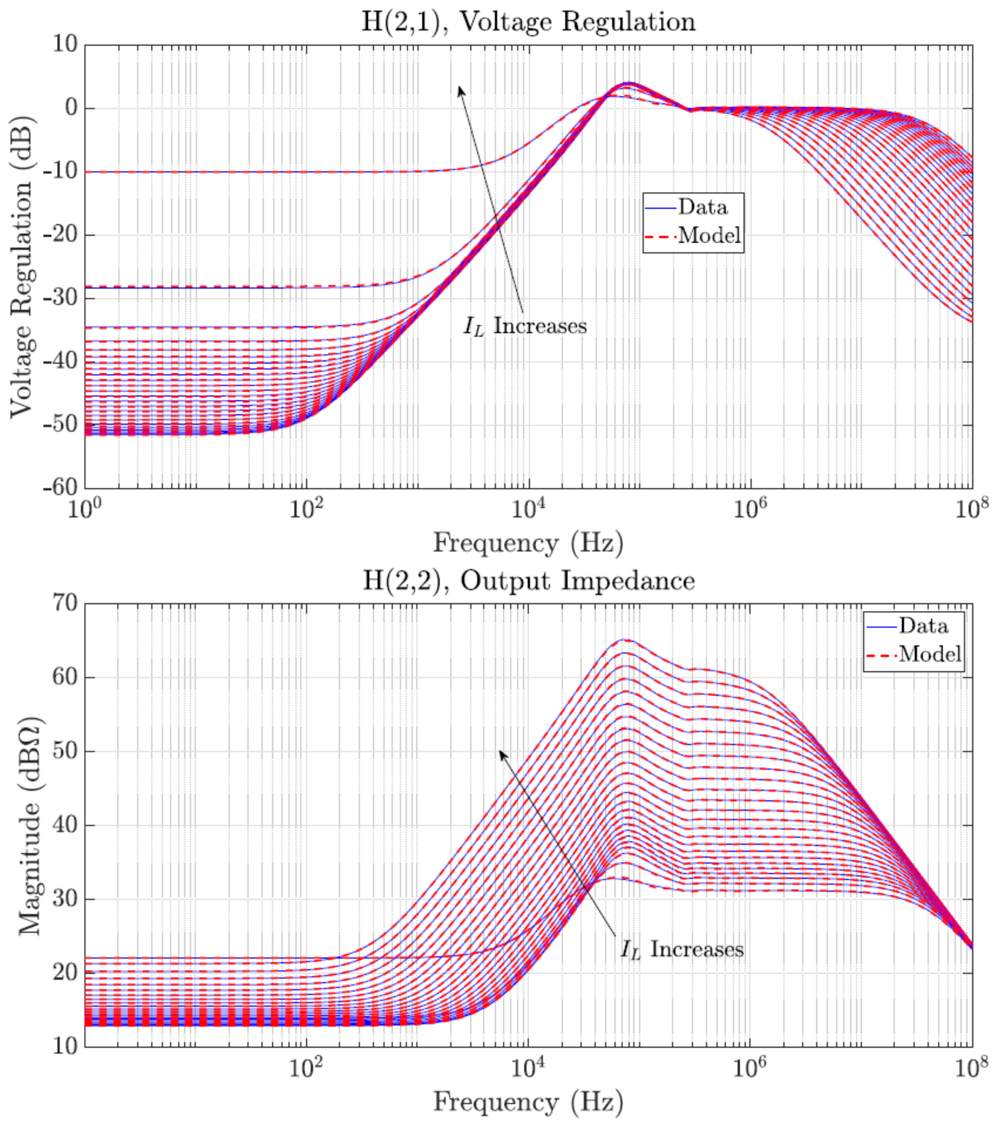
The LPV model performs much better then standard Linearized time invariant equivalent

A realistic test case



30 MB netlist !

$I_L \in [0, 10]$ mA



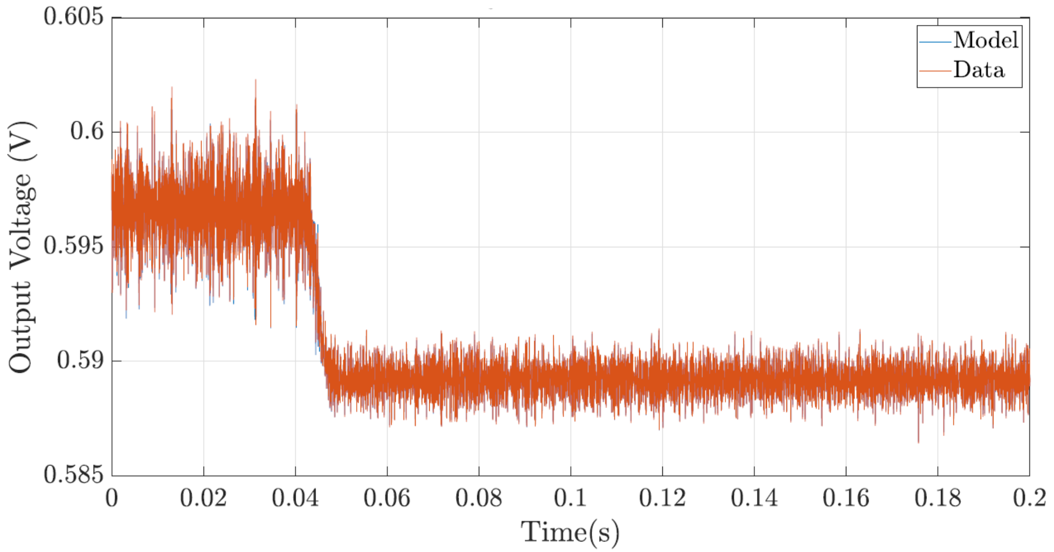
We built a 9th order model for large load current signals and input voltage $V_{DD} = 0.9$ V

Model generation time: 8.6 s

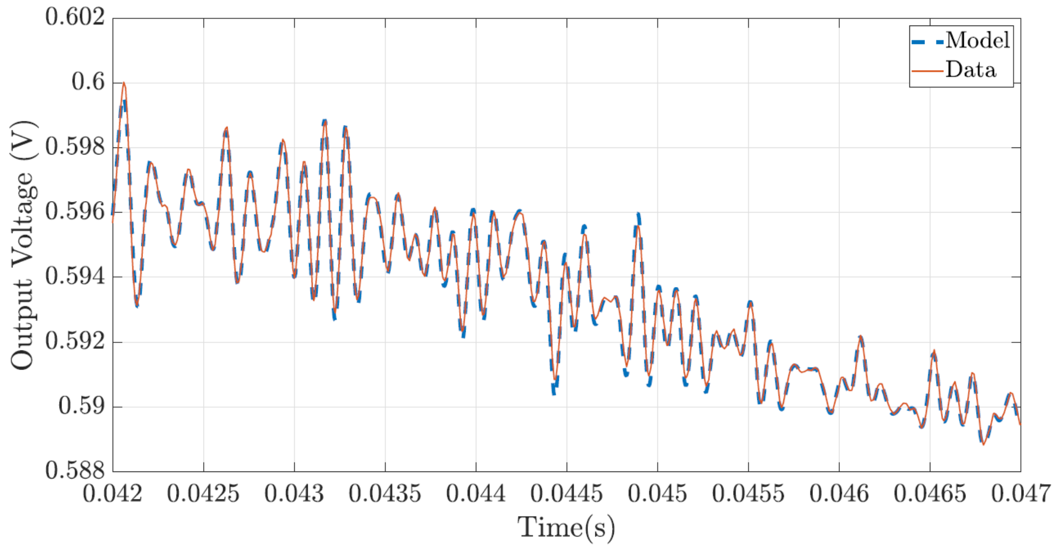
Transient simulation. Accuracy and speed up

- We simulate a load state transition from 0.5 to 2 mA, $\Delta t = 6\text{ms}$
- 0.2 mA small signal, noise with flat power spectrum over [1,10] kHz

Full time window



Zoom on transition

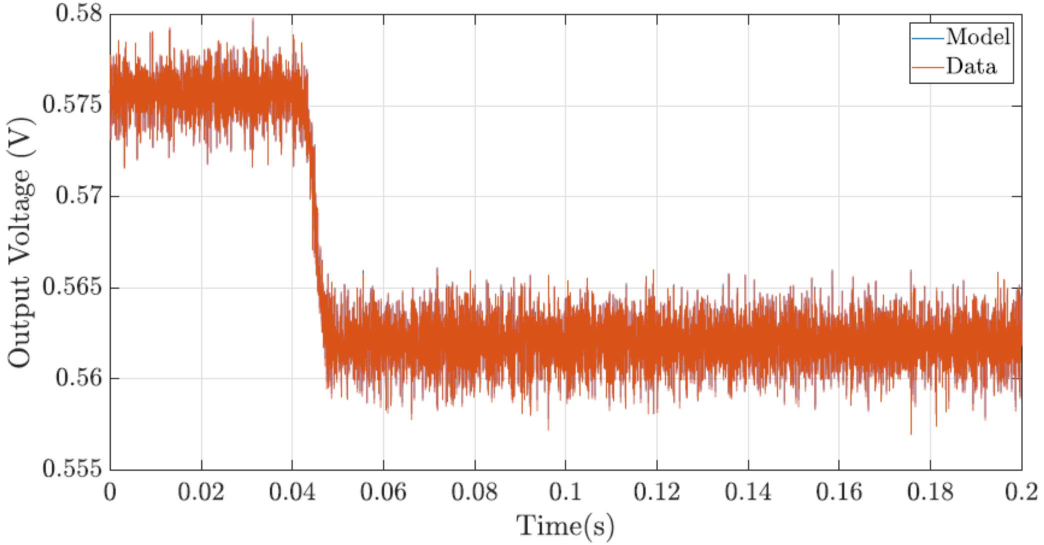


REFERENCE: 13 min, MODEL: 16 s → 50 × Speed-up!

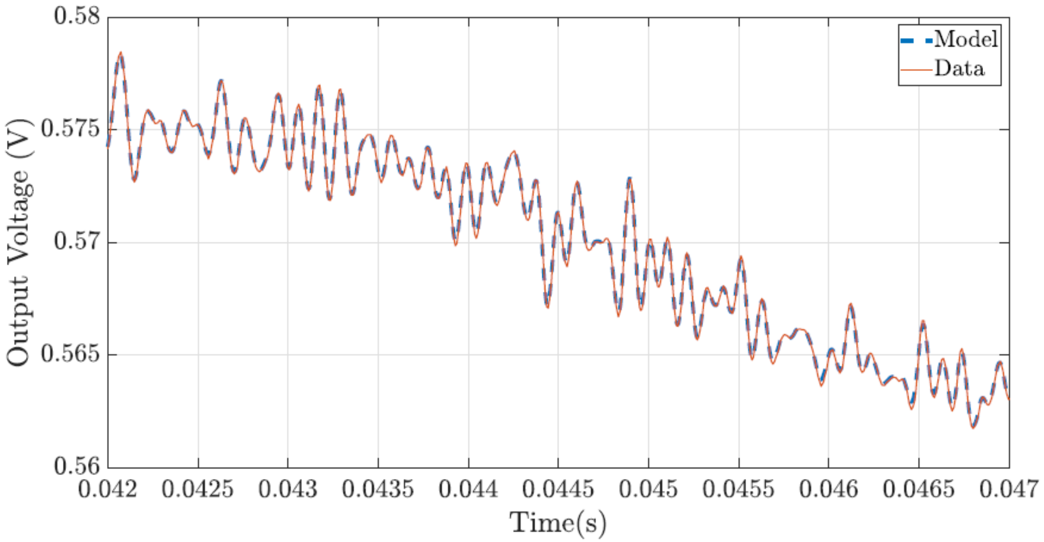
Transient simulation. Accuracy and speed up

- We simulate a load state transition from 5 to 8 mA, $\Delta t = 6\text{ms}$
- 0.5 mA small signal, noise with flat power spectrum over $[1,10]$ kHz

Full time window



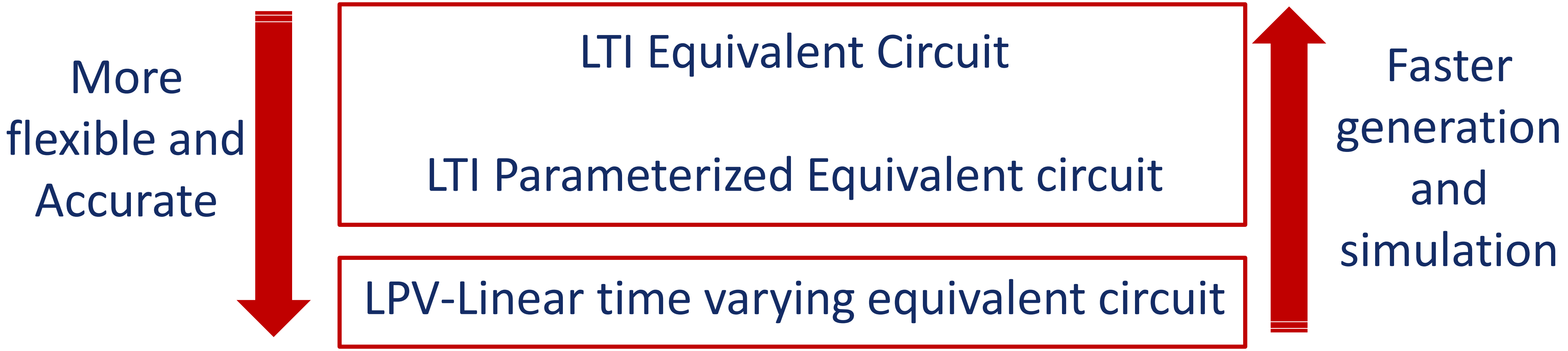
Zoom on transition



REFERENCE: 13 min, MODEL: 16 s → 50 × Speed-up!

Conclusions

- We discussed how linearized reduced order equivalent circuits can replace native mildly nonlinear CB descriptions for fast simulations
- Three fitting-based modeling approaches are possible depending on the required flexibility-vs-time requirements



- In all cases, models are accurate, stable, and generated via automated algorithms

Thank you!

Thanks for your attention!

Time for Q&A

Stability enforcement

The same criterion is not applicable for the proposed LPV model

The dynamic equations
$$\dot{x} = A(U_0(t))x$$
may be unstable
even if $A(U_0)$ is Hurwitz
for constant U_0

Stability can be ensured for any $U_0(t)$ trajectory through Lyapunov theory

$$\dot{x} = A(U_0(t))x$$
is asymptotically stable if
$$\exists\, P \succ 0\colon\ A^T(U_0)P + P\,A(U_0) \prec 0$$
$$\forall U_0 \in \mathcal{U}_0$$

Stability enforcement

$$\dot{x} = A(U_0(t))x$$

is asymptotically stable if

$$\exists\, P \succ 0:\ A^T(U_0)P + P\,A(U_0) \prec 0$$

$$\forall U_0 \in \mathcal{U}_0$$

$$H(s, U_0) = \frac{\sum_n \sum_\ell \mathrm{R}_{n,\ell}\, \zeta_\ell(U_0)\varphi_n(s)}{\sum_n \sum_\ell r_{n,\ell}\, \zeta_\ell(U_0)\varphi_n(s)} \quad \longleftrightarrow \quad \begin{aligned} \dot{x} &= A(U_0)x + B(U_0)u \\ y &= C(U_0)x + D(U_0)u \end{aligned}$$

The state matrix $A(U_0)$ depends on the parameterized denominator coefficients $r_{n,\ell}\, \zeta_\ell(U_0)$.

Find coefficients s.t. the Lyapunov condition is satisfied

Stability enforcement

Not numerically tractable

Parameterized LMI ⟷ Infinite number of constraints

Simpler **sufficient conditions** can be derived with suitable **polynomial basis**

$\zeta_\ell(U_0) \equiv b_{\ell,\bar{\ell}}(U_0)$ *The ℓ − th basis of Bernstein polynomials of degree $\bar{\ell}$*

Positivity

$$0 \leq b_{\ell,\bar{\ell}}(U_0) \leq 1 \;\; \forall\, \ell$$

Partition of unity

$$\sum_{\ell}^{\bar{\ell}} b_{\ell,\bar{\ell}}(U_0) = 1$$

Stability Enforcement

Under this particular parameterization we can tackle the problem

$$\exists\, P \succ 0:\ A^T(U_0)P + P\,A(U_0) \prec 0 \quad \forall U_0 \in \mathcal{U}_0$$

Implied by

$$\exists\, P \succ 0: \sum_{\ell}^{\bar{\ell}} b_{\ell,\bar{\ell}}(U_0) \cdot X_\ell(P, r_{n,\ell}) \prec 0 \quad \forall U_0 \in \mathcal{U}_0$$

1. Bases $b_{\ell,\bar{\ell}}(U_0)$ are always non-negative
2. The matrix coefficients $X_\ell(P, r_{n,\ell})$ depend linearly on the decision variables, meaning that:

A STANDARD LMI !

$$\exists\, P \succ 0: X_\ell(P, r_{n,\ell}) \prec 0\ \forall\, \ell$$

Implies

$$\exists\, P \succ 0:\ A^T(U_0)P + P\,A(U_0) \prec 0 \quad \forall U_0 \in \mathcal{U}_0$$