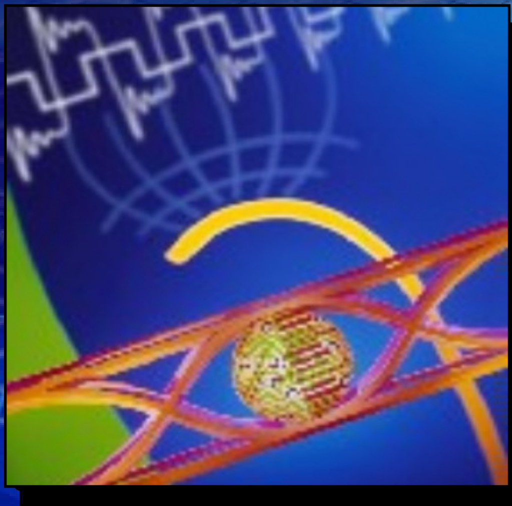
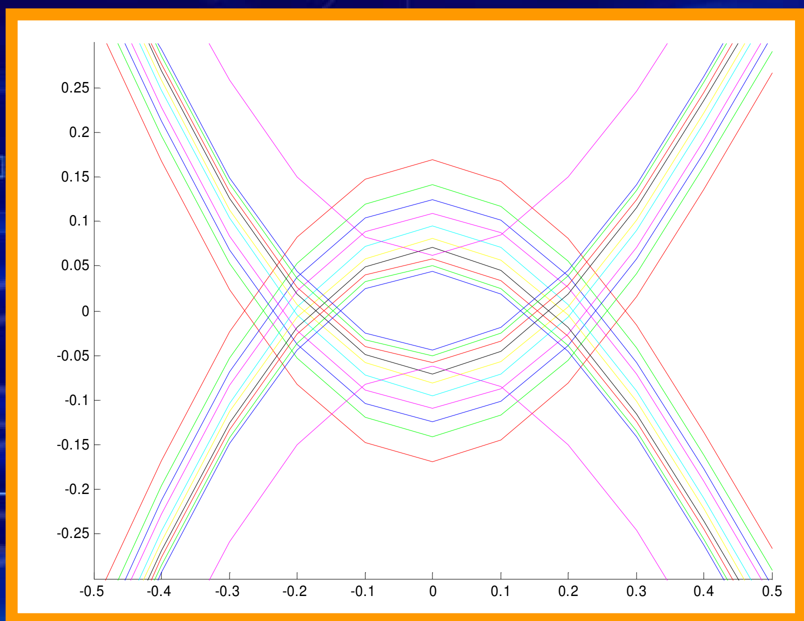
# Predicting BER to Very Low Probabilities

***IBIS Summit, DAC, Anaheim, CA***
***June 15, 2010***







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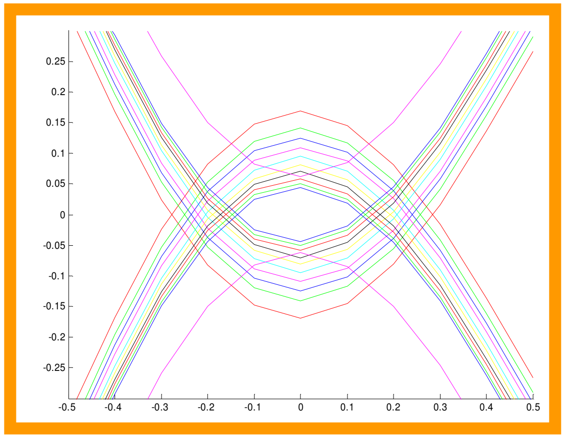


1. **Low BER in perspective**
2. **Peak Distortion Analysis (PDA) and Statistical ISI Analysis primer**
3. **Probability Density Functions (PDF) and extrapolation**
4. **Worst Sequence and BER**

# Putting low BER in perspective

- **Even with the most advanced technologies, we are limited in how long our simulations can be, consequently we are also limited in how low we can go with our BER predictions**

**Example:**

**If a certain 50 bit long sequence causes an error, the random sequence in which that sequence appears once must be at least $2^{50}$ bits long. This corresponds to BER = 1e-15. If we could simulate a million bits in one second(!), this would take ~35 years to run…**

**Crosstalk, noise and jitter adds significant complexity.**

- **The lowest achievable BER in a reasonable time is around 1e-12**

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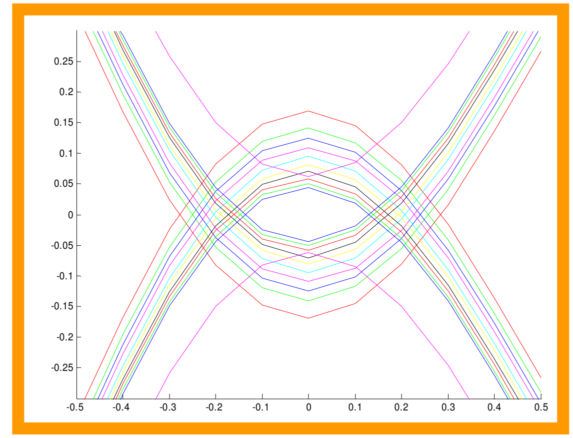


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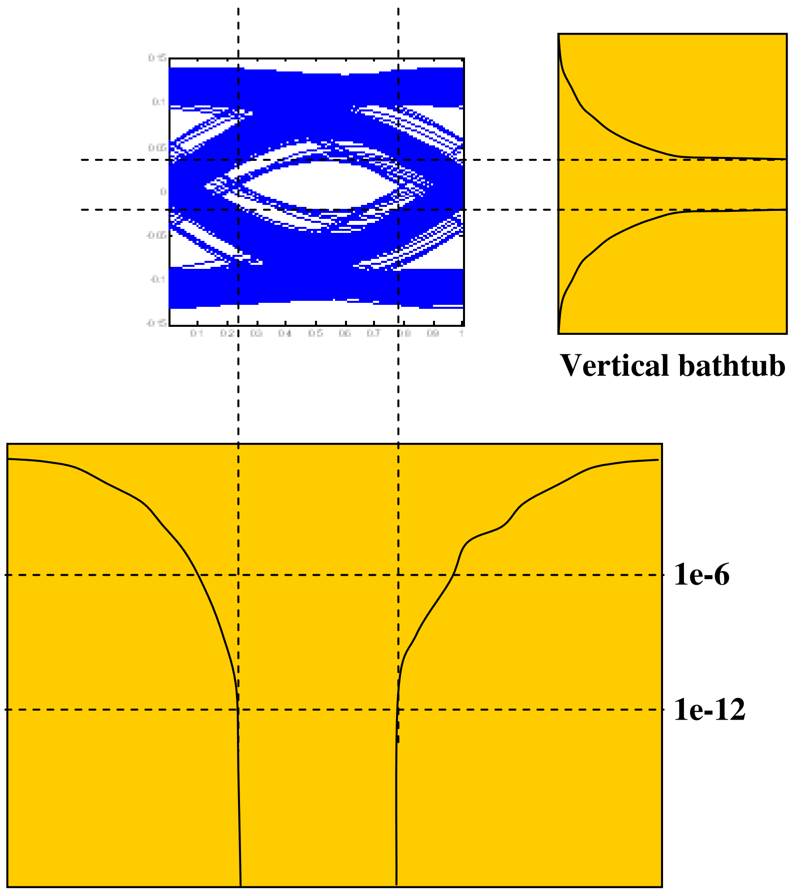
# PDA and Statistical ISI analysis primer

- **Peak Distortion Analysis (PDA) is used to find the worst case eye and/or the worst case bit sequence analytically**
- **Statistical ISI analysis is used to generate statistical eye diagrams from which Probability Density Functions (PDF) and Bit Error Rates (BER) can be obtained**
- **For more information see:**

**http://web.engr.oregonstate.edu/~pchiang/classes/ece679/osu_pres_April2007_Frank.ppt**

- **Let's first consider PDF-s which come from ISI and crosstalk alone without including uncorrelated or random jitter (those can be discussed another time…)**

# ISI and crosstalk PDF define jitter-less BER




Vertical bathtub

Horizontal bathtub

**Without random jitter, the smallest opening of the vertical or horizontal bathtub curve is defined by the worst case ISI effects (including encoding, DCD, asymmetry, crosstalk, etc…)**

**By reversed reasoning we may ask the question: Is it possible to calculate the lowest BER without knowing the worst case solution which defines the BER margins?**

**What is the probability where the BER walls become vertical? *Depending on ISI/crosstalk, this could be anywhere, 1e-6 or 1e-20***

**Can we describe the slopes of the bathtub curve by Gaussian or any other regular or extrapolatable dependence or function? *NO!***

**Can we do that by adding random jitter to the same model? *NO!***

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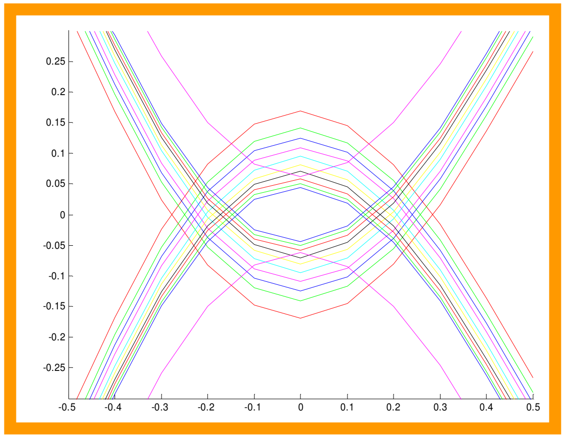


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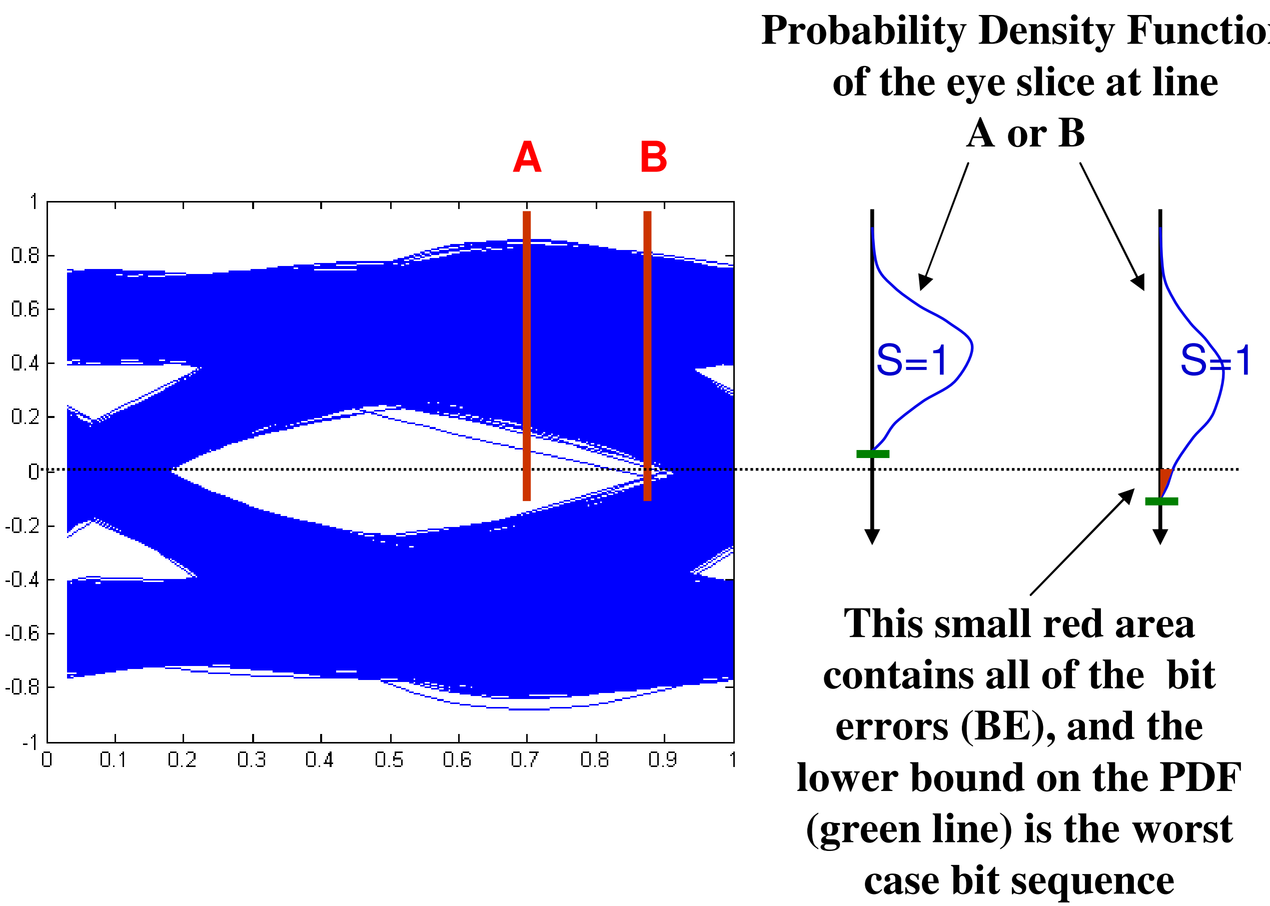
# The importance of the PDF tails

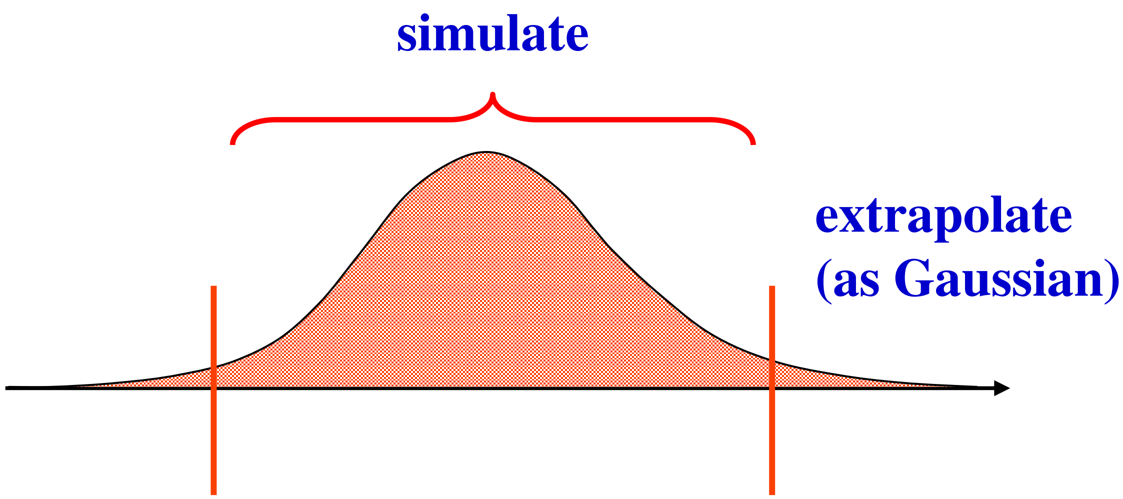



From here we can see how important it is to generate an accurate tail for the PDF.

The lower bound of the PDF is determined by the worst bit patterns. Without knowing this bound, an accurate BER estimation is not possible.

# How can we find the tails of the PDF-s?

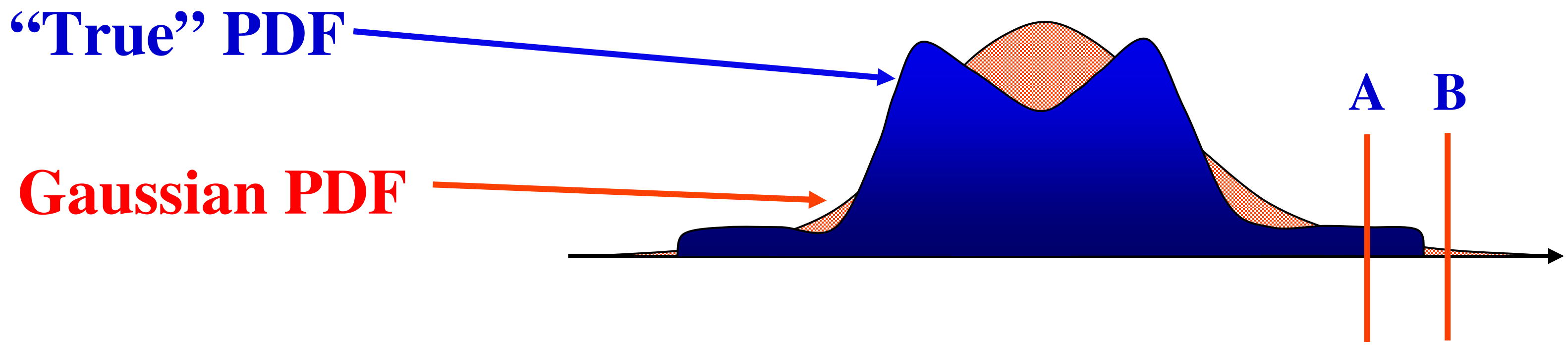
- **If we can afford to wait decades for the results, we could run long simulations**
- **We might try to apply extrapolation techniques, but *only if we know the type of the distribution*; otherwise the extrapolation could be misleading**



# What does a true PDF look like?

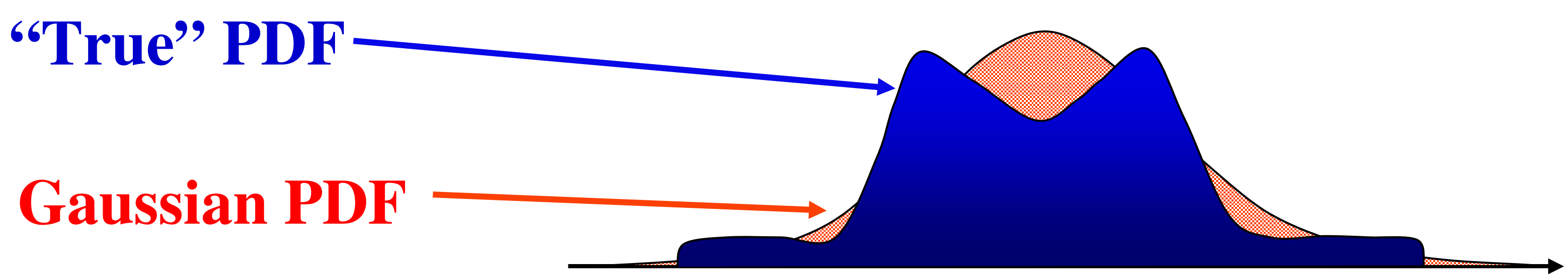
- **Depending on distribution type, two PDF-s with the same mean value and mean deviations may produce both under- (A) and over-estimation (B) of the BER**

  **The central limit Theorem states that the sum of many independent and identically distributed random variables tends to have Gaussian distribution. However, the cursors typically have very different magnitudes therefore the associated values are not identically distributed and the assumption of a Gaussian distribution may not be valid.**



# When does extrapolation work?

- In order to get a reasonably good prediction from an extrapolation, we need to know the type of the PDF
- Unfortunately, this information is most often not available to us
- This is why extrapolations based on an assumed PDF may not yield meaningful results



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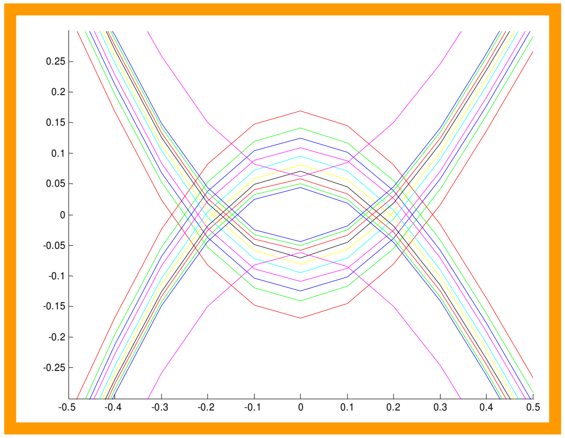


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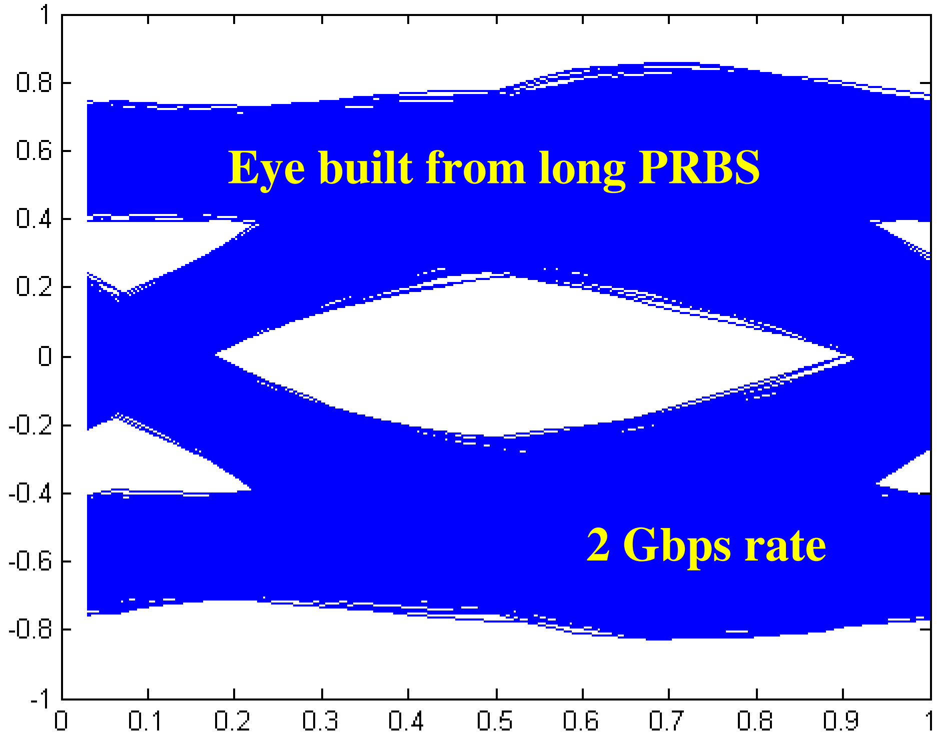
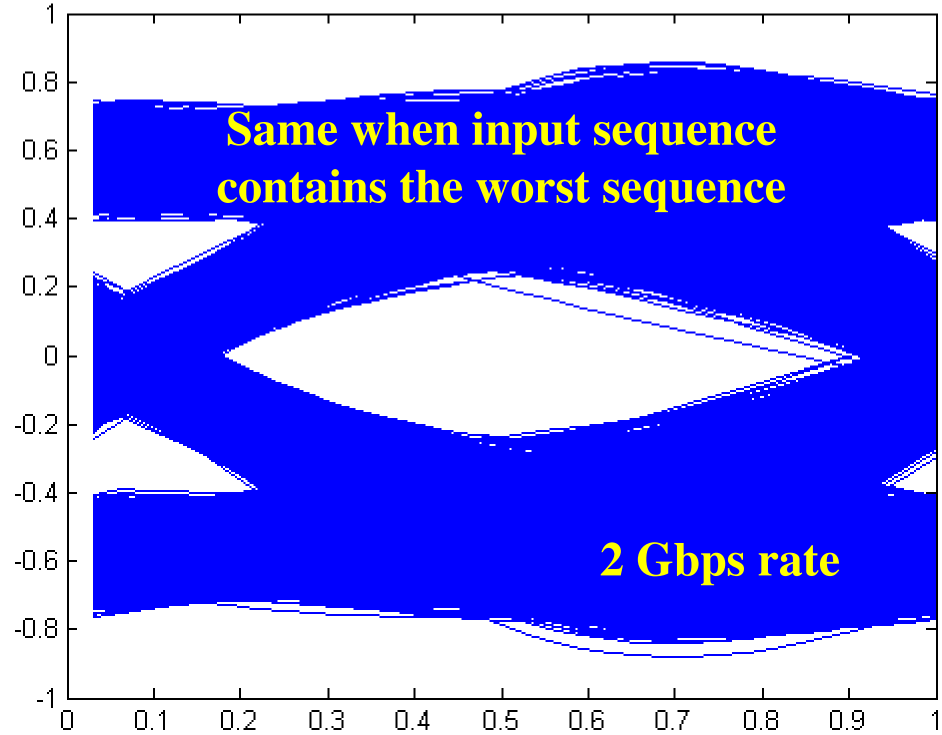
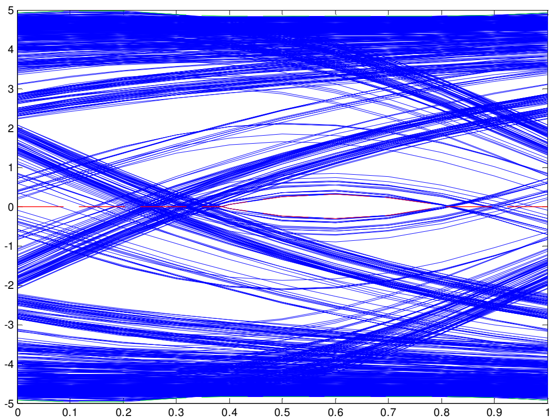
# How about Worst Sequence (WS)?

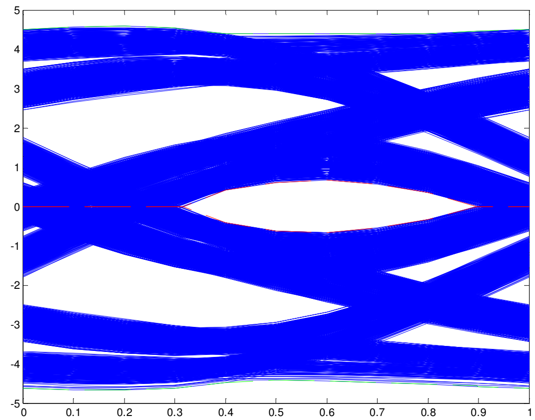
- The WS is unique for any combination of channel and pulse width
- If the WS is 50 bits long, the probability to find the WS (and worst eye) by applying a long random sequence is 1 in $2^{50}$ bits (or less than 1 in 1e+15)
- With analytical algorithms, it is possible to find a WS ***directly*** that generates the worst eye for a given channel and pulse width in a fraction of a second
- We can also find the WS directly for encoded data sequences, such as 8b10b

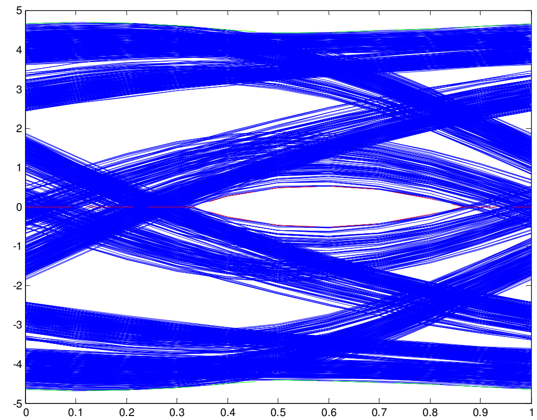


# Encoded data and WS prediction




**Unconstrained worst case sequence results in a closed eye (*pessimistic*)**

**A 100k-bit long *random* 8b10b input sequence results in an eye that is better than reality (*optimistic*)**

**A 400-bit long *worst* 8b10b input sequence yields a *realistic* worst case eye while complying with the protocol**

# Worst case solution and BER measure

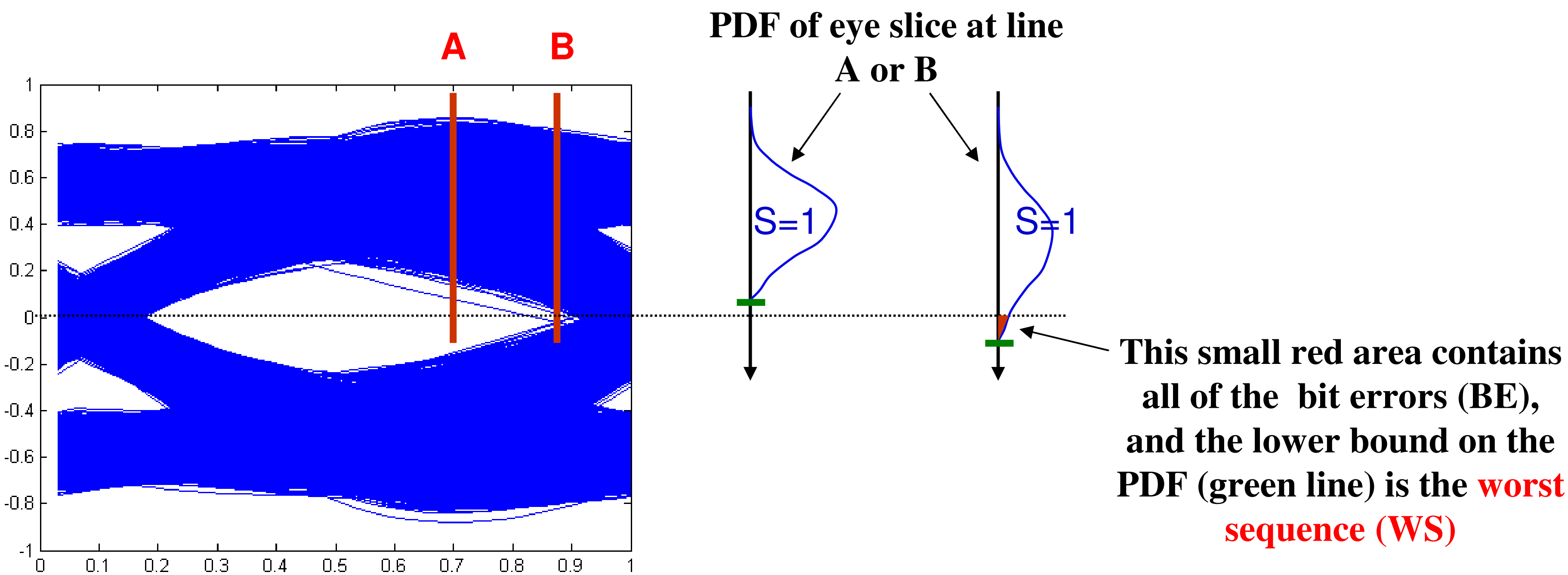
- **Typical objection against using worst case eye/BER is this:**

  **Worst case does not provide probability measure (or "statistical relief"), hence may produce over-pessimistic eye. "What we really need is BER, because the worst case solution may never happen in real life."**

- **However, this is not true. When forming a stressed eye, we can easily adjust the "probability depth" by considering a selected number of the major contributors and not allow the 'worst' solution go beyond a certain level of interest (e.g. 1e-15).**

# How does the WS help finding low BER?

- Remember, the end of the tail is defined by the WS, and
- The bit errors are all near the end of the tail

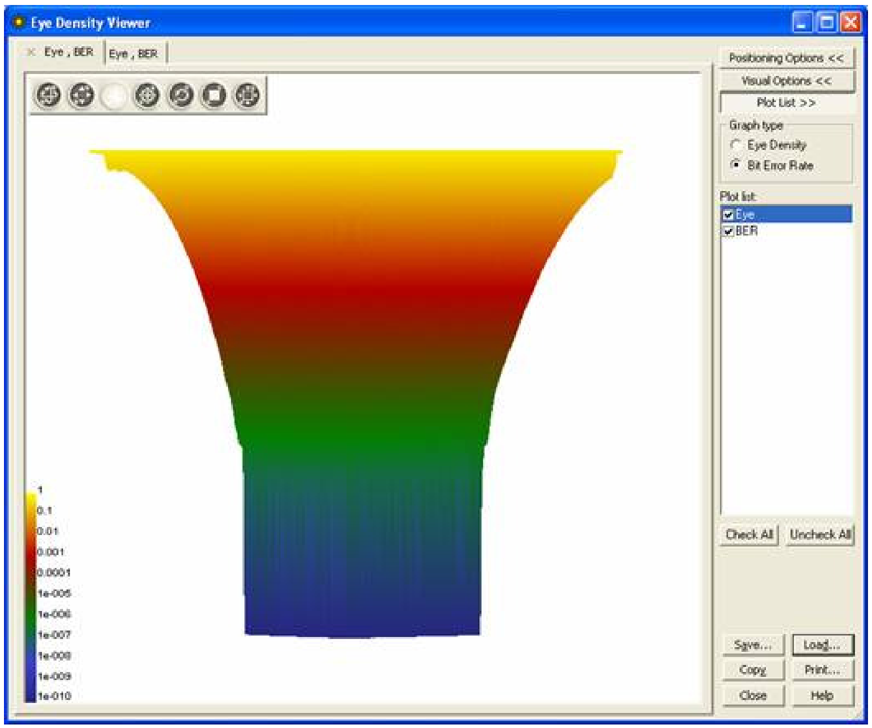


# Combining WS with IBIS-AMI simulations

- One of the inputs for the IBIS-AMI Time Domain (TD) simulations is a bit pattern
- If this bit pattern is not completely random, but is "biased" to contain worst case sequences, the statistical eye will include the waveforms which make up the tails of the PDF
- The tails of the PDF which are obtained this way are simulated (not estimated), consequently the BER results will be actual and accurate
- The non-LTI behavior of IBIS-AMI models poses a challenge. Instead of finding a fixed worst case signal/crosstalk pattern to stress the eye, the EDA platform utilizes an adjustable system with a feedback loop in which the output is analyzed periodically and the input stimulus is generated accordingly to achieve a maximally stressed statistical eye.

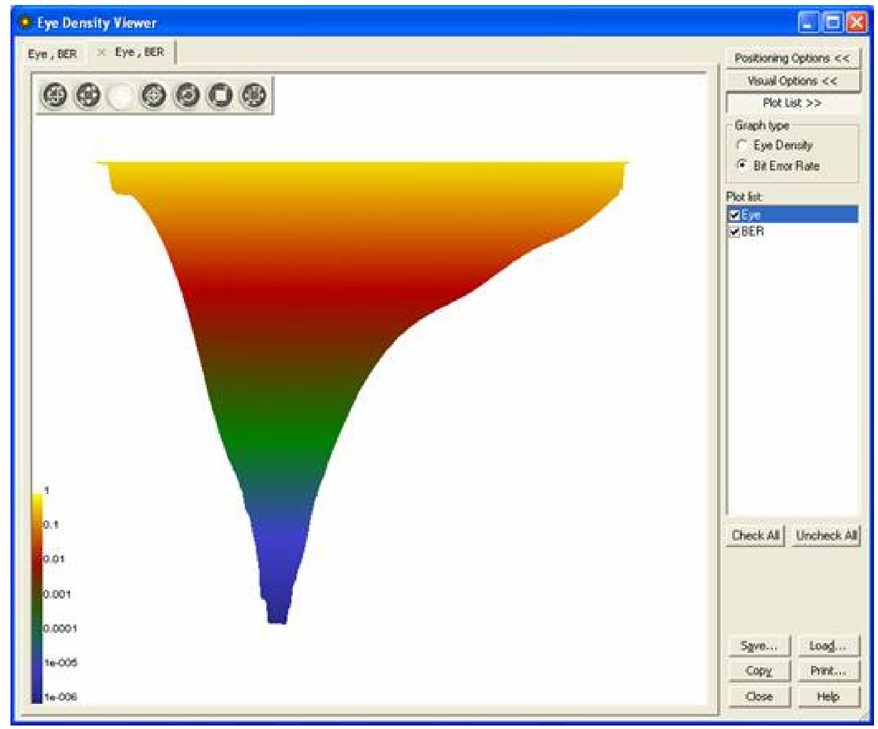
# IBIS-AMI results with WS stressing

**IBIS-AMI simulation with a normal PRBS stimulus**




An extrapolation of this plot will not indicate any eye closure

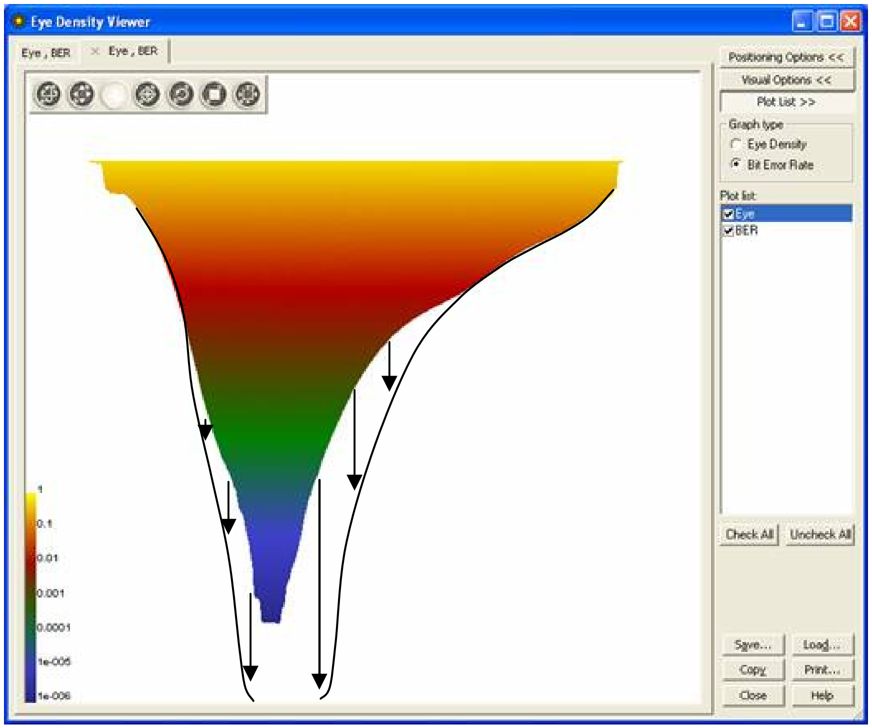
**The same IBIS-AMI simulation with a WS stressed stimulus**



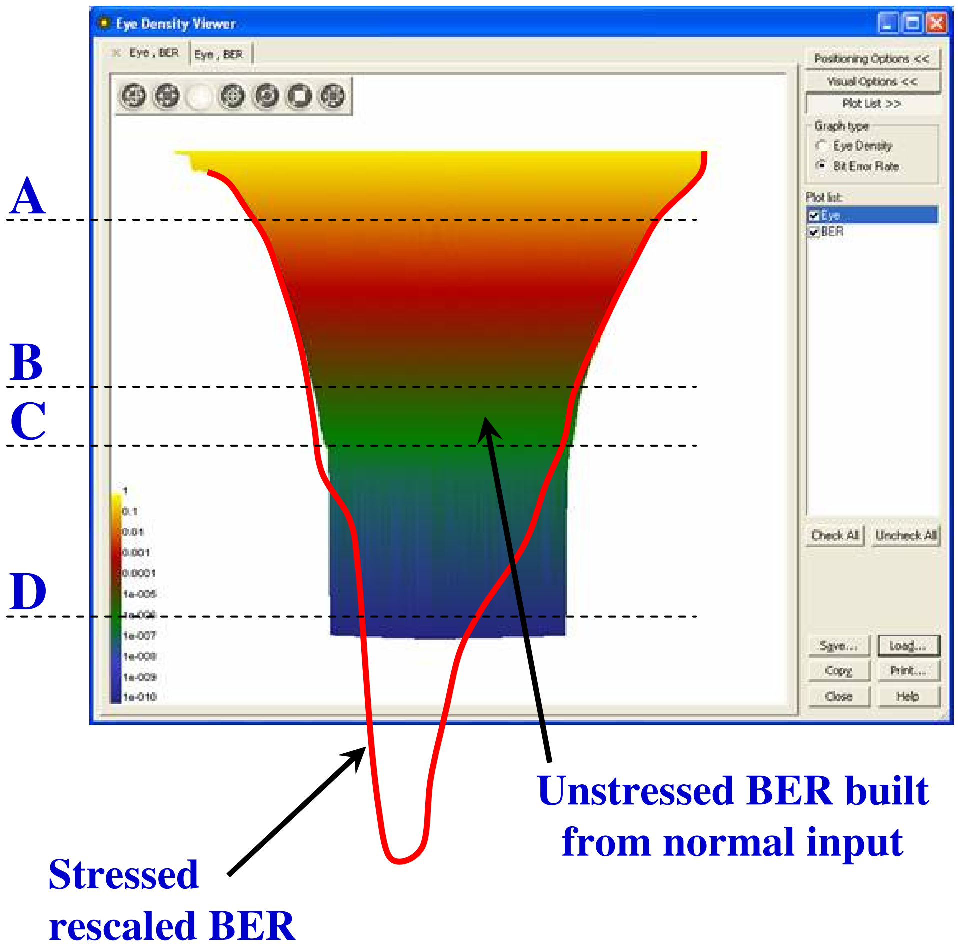

WS stressed simulation shows an alarming eye closure

# Scaling the results to get the actual BER

- **To obtain the actual BER probabilities, after the stressed eye/BER density plot has been created, its z-axis is rescaled according to the "selectivity" of the WS stressing algorithm**
- **Unlike extrapolation, this process does not produce anything that was not actually observed in the IBIS-AMI simulation**



# Comparing the results after rescaling




A. At this probability level both give correct prediction since the number of simulated bits provide sufficient sample size

B. At this probability level, 'normal' BER already suffers from granulation/noise due to insufficient statistical base; estimation is inaccurate. Below this level, the two BER plots demonstrate different behavior.

C. Below this level, 'normal' BER cannot show anything informative

D. Important low probability information is revealed by the stressed/rescaled BER

# Conclusions

- Attempts to predict low BER probabilities using extrapolation techniques can yield incorrect or misleading results
- Combining the Worst Sequence algorithm with algorithmic SERDES simulations can predict low BER probabilities reliably and accurately

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