

Modelling of Ground-Noise for Circuits with Short-Channel Transistors

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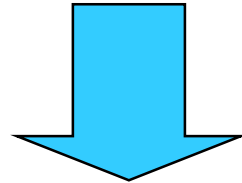


Overview

- **Motivation**
- **Definition of Ground-Noise**
- **Modelling of Ground-Noise**
 - **Ground-Noise model**
 - **Parameter determination**
- **Numerical example**
- **Summary**

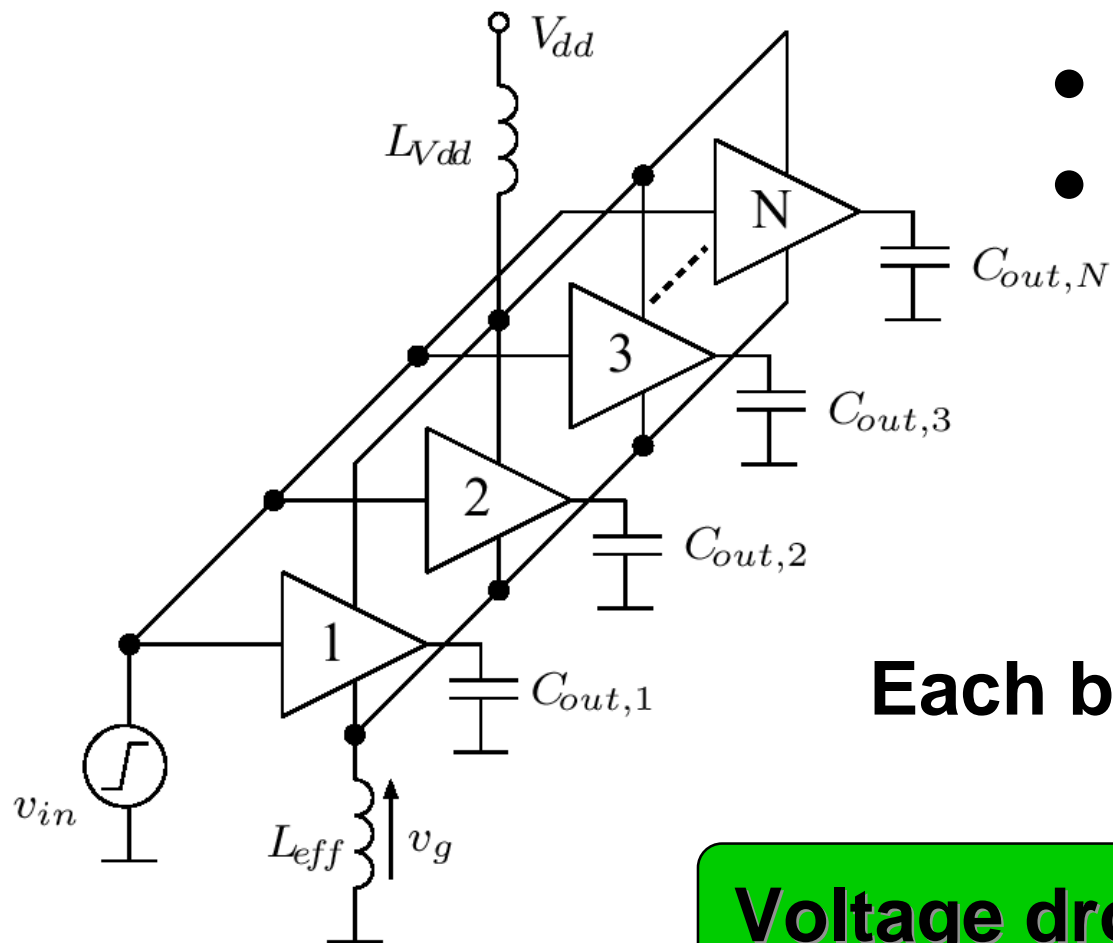
Motivation: **Problems and their solutions**

- Expensive measurements on finished product
- Time-consuming simulations of full circuit
- Overlap of different effects

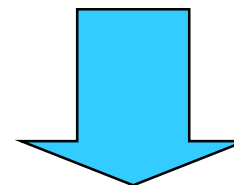


- Partitioning of circuit into functional blocks
- Description of single effect
- Error-free circuit function in consideration of one effect
- High-Level Simulations for complete circuit

Definition of Ground-Noise



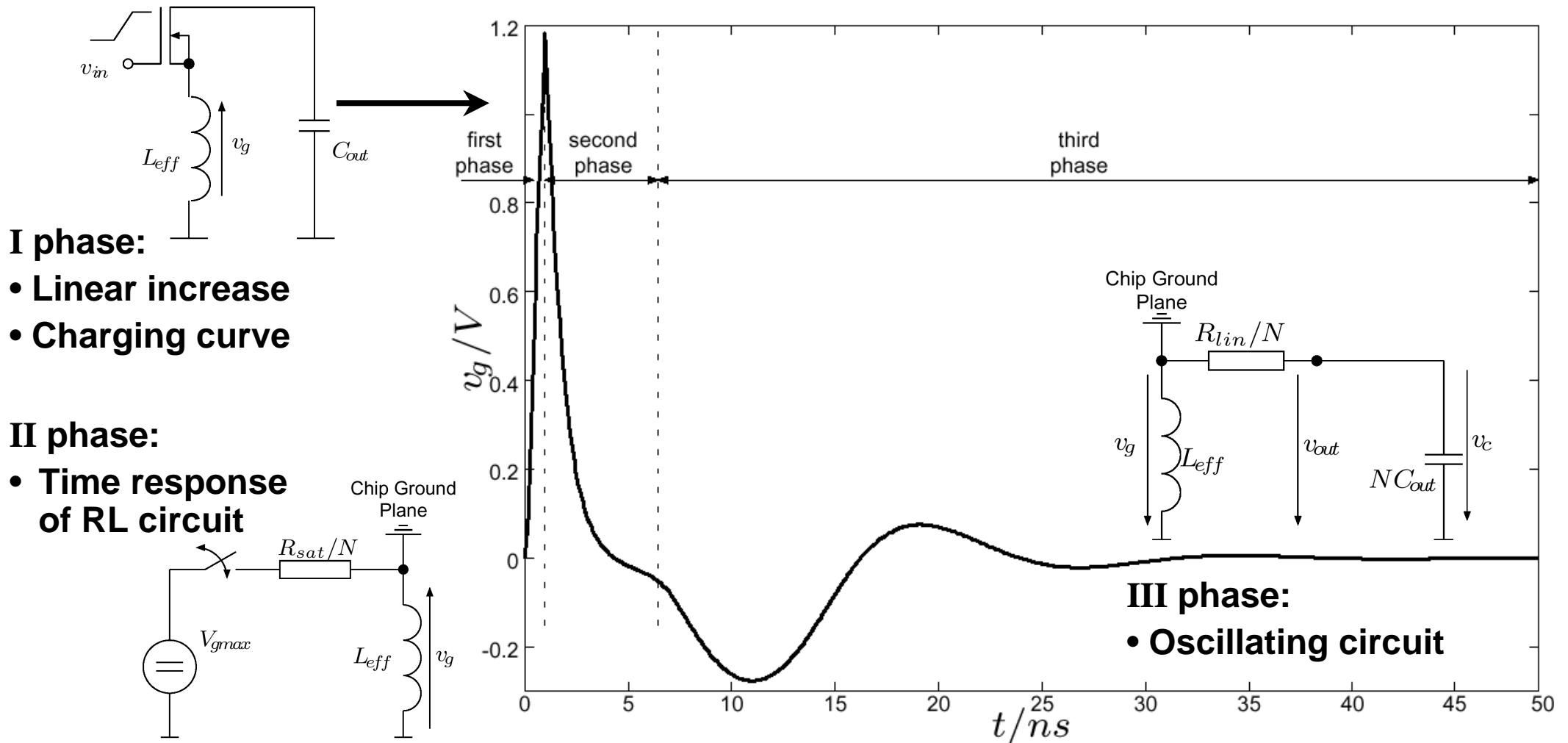
- The *effective inductance*
- N buffers switch simultaneously (High \rightarrow Low)



Each buffer can be simplified to a CMOS inverter

Voltage drop at the *effective inductance*

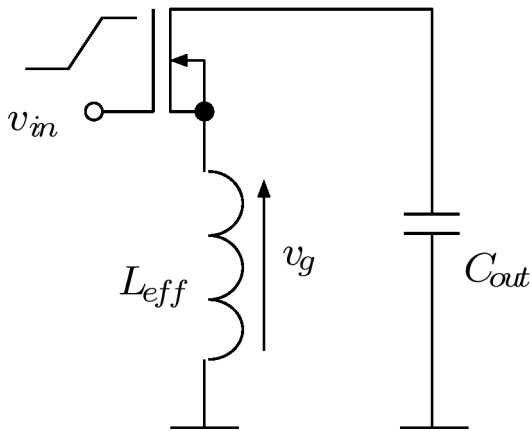
Modelling of Ground-Noise: **Split of the Ground-Noise signal**



Modelling of Ground-Noise: **Analysis of the first phase**

For signals with short rise/fall time:

- the load capacitance is relatively large
- during the transient process the n-channel transistor operates in the saturation area
- current through load capacitance is negligible in comparison with the transistor current
- maximal value of noise originates, when the input voltage originates its plateau
- noise voltage increases linear



Starting point:

$$v_g(t) = NL_{eff} \frac{di_d(t)}{dt}$$

BSIM3v3 mobility model for saturation current: $i_d = WC_{ox} v_{sat} \frac{(v_{gs} - V_{th})^2}{E_{sat} L + v_{gs} - V_{th}}$

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Modelling of Ground-Noise: **Analysis of the first phase**

$$V_{gmax} = WC_{ox}v_{sat}NL_{eff} \left(\left(\frac{2(a - V_{gmax})}{b - V_{gmax}} - \frac{(a - V_{gmax})^2}{(b - V_{gmax})^2} \right) \left(\frac{V_{dd}}{t_r} - \frac{V_{gmax}}{t_r - t_d} \right) \right)$$

$$a = V_{dd} - V_{th}$$

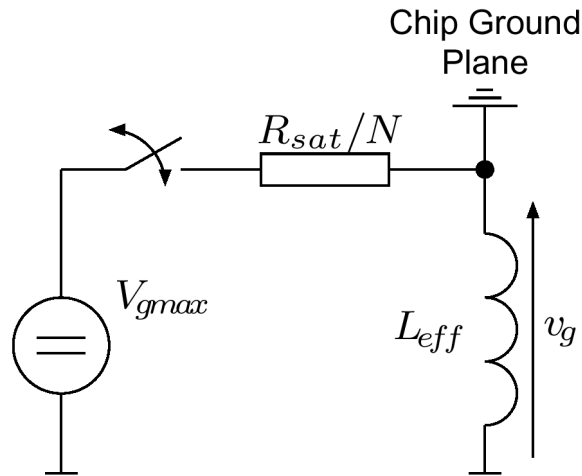
$$b = E_{sat}L + V_{dd} - V_{th}$$

V_{gmax} - noise voltage
 C_{ox} - oxide capacitance
 N - number of buffers
 V_{dd} - power supply voltage
 E_{sat} - electrical field in channel
 t_r - rise time of input voltage

W - channel width
 v_{sat} - saturation velocity of carrier
 L_{eff} - effective inductance
 V_{th} - threshold voltage
 L - channel length
 t_d - delay time



Modelling of Ground-Noise: **Analysis of the second phase**



Mathematical description of the circuit

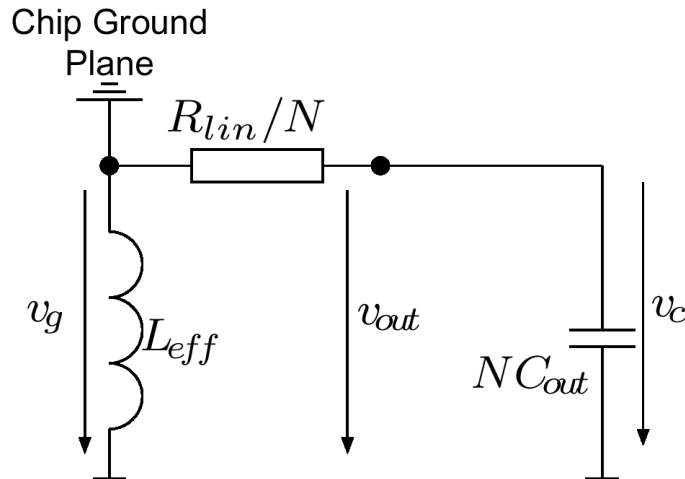
$$v_g(t) = \frac{R_{sat}}{N} i(t) + L_{eff} \frac{di(t)}{dt} = R_s i(t) + L_{eff} \frac{di(t)}{dt}$$

$$i(t) = \frac{V_{gmax}}{R_s} \left(1 - \exp^{-\frac{t}{\tau_L}} \right) \quad \tau_L = \frac{L_{eff}}{R_s}$$

Mathematical description of the voltage at the inductance:

$$v_g(t) = V_{gmax} - R_s i(t) = V_{gmax} \exp^{-\frac{t}{\tau_L}}$$

Modelling of Ground-Noise: **Analysis of the third phase**



Mathematical description of the circuit

$$L_{eff} C_{outtot} \frac{d^2 v_c(t)}{dt^2} + R_l C_{outtot} \frac{dv_c(t)}{dt} + v_c(t) = 0$$

$$C_{outtot} = NC_{out} \quad R_l = \frac{R_{lin}}{N}$$

$$\vartheta = \frac{R_l \sqrt{L_{eff} C_{outtot}}}{2L_{eff}}$$

Three cases for solution of differential equation depending on:

- $\vartheta > 1$: over-damped case - only for some configurations
- $\vartheta = 1$: critically damped case - the most uncommonly case
- $\vartheta < 1$: under-damped case - the most frequently case



Modelling of Ground-Noise: **Analysis of the third phase**

Under-damped case

$$v_g(t) = V_{dsat} \exp^{-\delta t} \cos(\varpi_d t - \Theta)$$

Damping constant:

$$\delta = \frac{R_l}{2L_{eff}}$$

Damping angle:

$$\Theta = \arctan\left(\frac{\delta}{\varpi_d}\right)$$

Eigen angular frequency:

$$\varpi_d = \frac{1}{\sqrt{L_{eff} C_{outtot}}} \sqrt{1 - \vartheta^2}$$



Ground-Noise model

$$v_g(t) = \begin{cases} 0 & \text{for } t < t_d \\ \frac{V_{gmax}}{t_r - t_d} t - \frac{V_{gmax}}{t_r - t_d} t_d & \text{for } t_d \leq t \leq t_r \\ V_{gmax} e^{-\frac{t-t_r-t_x}{\tau_L}} - K & \text{for } t_r < t < t_{23} \\ v_l(t - t_{s3}) & \text{for } t_{23} < t \end{cases}$$

K - voltage shift for the second phase

t_x - time shift for the second phase

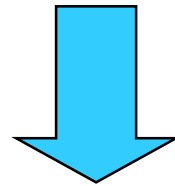
$v_l(t)$ - voltage on the inductance during the third phase

t_{s3} - time shift for the third phase

Parameter determination: **Split of all parameters**

- physical constants,
- transistor parameters, found in the parameter file of a given transistor model,
- circuit parameters, to be read in the schematic,
- parameters, which have to be determined using simulations,

Needed to calculate the maximal value of Ground-Noise due to $\frac{di}{dt}$



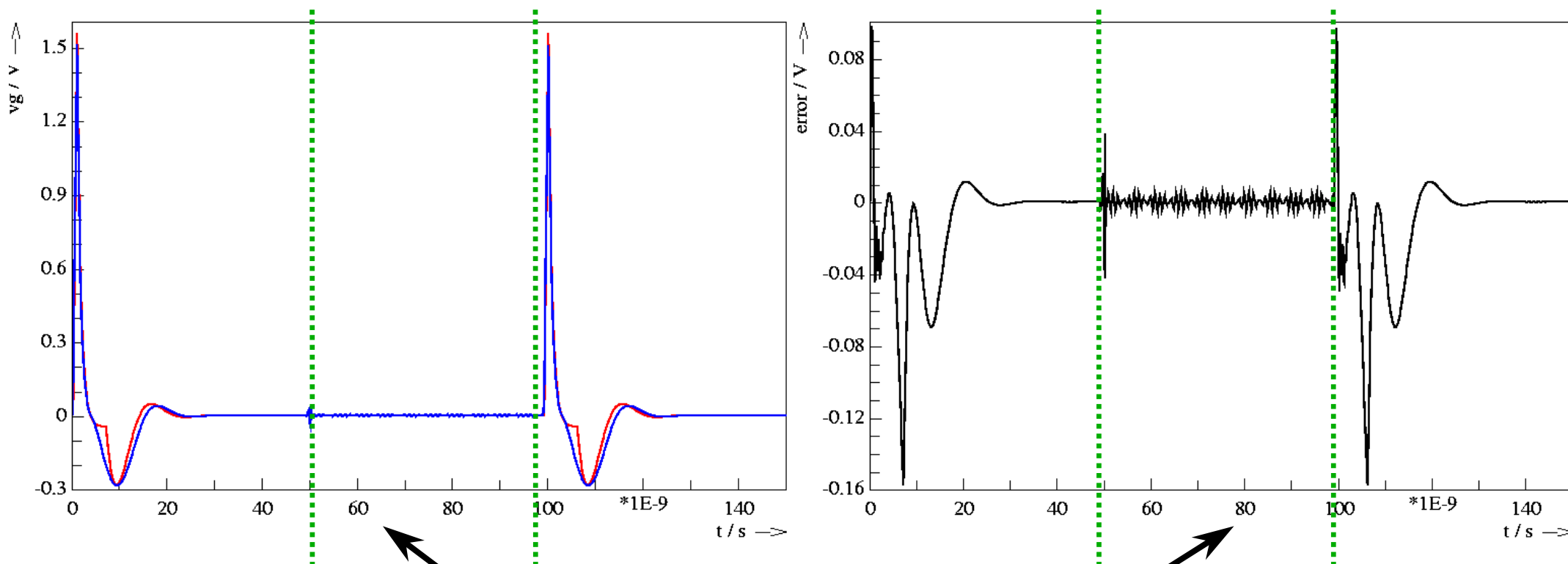
Can be taken from IBIS model

Numerical example I: **Circuit and signal parameters**

Comparison between **simulation** results and **calculation** results

number of buffers	(N)	= 8
frequency of input signal	(f)	= 10 MHz
rise time of input voltage	(t_{r_in})	= 1 ns
power supply voltage	(V_{dd})	= 3.3 V
effective inductance	(L_{eff})	= 10 nH
load capacitance	(C_{out})	= 50 pF
fall time of output voltage	(t_f)	= 6.85 ns

Numerical example I: In the time domain

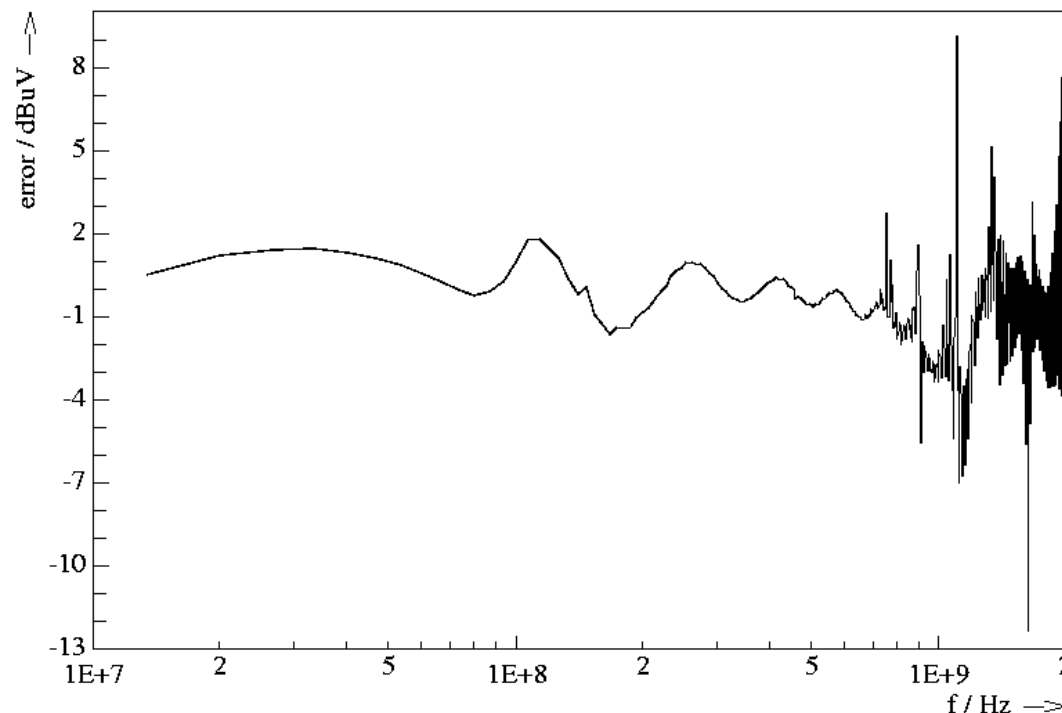
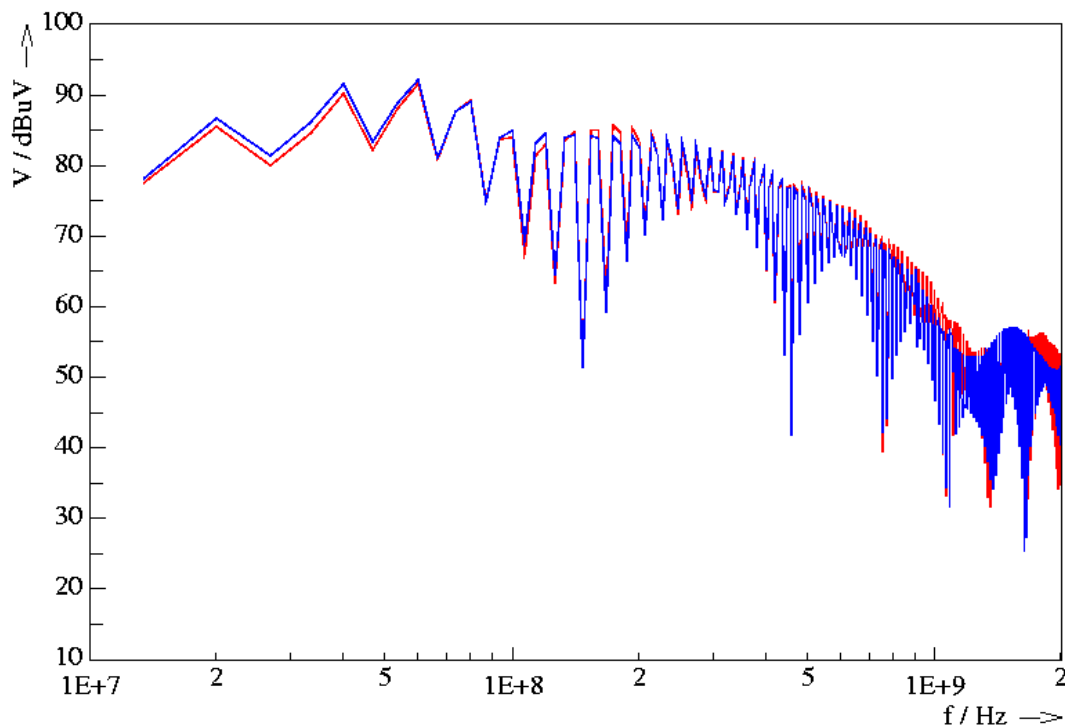


outside of the validity area

— full circuit simulation
— model

$$error = v_{simul} - v_{model}$$

Numerical example I: In the frequency domain



— full circuit simulation
— model

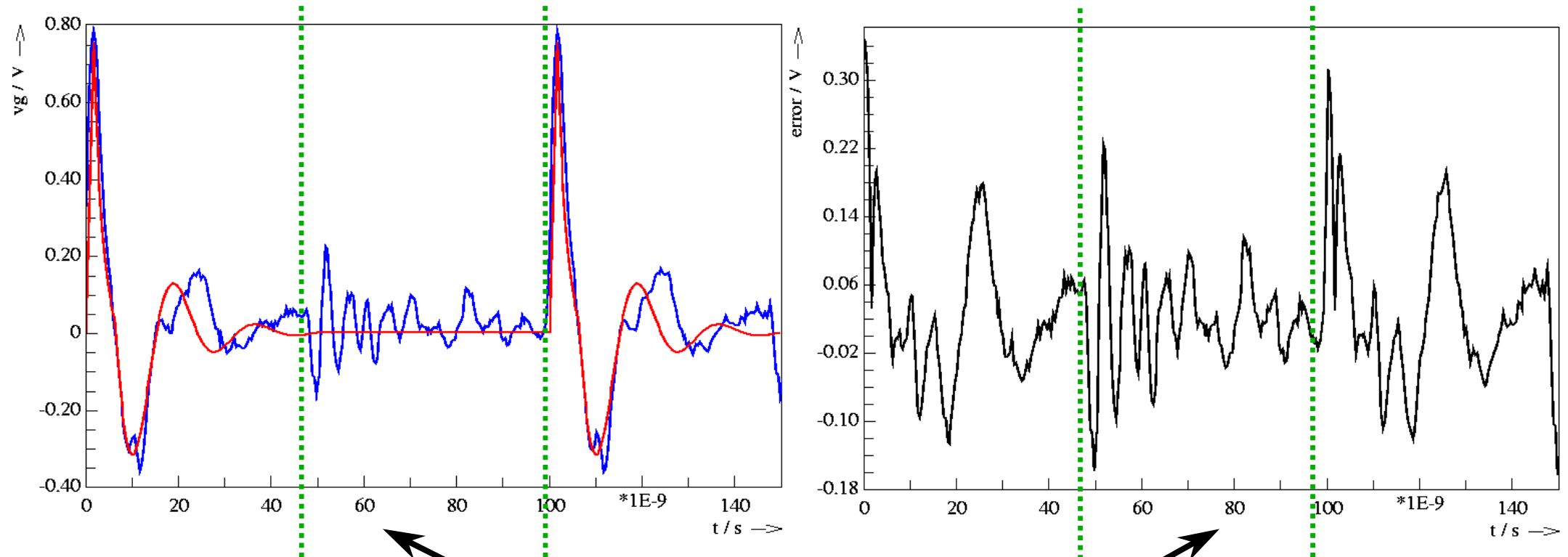
$$error = v_{simul} - v_{model}$$

Numerical example II: **Circuit and signal parameters**

Comparison between **measurements** and **calculation** results

number of buffers	(N)	= 8
frequency of input signal	(f)	= 10 MHz
rise time of input voltage	(t_{r_in})	= 2 ns
power supply voltage	(V_{dd})	= 3.0 V
effective inductance	(L_{eff})	= 18 nH (recalculated)
load capacitance	(C_{out})	= 47 pF
fall time of output voltage	(t_f)	= 5.00 ns

Numerical example II: In the time domain

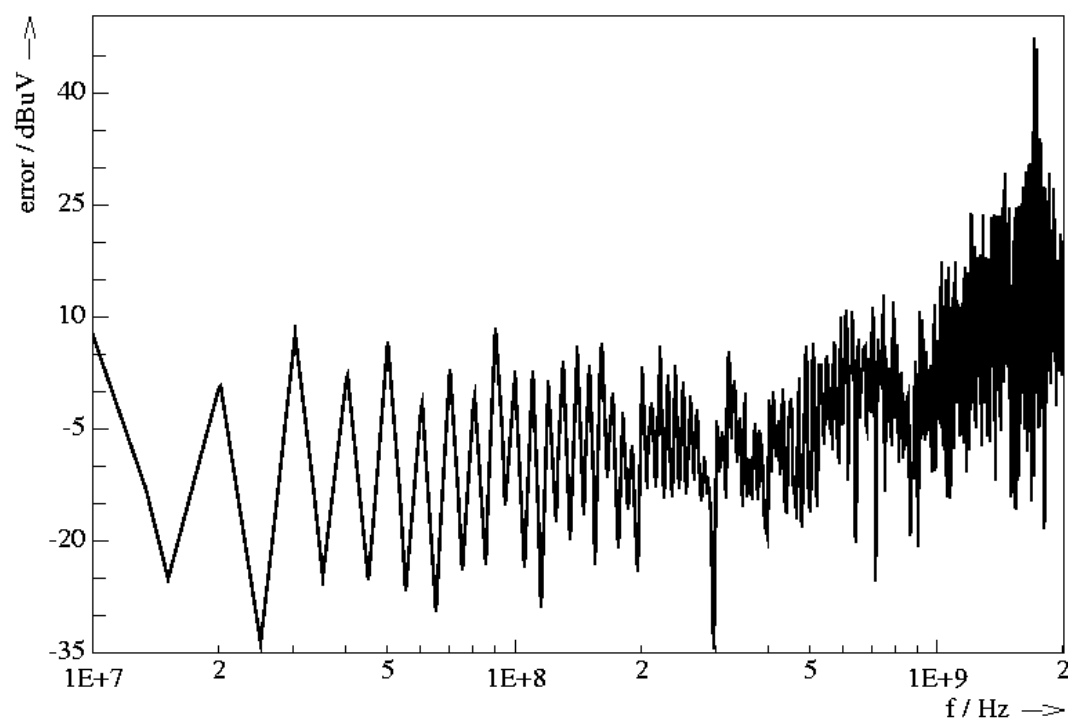
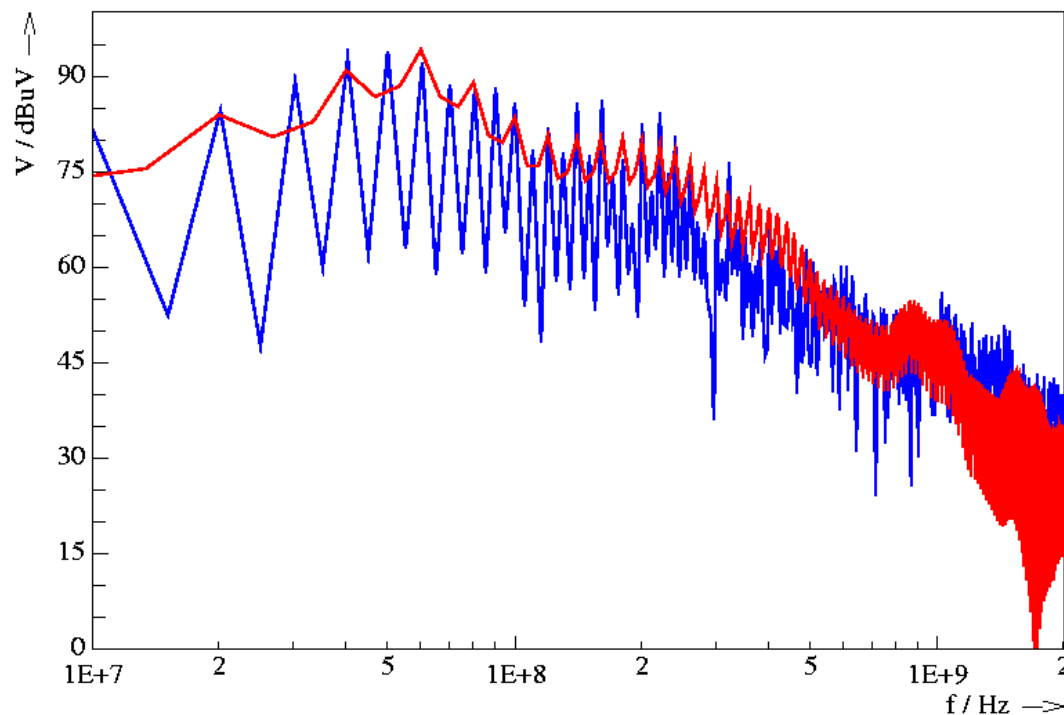


outside of the validity area

— measurements
— model

$$error = v_{measurements} - v_{model}$$

Numerical example II: In the frequency domain



— measurements
— model

$$error = v_{measurements} - v_{model}$$

Summary

- **Motivation**
- **Definition of Ground-Noise**
- **Modelling of Ground-Noise**
- **Determination of parameters**
- **Accuracy of model**
 - **model \leftrightarrow simulations**
 - **model \leftrightarrow measurements**