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# BATHTUB EXTRAPOLATION OF IBIS-AMI TIMING JITTER

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# ABSTRACT

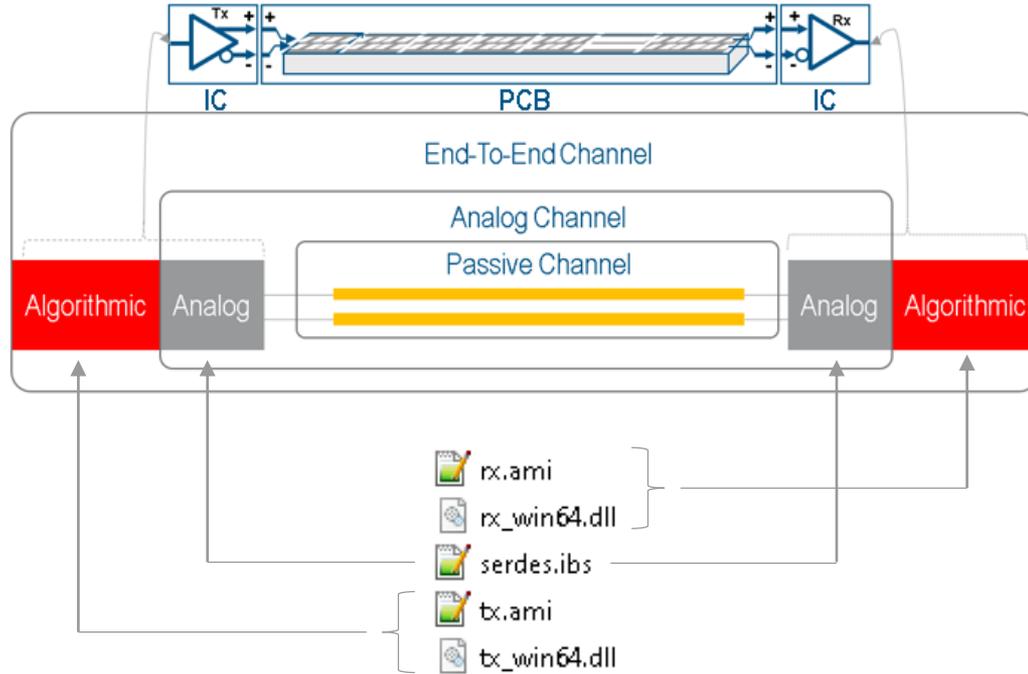
IBIS Algorithmic Modeling Interface (IBIS-AMI) is widely accepted in the industry to model the analog and algorithmic part of a Serializer/Deserializer (SerDes) PHY [1]. The simulation workflow of IBIS-AMI including deterministic jitter (DJ) and random jitter (RJ) effects is introduced. Dual-Dirac model [2] is used to extrapolate the bathtub to obtain the eye width at a low bit error rate (BER), and its accuracy are validated by comparing to the convolution results of DJ and RJ.

# OVERVIEW

1. IBIS-AMI Time-Domain Workflow
2. Jitter and Noise in IBIS-AMI
3. Bathtub Extrapolation
4. Summary

# 1. IBIS-AMI TIME-DOMAIN WORKFLOW (1/3)

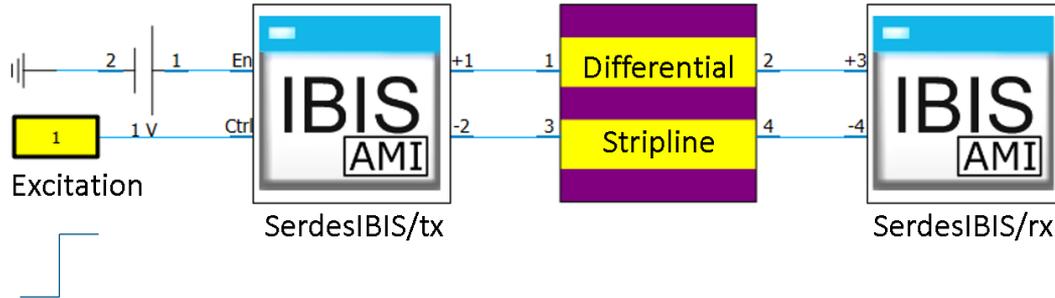
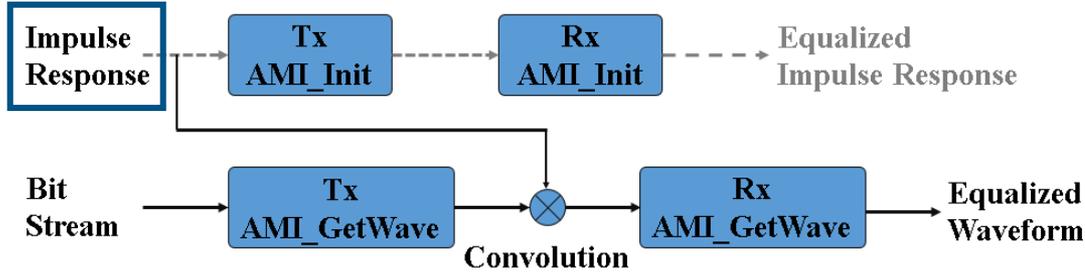
## SerDes Channel [1]



- A SerDes channel consists of an analog part and an algorithmic part.
- The analog part contains the passive channel (e.g. a differential pair) and analog I/O buffer models, which are modeled by the .ibs file [1].
- The algorithmic part contains the functions implemented in the .dll files [1].
- The .ami files are in ASCII format and readable. They are used to provide information about the encrypted functions in the .dll files [1].

# 1. IBIS-AMI TIME-DOMAIN WORKFLOW (2/3)

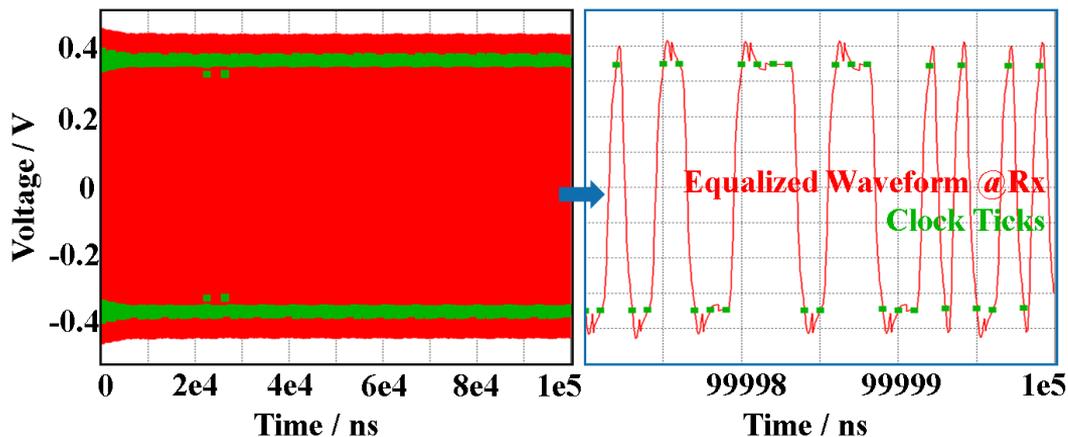
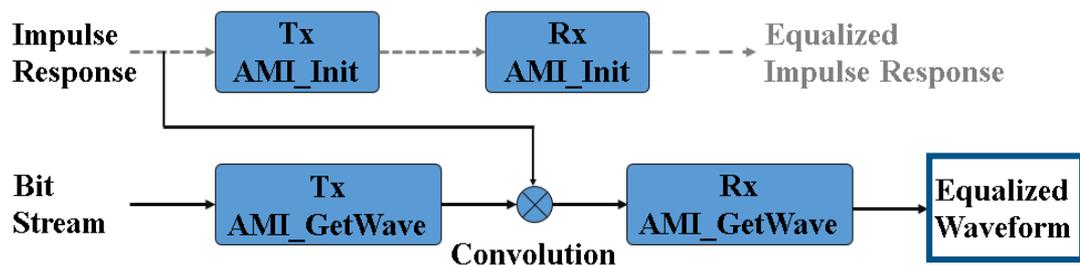
Impulse Response [3] [4]



- There could exist 4 functions in the .dll files, as shown in the upper left figure.
- The AMI\_Init functions are not directly involved in the time-domain workflow for a dual-dual combination, as shown in the left figure [3].
- Impulse response of the analog channel can be obtained by taking derivative of the analog channel's step response, which is the received signal at Rx with a step excitation as shown left [4].

# 1. IBIS-AMI TIME-DOMAIN WORKFLOW (3/3)

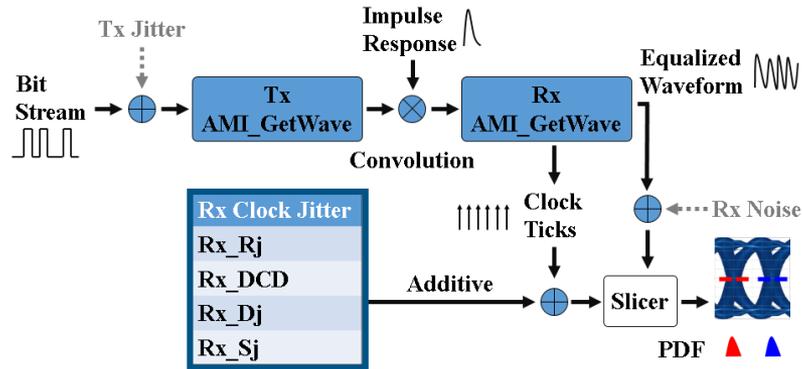
Equalized Waveform [3] [4]



- Rx AMI\_GetWave function returns the equalized waveform sample by sample [1] [3].
- Besides the waveform, clock ticks are also returned, as shown in the left figure [4].
- The bit time or UI is 0.1 ns.
- 1e6 bits are simulated.

## 2. JITTER AND NOISE IN IBIS-AMI (1/4)

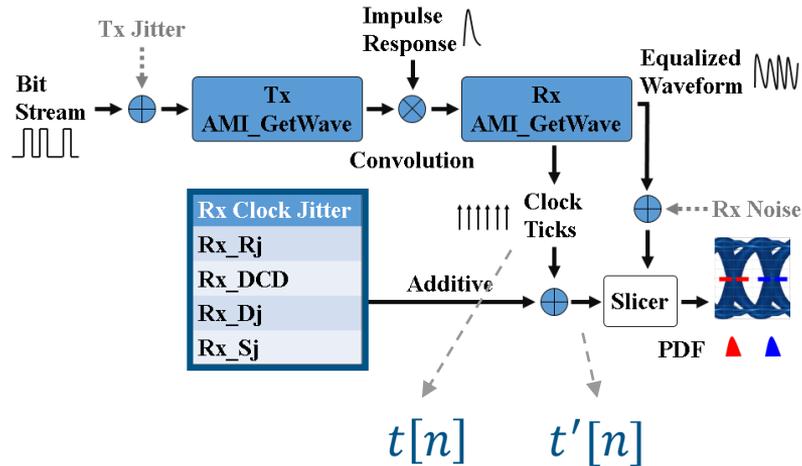
Jitter and Noise in Time-Domain Flow [1]



- IBIS-AMI workflow including jitter and noise is presented left [1].
- Only Rx clock jitter is considered in the work.
- With the modified clock ticks by the Rx clock jitter and equalized waveform, a statistical eye diagram can be generated.
- Jitter PDF can be extracted from the eye diagram.

## 2. JITTER AND NOISE IN IBIS-AMI (2/4)

### Jitter and Noise in Time-Domain Flow [1]



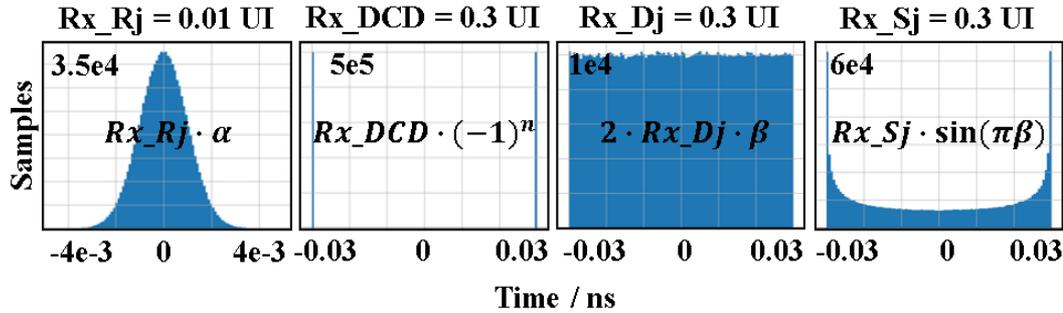
$$t'[n] = t[n] + Rx\_Rj \cdot \alpha + 2 \cdot Rx\_Dj \cdot \beta + Rx\_DCD \cdot (-1)^n + Rx\_Sj \cdot \sin(\pi\beta)$$

- $t[n]$  is the time of the n-th clock tick returned by the Rx AMI\_GetWave function;
- $t'[n]$  stands for the n-th actual clock time including all the jitter effects;
- $\alpha$  is a Gaussian random variable centered at 0.0 with a standard deviation of 1.0;
- $\beta$  is a random variable with white uniform distribution in the range of [-0.5, 0.5];
- $Rx\_Rj$  is the standard deviation of a Gaussian phase noise;
- $Rx\_Dj$  is the worst case half peak to peak variation of the recovered clock;
- $Rx\_DCD$  is the half of the peak to peak variation of a clock duty cycle distortion;
- $Rx\_Sj$  is the half of the peak to peak variation of a sinusoidal phase noise [1].

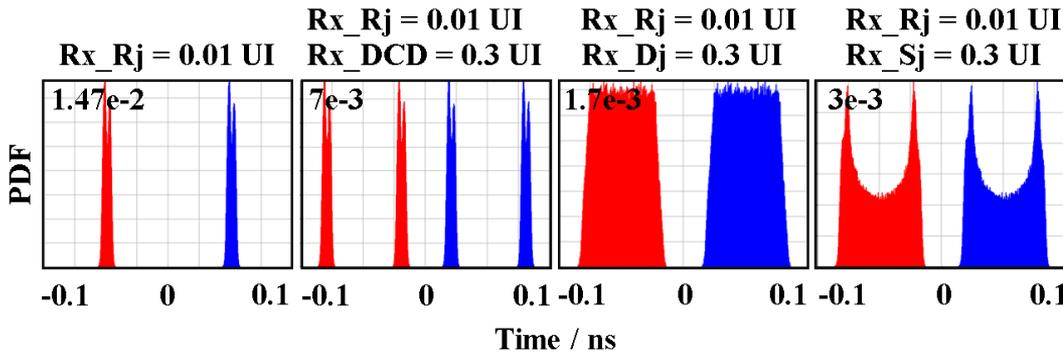
## 2. JITTER AND NOISE IN IBIS-AMI (3/4)

Jitter Profiles [5]

### Histograms



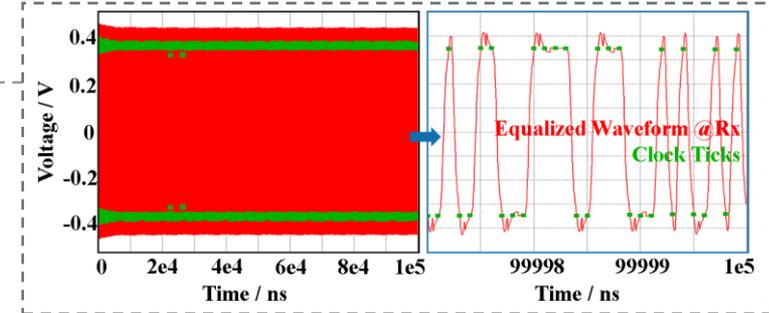
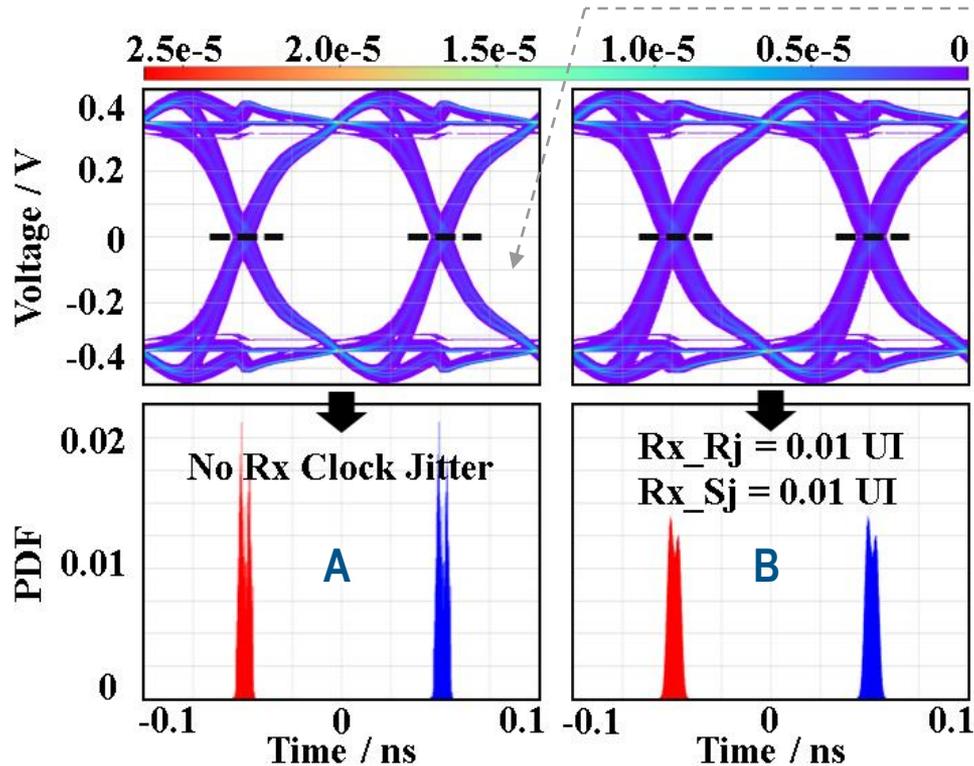
### Jitter PDF



- Histograms of individual jitters.
- $1e6$  samples and 100 bins are used in the jitter histograms for illustration purpose.
- Individual effects of the Rx clock jitters can be observed on jitter PDF extracted from the statistical eye diagram.
- 2000 x-bins and 1000 y-bins are used in the jitter PDF.

## 2. JITTER AND NOISE IN IBIS-AMI (4/4)

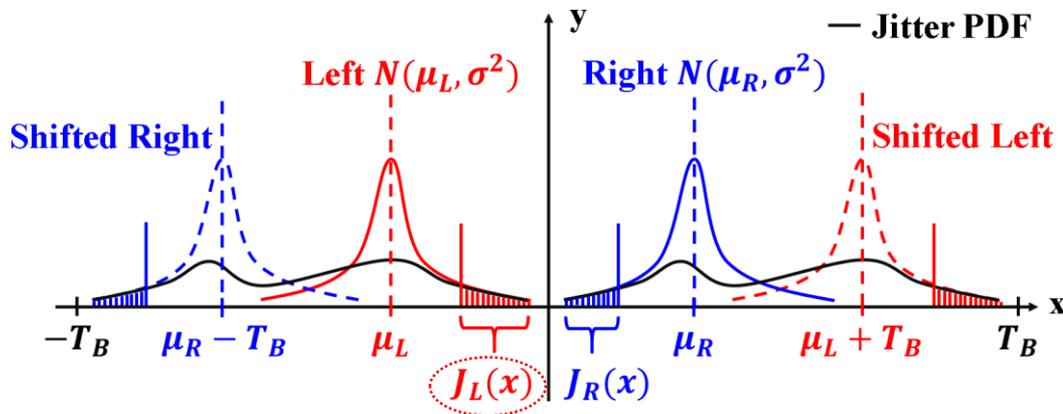
Eye Diagram and Extracted Jitter PDF [4] [5]



- Comparison of A and B shows the effects of the Rx clock jitter on the eye diagram and jitter PDF.
- **Example A** is created with the waveform of the previous slide.
- **Example B** is used for results' validation in the following slides.

# 3. BATHTUB EXTRAPOLATION (1/7)

Dual-Dirac Model [2]



Dual-Dirac model assumes that RJ dominates in the outer edge of jitter PDF, and follows a Gaussian distribution [2].

- In dashed areas, the jitter PDF is assumed to follow a normal distribution [2]:

$$J_L(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x - \mu_L)^2}{2\sigma^2}\right)$$

- Then bit error rate can be derived:

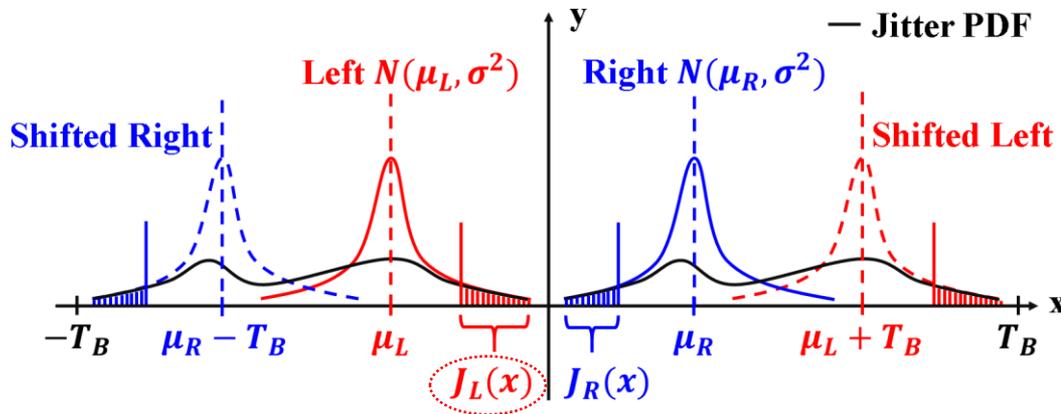
$$\begin{aligned} BER_L(x) &= \rho_T \cdot \int_{-\infty}^{+\infty} J_L(x') dx' \\ &= \rho_T \cdot \int_{\frac{x - \mu_L}{\sqrt{2}\sigma}}^{+\infty} \frac{1}{\sqrt{\pi}} \exp(-t^2) dt \end{aligned}$$

- If we let:  $Q_L(x) = \frac{x - \mu_L}{\sigma}$ ,
- Substitute into the equation above and obtain:

$$BER_L(Q_L) = \rho_T \cdot \int_{\frac{Q_L}{\sqrt{2}}}^{+\infty} \frac{1}{\sqrt{\pi}} \exp(-t^2) dt$$

# 3. BATHTUB EXTRAPOLATION (2/7)

Dual-Dirac Model [2]



Dual-Dirac model assumes that RJ dominates in the outer edge of jitter PDF, and follows a Gaussian distribution [2].

- The complementary error function  $\text{erfc}(x)$  and error function  $\text{erf}(x)$  are defined as:

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{+\infty} \exp(-t^2) dt$$

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2) dt = 1 - \text{erfc}(x)$$

- So  $BER_L(Q_L)$  can be rewritten as:

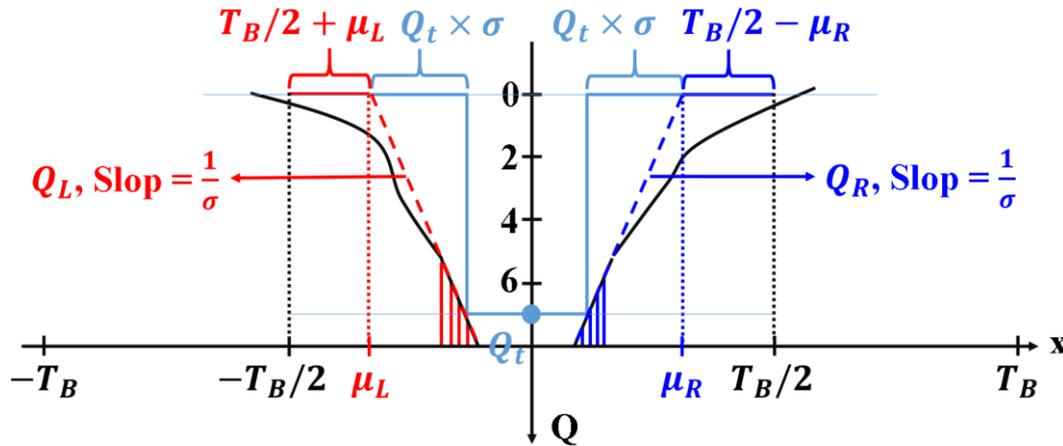
$$\begin{aligned} BER_L(Q_L) &= \frac{\rho_T}{2} \cdot \text{erfc}\left(\frac{Q_L}{\sqrt{2}}\right) \\ &= \frac{\rho_T}{2} \cdot \left(1 - \text{erf}\left(\frac{Q_L}{\sqrt{2}}\right)\right) \end{aligned}$$

- Similar results can be derived for  $J_R(x)$ , hence we remove the subscript of Q and obtain in general:

$$Q(BER) = \sqrt{2} \cdot \text{erf}^{-1}\left(1 - \frac{2BER}{\rho_T}\right)$$

# 3. BATHTUB EXTRAPOLATION (3/7)

Q Function [2]



- Even though the Q function is derived for the dashed area, it can be applied to the full region of an arbitrary jitter PDF [2]:

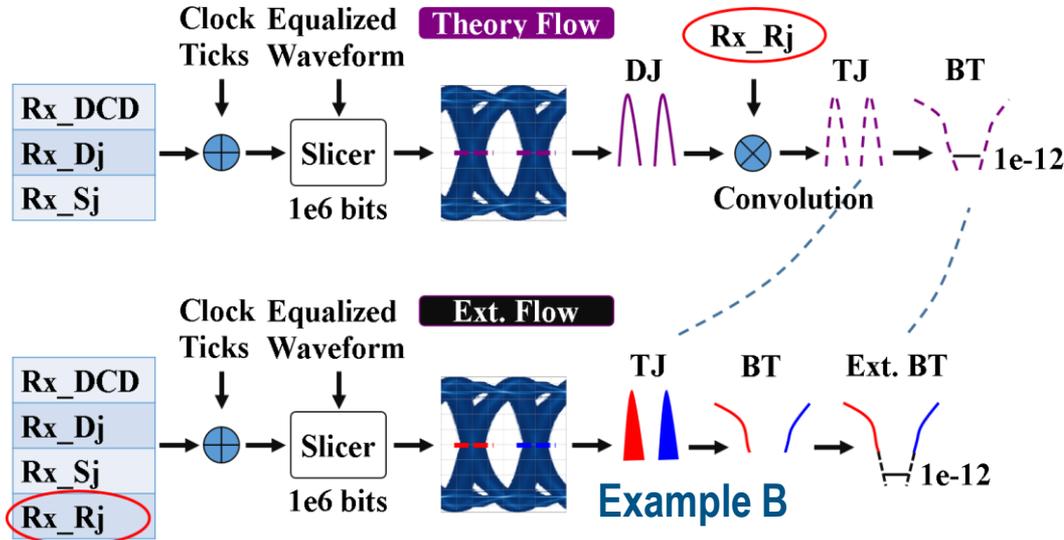
$$Q(BER) = \sqrt{2} \cdot \text{erf}^{-1} \left( 1 - \frac{2BER}{\rho_T} \right)$$

- If RJ dominates and follows normal distribution for a specific region, the Q function will be a linear function.
- The total jitter at the target BER is:  
 $T_J(BER_t) = 2Q_t \times \sigma + \mu_L - \mu_R + T_B$

DJ: Deterministic Jitter  
RJ: Random Jitter  
Ext.: Extrapolation

# 3. BATHTUB EXTRAPOLATION (4/7)

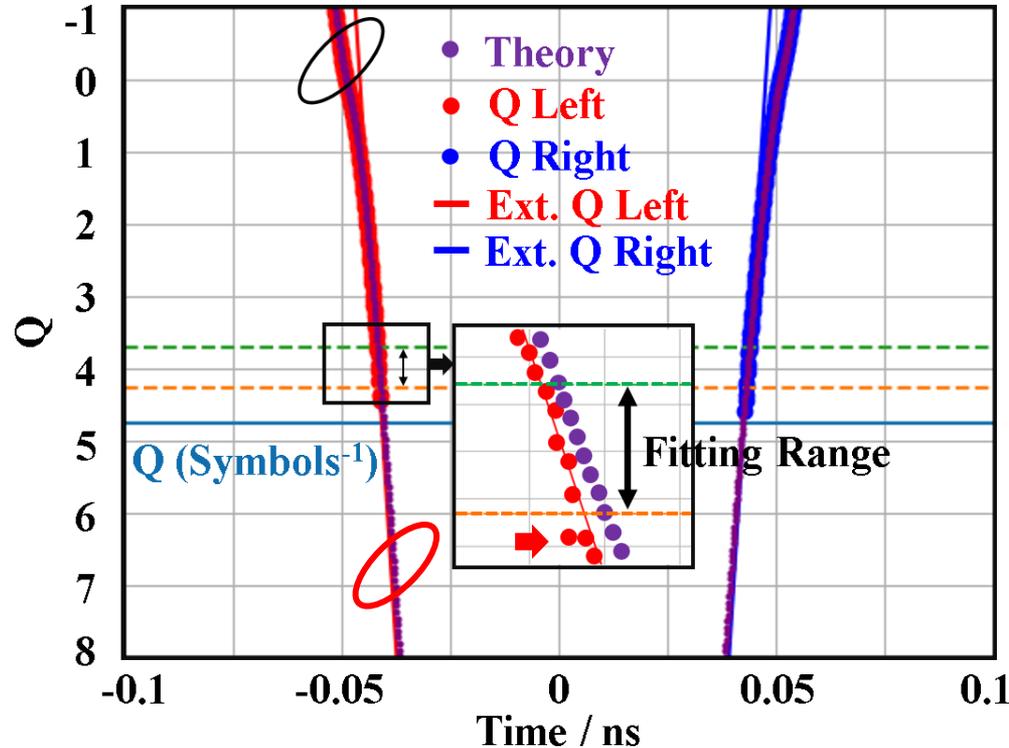
## Theory and Extrapolation Flow



- The only difference between both flows is how to apply the Rx\_Rj of random jitter.
- In the Theory Flow, the total jitter @ 1e-12 is obtained by convolving DJ and RJ.
- In the Ext. Flow, the Rx\_Rj is applied to the clock ticks in time-domain. The total jitter @ 1e-12 is obtained by extrapolating the bathtub.

### 3. BATHTUB EXTRAPOLATION (5/7)

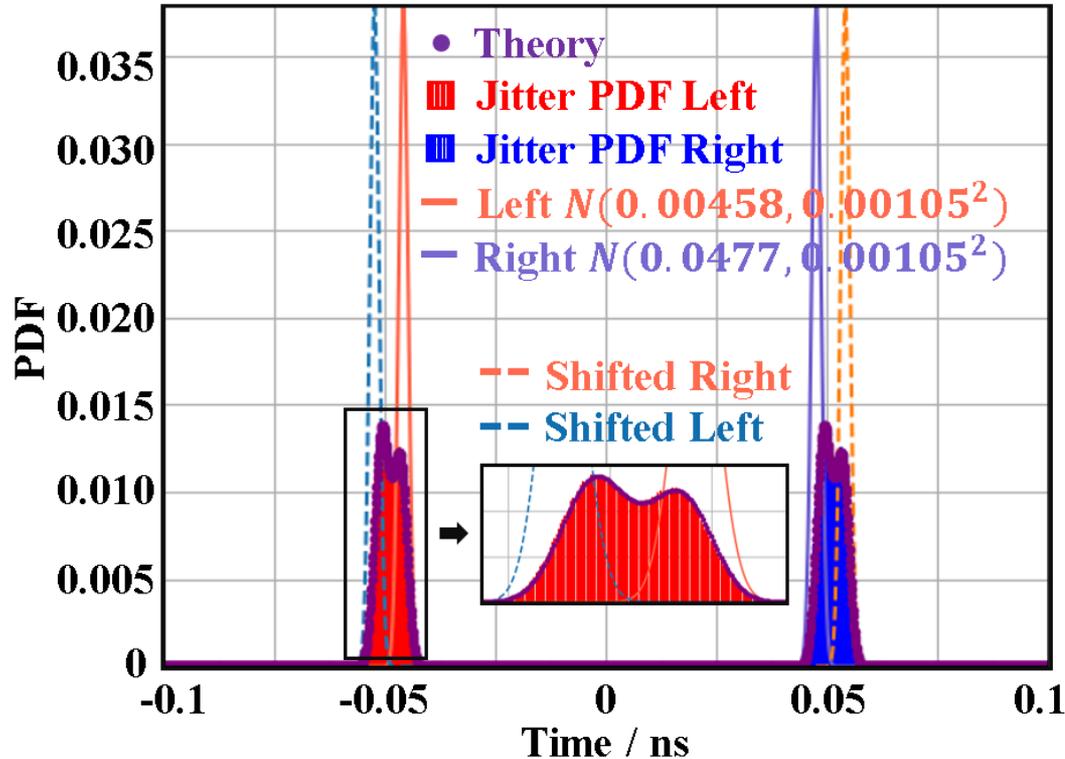
Comparison: Q [2] [5]



- Dual-Dirac model is NOT supposed to match the overall jitter PDF (see the black circle), but should only match around the outer edge (see the red circle) [2].
- The fitting range is critical. If it's too low (large Q, small BER), the samples have significant noise (see the red arrow). If too high, it's close to DJ, where RJ doesn't dominate and the assumption of Dual-Dirac model is violated.

### 3. BATHTUB EXTRAPOLATION (6/7)

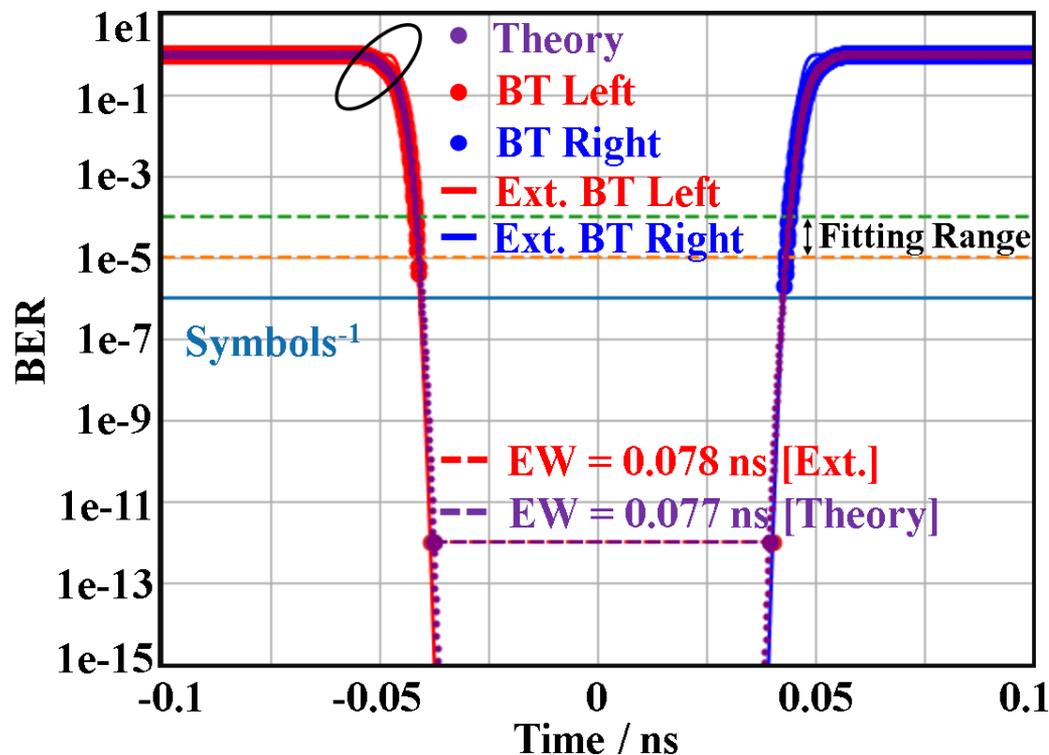
Comparison: PDF [5]



- The jitter PDF obtained in both flows match around the middle region of the jitter, as expected. This can be also observed in the previous Q function.
- The extrapolated standard deviation is 0.00105, which is very close to the theory 0.001.

### 3. BATHTUB EXTRAPOLATION (7/7)

Comparison: BT [5]



- In this work, the fitting range is chosen as  $[10, 100] * \text{symbols}^{-1}$  (the number of symbols in time domain simulation)
- Symbols =  $1e6$ , so the fitting range is  $[1e-5, 1e-4]$
- The extrapolated eye width is  $0.077 \text{ ns}$ , which is very close to the theory  $0.078 \text{ ns}$ .

## 4. SUMMARY

- IBIS-AMI supports parameters for jitter and noise. The model creators can use them to specify individual jitters, which have different histograms and mechanisms [1].
- The extrapolated results of 1e6 bits (e.g. the EW @1e-12) match the theory, which calculates the convolution of DJ and RJ.
- The fitting range is critical to choose. It can be neither too high nor too low [2].
- For large DJ, the fitting range should be as low as possible, so that RJ dominates. However it requires more simulated symbols and therefore longer simulation time.

# REFERENCES

- [1] IBIS Open Forum, <https://ibis.org/>.
- [2] R. Stephens, "Jitter Analysis: The dual-Dirac Model, RJ/DJ, and Q-Scale", Agilent Technologies, 2004.
- [3] D. Burns, J. Madsen, T. Westerhoff and W. Katz, "Understanding IBIS-AMI simulation", *DesignCon 2015*.
- [4] Dassault Systèmes, <https://www.3ds.com/>.
- [5] J. D. Hunter, "Matplotlib: A 2D Graphics Environment", *Computing in Science & Engineering*, vol. 9, no. 3, pp. 90-95, 2007.

# Thanks!