Statistical Coverage in SI System Simulations and Implications for Models

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Agenda

- What You Should Take Away
- The Challenge of Complete Coverage
- Using Statistics to Improve Coverage
  - One Approach: Response Surface Modeling
- Using Statistics to Assess Risk
  - One Approach: DPM
- Implications for the IBIS Community
- Summary
- References
- Q & A

Thanks to Tommy Cheung of Intel for significant source material

Caution!

The word “statistics” here does not refer to analysis of SerDes buffer data processing algorithms or performance

Here, “statistics” refers to the broader science of using numerical data to make statements or inferences about groups

What You Should Take Away

Designers, model makers, etc. need to understand where statistical assumptions are made

Statistics can be used to maximize efficiency in design

- This covers both analysis time and design cost

Avoiding statistics can cause over-design or under-design

For complete, informed SI coverage, consider making explicit use of several statistical concepts...

- Parameter distributions (e.g., PCB impedance in volume manufacturing)
- Defect tolerances
- Confidence levels

The IBIS community should encourage and design for use of statistical concepts and data

Review: Objectives of SI Simulation

Generally, signal integrity (SI) simulations are performed to:

- Characterize or establish the sensitivity of analog signal quality outputs or goals to design parameters
  - *e.g., how long can my traces get before the signal vanishes?*
- Find the worst-case topology or design situation in a platform
  - *"Worst-case" can refer to many outputs: timing, voltage, etc.*

Customers often have multiple goals

- Ensure positive margins across the design & manufacturing
- Perform cost reductions or optimizations

Simulations mean modeling the system response to establish input-output relationships, then examining or testing those relationships

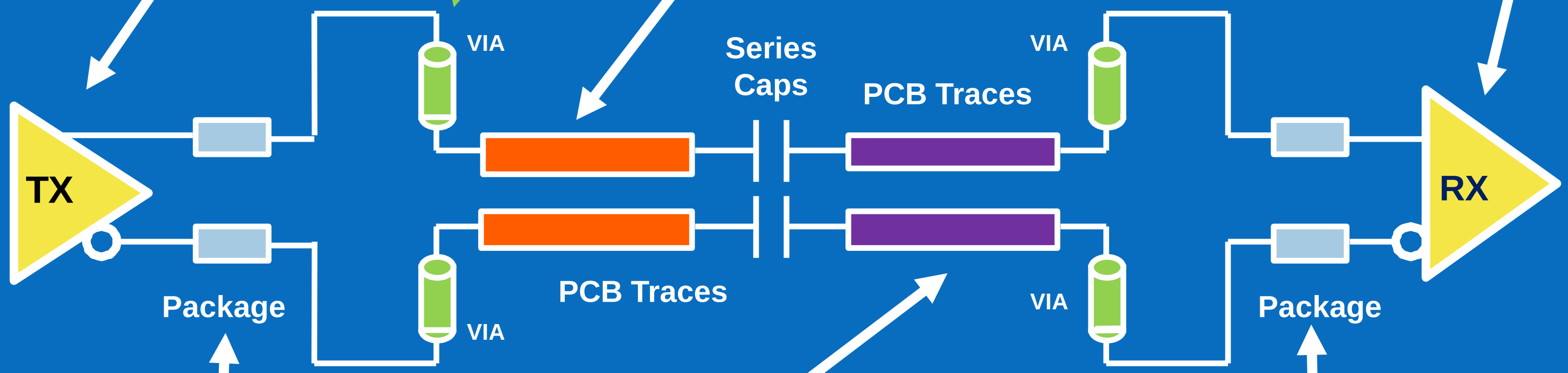
Design Parameters in a SerDes System

TX Buffer
- Impedance
- “Corner”

Via

Board Trace, Section 1
- Dielectric thickness, loss tangent, Er
- Trace width, spacing, thickness

RX Buffer
- Impedance
- “Corner”



TX Package
- Length
- Impedance
- “Corner”

Board Trace, Section 2
- Dielectric thickness, loss tangent, Er
- Trace width, spacing, thickness

RX Package
- Length
- Impedance
- “Corner”

Note the number of parameters. How do you cover all these? How do you determine formal relationships between inputs and outputs?

Investigating a SerDes Solution Space

Design Parameter	Specific Variations
Transmitter	
Termination Resistance	3
Termination Capacitance	3
Receiver	
Termination Resistance	3
Termination Capacitance	3
Transmitter Package	
Routing Length	3
Differential Impedance	3
Receiver Package	
Routing Length	3
Differential Impedance	3
Trace Cross-Section	
Section 1 Dielectric Constant	3
Section 1 Loss Tangent	3
Section 1 Trace Width	3
Section 1 Within-pair Spacing	3
Section 1 Pair-to-pair Spacing	3
Section 1 Dielectric Constant	3
Section 2 Loss Tangent	3
Section 2 Trace Width	3
Section 2 Within-pair Spacing	3
Section 2 Pair-to-pair Spacing	3
Routing	
Section 1 Length	3
Section 2 Length	3

Specific variations are the values each parameter can take in a simulation

For example
Dielectric range = 3 – 4 mils
3 variations in that range = 3.0, 3.5, 4.0

Exhaustive simulation means covering every combination
(variation * variation *...)

59,049

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Investigating the Solution Space

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Section 1 Trace Width	3
Section 1 Within-pair Spacing	3
Section 1 Pair-to-pair Spacing	3
Section 1 Dielectric Constant	3
Section 2 Loss Tangent	3
Section 2 Trace Width	3
Section 2 Within-pair Spacing	3
Section 2 Pair-to-pair Spacing	3
Routing	
Section 1 Length	3
Section 2 Length	3

Exhaustive or “grid” simulation covers *every* case

Permitting variations to cover minimum, typical and maximum *values* for each parameter results in…

3.48 billion simulations

This is not a complete or sufficient list… and the ranges themselves may require iteration

Exhaustive simulation is impossible.

Even grid simulation doesn’t establish relationships.

Is there a better way?

Finding Relationships with Few Cases

Problem: too many cases for unknown relationships

- Exhaustive simulation takes too much simulation time and resources
- Random simulations or guesses can miss critical behaviors and may not rigorously define relationships – this makes predictions difficult!

Solution: Use statistical methods

- One method: Response Surface Modeling (RSM) – see references
- Select cases and create a predictive model using least-squares fitting

Advantages of Statistical Methods Like RSM

- Well-known: used in manufacturing and the social sciences
- Efficient: can reduce simulation burden significantly
- Predictive: you get definitive, mathematical relationships
- Comprehensive: more than just a one-dimensional sensitivity analysis
- Quantify risk: provide explicit confidence levels

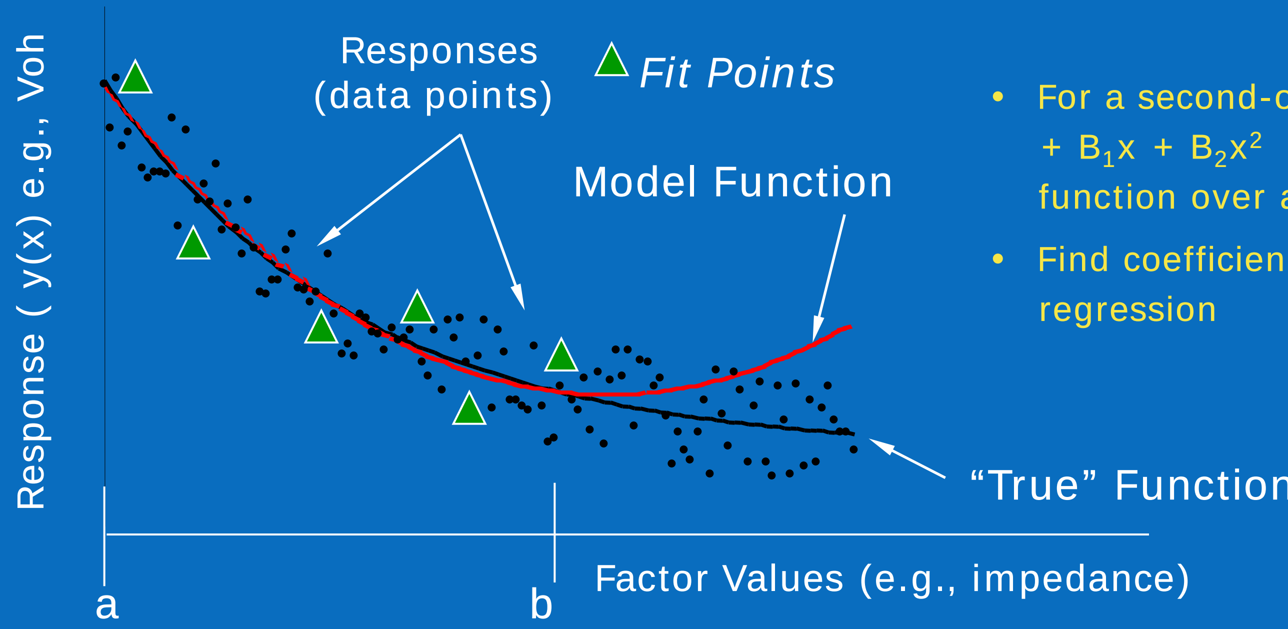
A second-order fit of 20 variables can reduce coverage burden from 3.48 billion cases to 256 cases

Empirical Modeling

Replaces a complicated analytic response function with a simpler *empirical* function of the input variables

Predicts the response for given factors *without re-running simulations*

Significance of the results can be evaluated from statistics on the fit



- For a second-order fit, model $F(x) = B_o + B_1x + B_2x^2$ approximates the true function over a limited range
- Find coefficients (B_i) using least squares regression

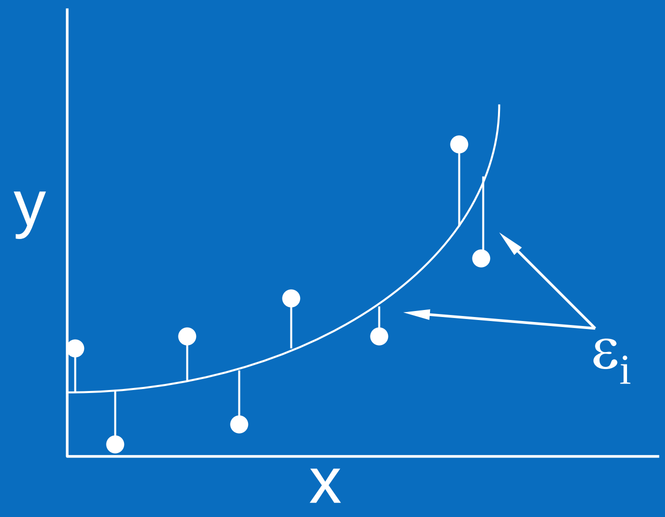
Fit a simpler statistical model to data from a more complex analytic SI computer model (e.g., SPICE)

Case Inputs and Evaluating Outputs

Cases are selected based on assumed fit relationship

- Select the input parameters and combinations to run based on fit needs
- Engineering judgment and screening runs are critical here

Quality of fit statistics indicate quality of assumptions about fit relationship and input parameters



Some example fit quality checks:

Residuals: Difference between actual and predicted results

Rsq: How much of the output variation is explained by input variations?

RMSE: Root Mean Squared Error – RMS value of predicted vs. actual error

Statistics texts provide many others…

Outcome is a simplified "model of a model" with known quality assessments and confidence intervals for the fit

How Can We Use the Model-of-a-Model?

RSM provides a simplified set of equations that predicts the system behavior

This is essentially a “simulator in a spreadsheet”

- Equations could be large and/or complex
- Quicker with faster computers and more capable math tools

We can then make fast “what-if” predictions within the valid ranges of this set

- Simulate large numbers of cases – potentially millions
- Examine non-uniform variations in input and resulting outputs (e.g. Monte Carlo)

Results approximate output of full simulations

- As accurate as the original prediction fits were accurate

Relatively few simulations can generate millions of results

RSM -> Prediction-Based Monte Carlo

Monte Carlo | **Prediction Formulas**

	DIMM1_ER	CSPkg_TRWIDTH	MB_TR_WIDTH	MB_SP	MBBrk2CHA_LENGTH	Pred Formula Voh	Pred Formula Total Derated System Skew
1	3.81749905	29.9286732	5	20	4.7	0.97111463	-0.2191045
2	4.14071353	26.8277443	6.66666667	18.3333333	0.9	1.03127234	-0.2472544
3	4.06055011	27.6148573	6.66666667	18.3333333	3.2	1.0196399	-0.2501578
4	3.81029692	26.0763189	5	20	3.15	0.98158417	-0.253558
5	3.97294708	28.4427032	5	20	0.8	1.00961271	-0.2421223
6	4.21150309	29.5636414	8.33333333	16.6666667	2.55	1.00643079	-0.2442386
7	3.93506332	27.9611728	10	15	4.4	1.02940844	-0.2715204
8	4.06282691	30.0494808	5	20	4.8	0.99547099	-0.259795
9	4.01186288	25.9681414	10	15	3.3	1.03314472	-0.2503127
10	4.00780808	30.9832494	6.66666667	18.3333333	3.7	0.9962155	-0.2289716

⋮

Millions of cases

Use RSM formulas in spreadsheet-style analyses to…

- Examine new situations not simulated
- Find minor max margins (worst/best case)

The Problem of Worst Case Analysis

Worst-case analysis or design assumes

- All parameters have an equal likelihood of being at the extremes of their ranges at any given time
- Therefore, design for positive margins where a system has all parameters at the extremes *simultaneously*…
- … regardless of how likely that is!

Parameter	Min	Typ	Max	Units
TX Resistance	75	85	95	Ω
TX Capacitance	0.85	1.00		
Board 1 Trace Impedance	72.25	85		
Board 2 Trace Impedance	72.25	85		
RX Resistance				
RX Capacitance				

As likely as this situation?

Parameter	Min	Typ	Max	Units
TX Resistance	75	85	95	Ω
TX Capacitance	0.85	1.00	1.15	pF
		85	97.75	Ω
		85	97.75	Ω
		85	95	Ω
		0.8	0.9	pF

Parameter	Min	Typ	Max	Units
TX Resistance	75	85	95	Ω
TX Capacitance	0.85	1.00	1.15	pF
Board 1 Trace Impedance	72.25	85	97.75	Ω
Board 2 Trace Impedance	72.25	85	97.75	Ω
RX Resistance	75	85	95	Ω
RX Capacitance	0.7	0.8	0.9	pF

Or this?

The Statistical Problem

In simulation, every parameter is known and controllable for any given system

In manufacturing, few parameters are known or controllable for any given system

- But the behavior of the *whole population* of cases is predictable *statistically* by *sampling*

Faster interfaces are now fault-tolerant

- e.g., BER = bit *error* rate – data errors are expected!
- The only way to predict or analyze this is through statistics

Statistical simulation predicts what design parameters affect the likelihood of errors

Modern designs limit errors, instead of eliminating them. Design teams and their management must get comfortable with *using* this concept!

DPM Concepts

DPM = Defects Per Million

- Of every million units, how many do not meet requirements?
  - *"Unit" can be a single parameter instance instead of an entire physical object or product*
- A statistical metric for a process
- Useful (and used) in manufacturing

DPM assumes...

- Some number of units will fail to satisfy a given criterion
- Parameters of interest have distributions (e.g., normal, uniform)
- Probabilities are involved
  - *Characterize populations by analyzing representative samples*

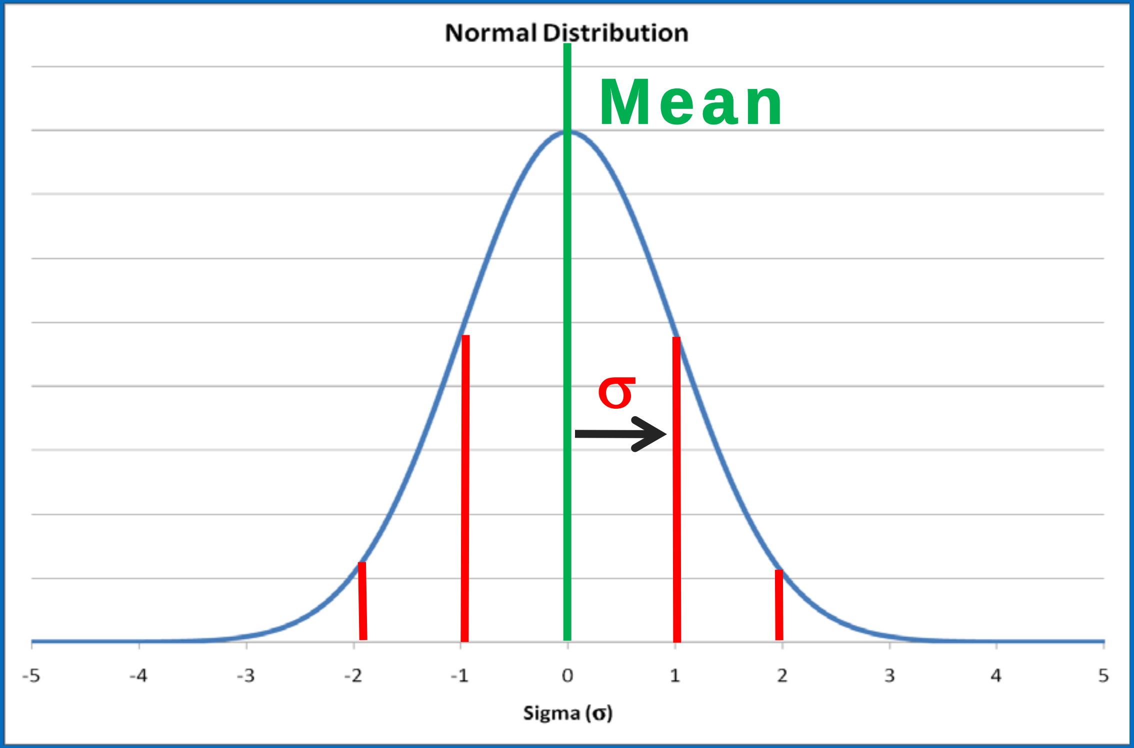
DPM goes beyond absolute worst-case!

Lightning Probability Review

Mean (μ) – average or sum of all measurements divided by the number of measurements

- For a normal distribution, the **mean** is also the **mode** or most often occurring value

Standard deviation (σ) – indication of the width or distribution of values within the curve



Each σ away from mean covers a fixed portion of the distribution…

$\pm$ 1 σ = 68.27%
$\pm$ 2 σ = 95.45%
$\pm$ 3 σ = 99.73%

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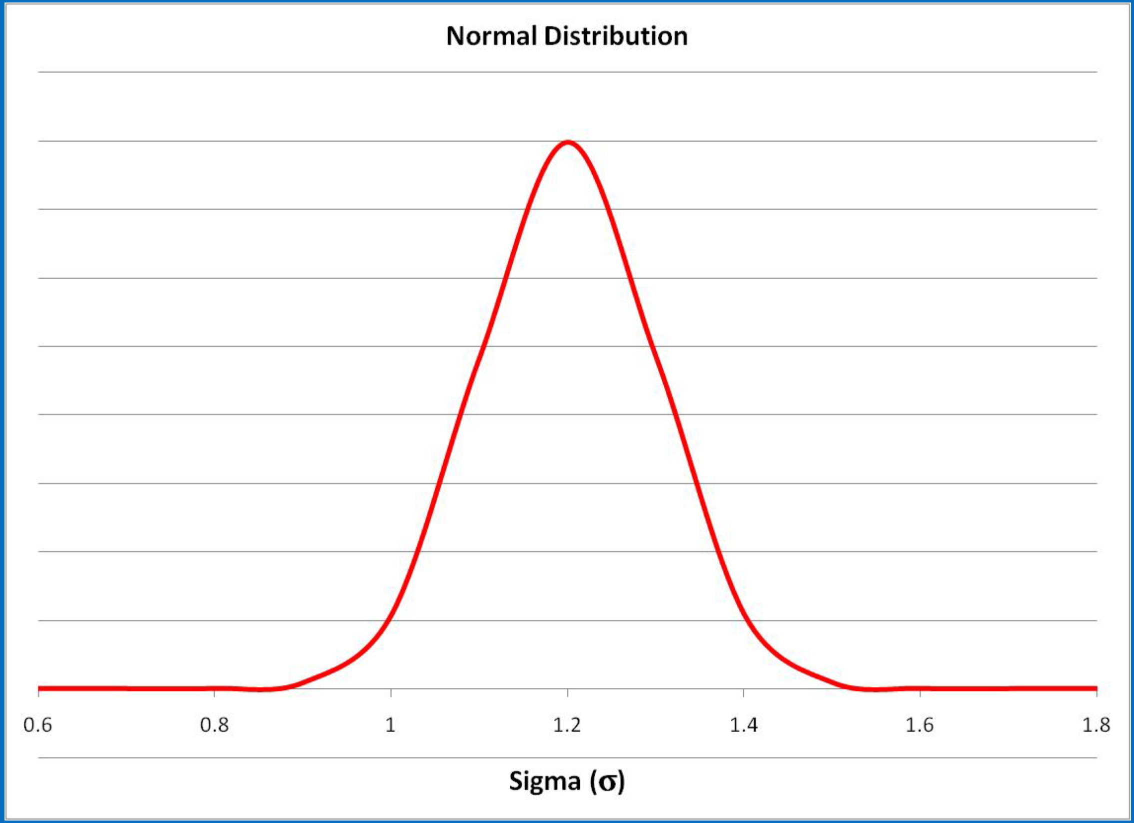
$\pm$ 6 σ = 99.999…%

A DPM Example

A factory making paper clips...

- The metal in the clips must be larger than a certain diameter to



On average, the clips are 1.2 mm in diameter

95% of them are within 0.2 mm of the average diameter

Of every million clips, how many will not meet the specification?

DPM Calculation

DPM is calculated using the Cumulative Distribution Function (CDF)

- The CDF is the number of units under the probability density curve within a given range

For a single parameter, with a normal distribution…

- Expected value or mean of μ
- Standard deviation of σ
- Lower specification limit of LSL

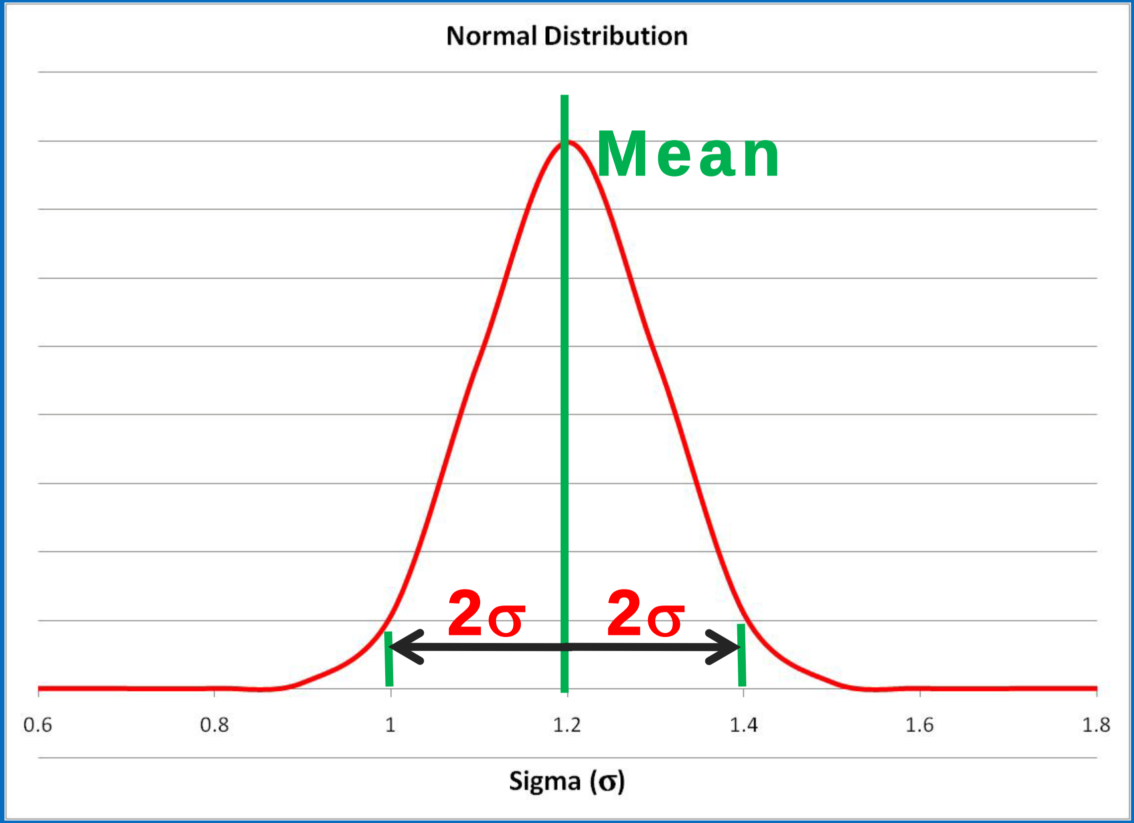
$$D_{ppm} = 1{,}000{,}000 * \frac{1 + erf\left[\frac{(LSL - \mu)}{\sigma\sqrt{2}}\right]}{2}$$

- Hall/Heck, p. 630 (equation has incorrect division by 1e6)
- This can be calculated in Microsoft Excel* using NORMDIST
  - *=NORMDIST(LSL, μ, σ,TRUE)*1000000*
- Other tools usually have similar functions

Applying DPM to an Example

A factory making paper clips…

- The metal in the clips must be larger than a certain diameter to prevent breaking (say 0.8 mm)



μ = 1.2 mm

2 σ = 0.2 mm (95% are within 0.2 mm)

σ = 0.1 mm

Of every million clips, how many are below 0.8 mm in diameter?

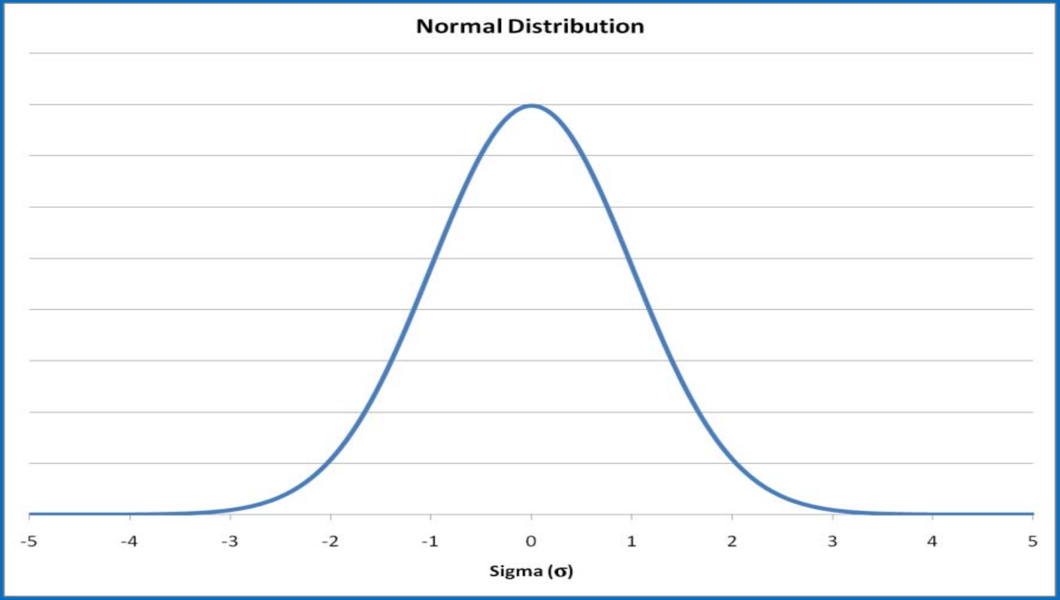
DPM = 32

Applying DPM

Manufacturing must assume some DPM

Many/most processes in nature and manufacturing are normal

- Follow the normal or Gaussian probability distribution “bell curve”



How many DPM is acceptable *depends*...

- e.g., for hard drive mechanical components, a DPM of 150 may result in customer calls to the CEO
  – *Wayne Fortun, CEO, Hutchinson Technology - July 10, 2009, Boston*

Putting RSM and DPM Together…

You can calculate DPM from RSM model fits

- Fit the model and obtain the RSM prediction formulae
- Set the distributions on the input parameters
- Run a statistical analysis on 1 million cases
- Examine the output (e.g., eye height, eye width)
- Number of cases violating your requirement = DPM

Available statistical tools should have this capability

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1e6

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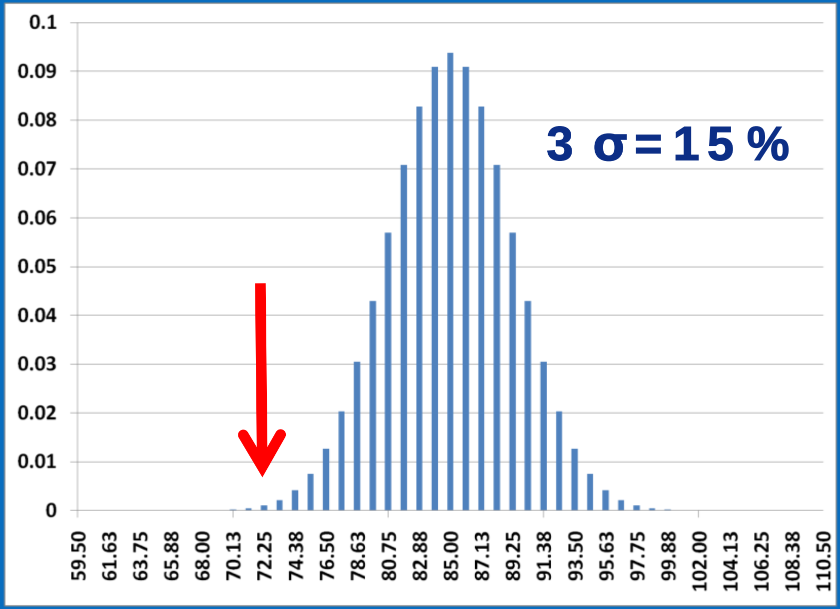
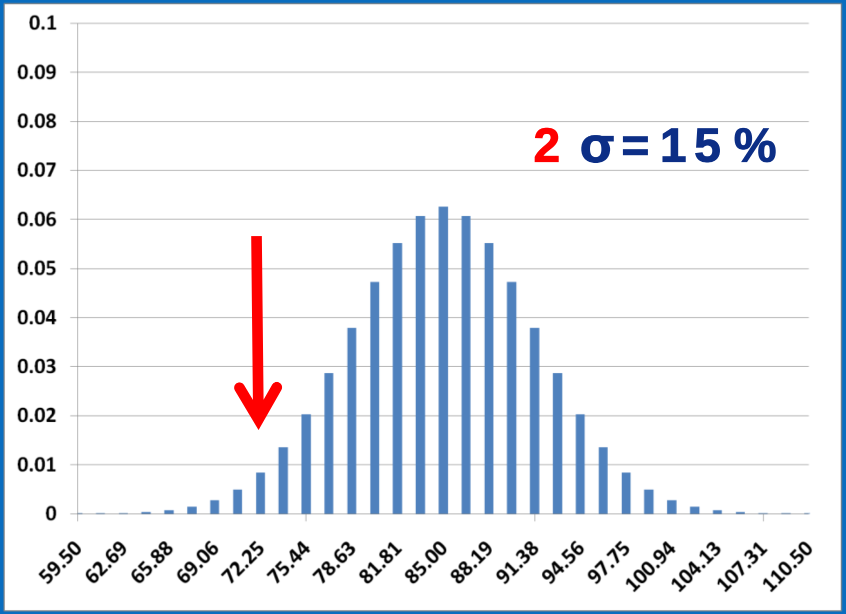
Impact of Statistics

An example using PCB impedance and tolerance

- Assume a differential target of 85 Ω ± 15%
- How many sigma does "± 15%" represent in production?
- What σ should I use in simulation? Does it vary per manufacturer?
- How probable is seeing a board impedance of 72.25 Ω?
  - *8 times more likely at 2 σ=15% than 3 σ=15% (normal dist.)*
  - *If 72.25 is my design limit, which manufacturer is more risky?*





Implications for the IBIS Community

How useful is the concept of "corner" in a statistical sense?

- Is the IBIS approach of using "typ/min/max" *categories* granular enough for statistical analysis?
- Should we consider variable numeric parameters instead?

If you produce and/or release models, are they configured to allow statistical variation?

- Applies especially to buffers, packages and PCB traces

Are you prepared to use and/or support statistical design inputs and evaluation criteria (e.g., DPM)?

Do you currently support statistical analysis in what you provide to your customers and partners? In your design flow?

Summary

Designers, model makers, etc. need to understand where statistical assumptions are made

Statistics can be used to maximize efficiency in design

- This covers both analysis time and design cost

Avoiding statistics can cause over- or under-design

For complete, informed SI coverage, consider making explicit use of several statistical concepts...

- Input distributions (e.g., PCB Z in volume manufacturing)
- Defect tolerances
- Confidence levels

The IBIS community should encourage and design for use of statistical concepts and data

References

Signal Integrity and Statistics – Primary sources for this presentation

- Hall, Stephen and Howard Heck, *Advanced Signal Integrity for High-Speed Digital Designs*, 2009: Wiley-IEEE Press
- Norman, Adam et al, Application of Design of Experiments (DOE) Methods to High-Speed Interconnect Validation, *Proceedings of the 12th IEEE Topical Meting on Electrical Performance of Electronic Packaging*, 2003: IEEE

General Statistics References

- Urdan, Timothy C., *Statistics in Plain English* (2nd edition), 2009: LEA
- Box, George E.P., Hunter, William G., and Hunter, J. Stuart, *Statistics For Experimenters: An Introduction to Design, Data Analysis and Model Building*, 1978: J.W. Wiley
- Hamming, R.W., *Numerical Methods for Scientists and Engineers*, 2nd ed., 1986: Dover
- Meyers, Raymond H., and Montgomery, Douglas C., *Response Surface Methodology: Process and Product Optimization Using Designed Experiments*, 1995: J.W. Wiley
- Many books, particularly on "Six Sigma" management techniques are available

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BACKUP

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