Mixed mode parameter support: definitions and transformations

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Linear multiports. Ports and pins (terminals)

Any Touchstone data file describes linear multiport system by means of NxN frequency-dependent matrix F (could be S, Y, Z or other type parameter matrix)

This matrix links 'input' and 'output' variable vectors, **X** and **U** of similar size (such as current, voltage, incident or reflected wave):

$$\mathbf{U} = \mathbf{F}\,\mathbf{X}$$

Each vector component (variable) corresponds to a particular "port" of the multiport system

The set of port variables in the input and output vector (**X** and **U**) must be the same, identically ordered and without repetitions (no identical variables)

If the variables in **X** and **U** are differently ordered, then matrix F lacks some inherent properties (such as symmetry, diagonal dominance or positiveness of real part, etc.)

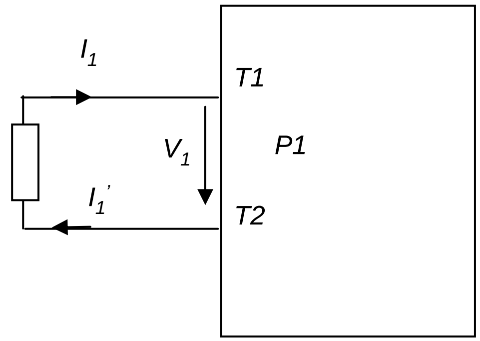
If there are repetitions or if port variables are not independent, the matrix becomes singular by structure (redundant)

What is the port?

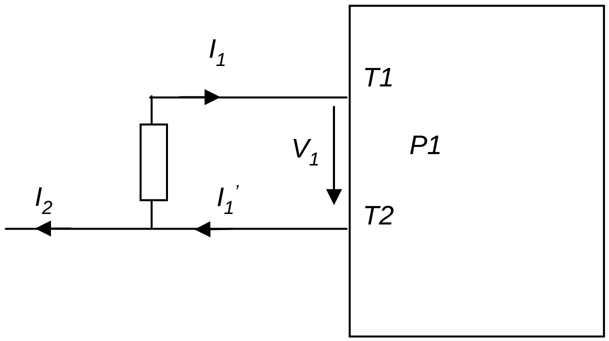
Port is the place where we attach two terminal source or load to the multiport system. Port is made from pair of pins. Each port is defined by a single port current and voltage, or incident and reflected wave. The port current flows between port pins. The port voltage is measured between the same pins, in the same direction.

“Direct” and “return” currents must be identical.

Proper connection of multiports should also satisfy above requirement (regularity).

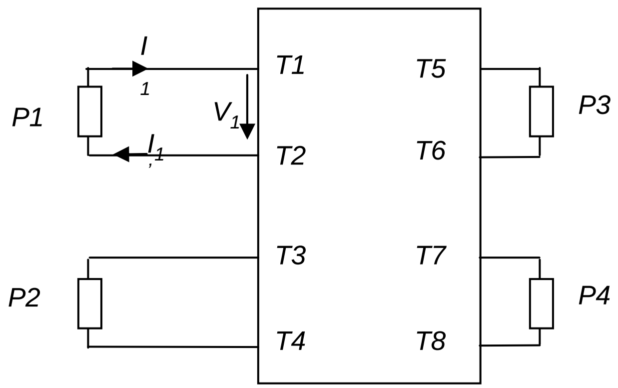


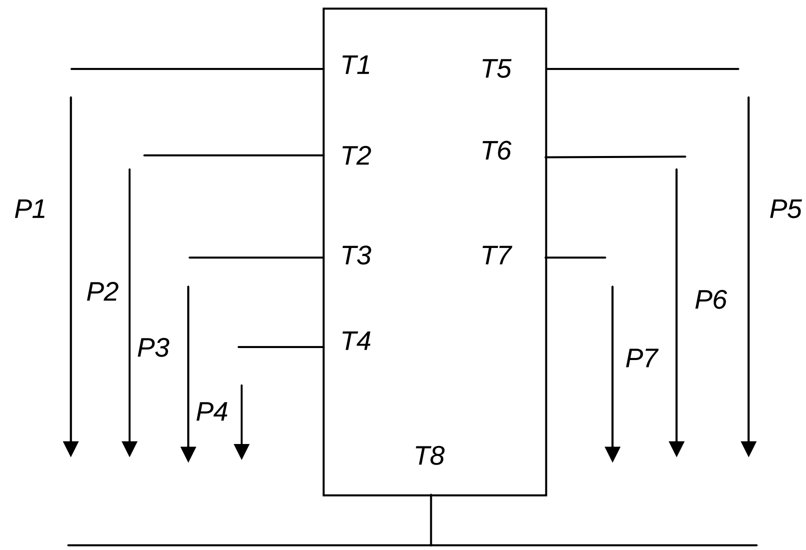
Valid port usage, consistent with port definition



Inconsistent port usage ($I_1 \neq I_1'$), simulation results could be wrong

How many independent ports we can define?





If multiport has T terminals (pins) then # of ports should not be less than T/2 or more than T-1.

For example, if we define less than 4 ports for 8-terminal model, some of terminals will not be used.

If we define more than 7 ports, some of the variables will become linearly dependent

If we define less than 7 ports (T-1), some limitations are imposed on possible multiport connection

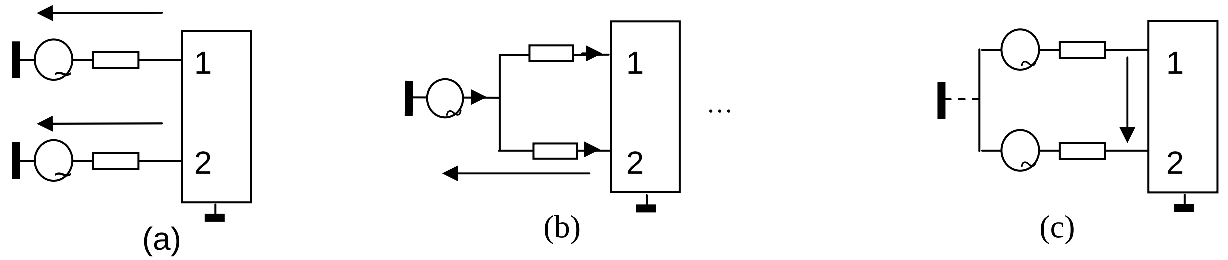
What are requirements to mixed mode ports?

(Mixed Mode = Differential Mode + Common Mode)

Origination of the differential and common mode concept…

(a) Consider two ports with common reference pin (denoted as “ground”).

Each port is connected to ideal voltage source through the resistor (reference impedance)



(b) If the applied external voltages and reference resistances are equal, and the 2-port model is symmetric (ports have identical input characteristics), the currents in both branches are identical. Hence, the resistances can be connected in parallel and the 2-port model replaced by equivalent 1-port model. We call it ‘common mode port’.

(c) If we take symmetrical model, but apply opposite voltages, the entire current circulates inside the contour; common ‘ground’ becomes disconnected. Again, we can define 1-port model by considering only signal pins of the original ports. This is differential port.

(d) If the original model is not symmetric, the common mode partially converts into differential (port voltages and currents are not identical, difference current circulates inside the contour). Similarly, differential mode partly converts into common. Some nonzero current flows into common ‘ground’.

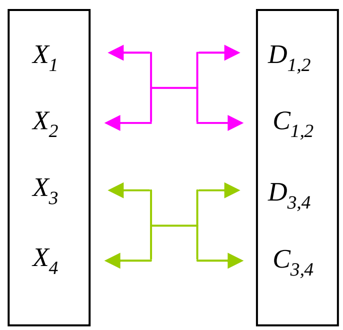
Note that above considerations require that (A) **original ports have common reference pin** and (B) **have identical reference impedances**.

What are requirements to mixed mode ports?

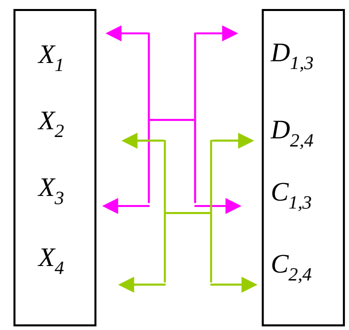
- Each pair of SE or standard mode ports, having common reference pin and identical normalizing impedance can be converted into another 2-port model, with differential and common mode ports. Together, they constitute a mixed mode model. Complexity and number of variables is retained

- Common reference pin is required for every port pair converted into mixed mode (MM). However, different pairs may have different reference pins and reference impedances.

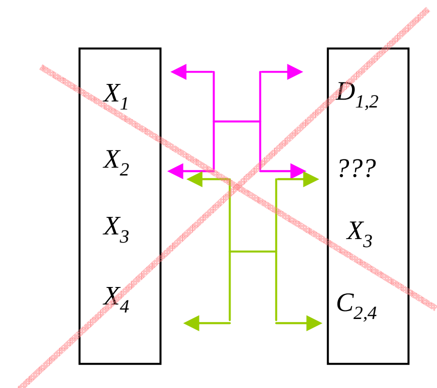
- Each standard port cannot participate in more than one standard-to-mixed mode conversion. Mappings (a) and (b) are legal, (c) is not.



(a)

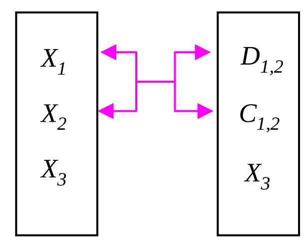


(b)



(c)

- Some of the ports may still remain in ‘standard mode’, others converted into MM (d). If standard port was used to create mixed mode port, it cannot remain in the standard mode.



(d)

Types and indexes in variables define the content of the parameter matrix

Input vector content:

D1,3	D2,4	C1,3	C2,4

Output Vector content:

D1,3
D2,4
C1,3
C2,4

<D1,3 D1,3>	<D1,3 D2,4>	<D1,3 C1,3>	<D1,3 C2,4>
<D2,4 D1,3>	<D2,4 D2,4>	<D2,4 C1,3>	<D2,4 C2,4>
<C1,3 D1,3>	<C1,3 D2,4>	<C1,3 C1,3>	<C1,3 C2,4>
<C2,4 D1,3>	<C2,4 D2,4>	<C2,4 C1,3>	<C2,4 C2,4>

We only define this

Matrix content

We don't need to define this

Matrix always retains structural symmetry

Mixed mode order keyword & definition

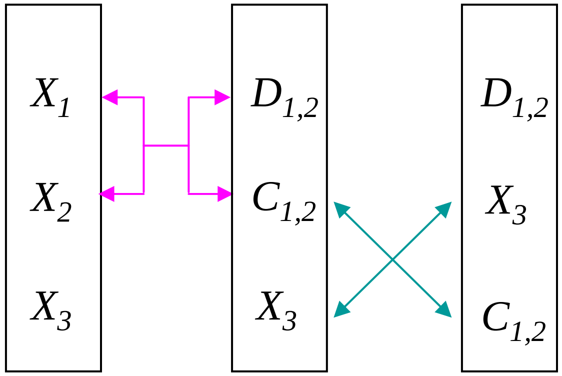
In Touchstone 2.0 format specification we allow mixed mode and single ended ports co-exist

[Mixed-Mode Order]

S<port> - single ended port with its original number
C<port>,<port> - common mode port originated from two other ports
D<port>,<port> - differential port, originated from two other single ports

[Reference] 20.0 20.0 50.0

[Mixed-Mode Order] D1,2 S3 C1,2

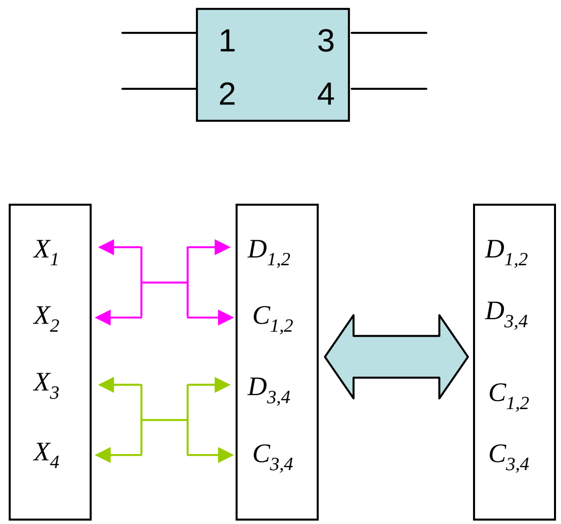


Compact definition, sufficient to track forward and hence required reverse transformation
In most cases, we need to convert the matrix back into its standard form, with variables properly ordered

Reference keyword – if present – refers to properly ordered single ended ports

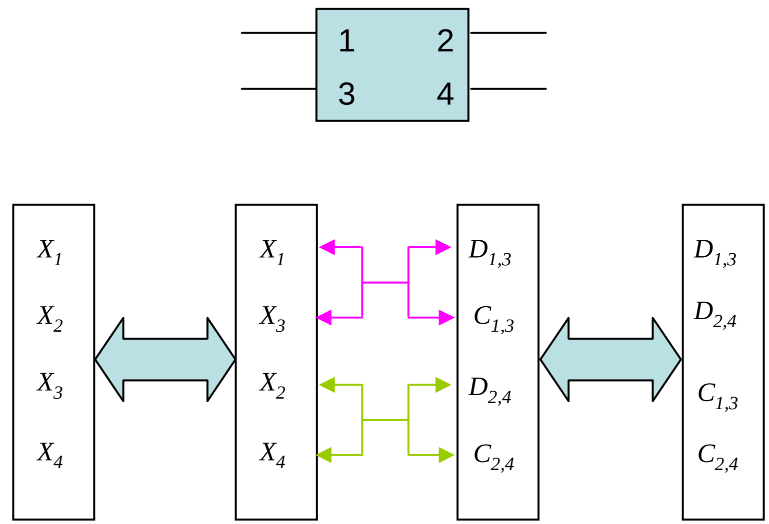
Transitions between standard and mixed mode variables

Example 1



conversion permutation

Example 2



permutation conversion permutation

1. In general, we perform permutation of STD parameters, block-wise conversion of pairs into MM parameters, and permutation of MM parameters
2. Reverse transition contains similar steps (exact mirror)
3. If we know port indexes (original STD port numbers) unambiguous reverse transition is possible

How this can be made mathematically?

Permutation applied to vectors and matrices

Row shows new position

Column shows previous position in the vector

Input vector permutation:

$$X_P = P * X_0$$

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_3 \\ X_2 \\ X_4 \\ X_1 \end{bmatrix}$$

Output vector permutation:

$$U_P = P * U_0$$

Matrix is non-singular
Has exactly one '1' in every row and column
Transpose is its inverse: $P^t = P^{-1}$

Parameter matrix permutation: $$F_P = P\, F_0\, P^t \qquad (1)$$

Requires left and right multiplication on permutation matrix and its transpose, to identically reorder rows and columns (i.e. symmetry is preserved)

Permutation applies to all types of vectors and matrices identically (a, b, V, I and S, Y, Z)

Mode conversion for a pair of ports

S-parameters: $b = S\,a$, $b_{mm} = S_{mm}\,a_{mm}$

Single ended incident wave vector to MM incident wave vector:

$$\begin{bmatrix} a_{d1,2} \\ a_{c1,2} \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma \\ \gamma & \gamma \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

where $\gamma = \dfrac{1}{\sqrt{2}}$

MM incident wave vector to SE or standard vector:

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \gamma & \gamma \\ -\gamma & \gamma \end{bmatrix} \begin{bmatrix} a_{d1,2} \\ a_{c1,2} \end{bmatrix}$$

Single ended reflected wave vector to MM reflected wave vector (identical to above):

$$\begin{bmatrix} b_{d1,2} \\ b_{c1,2} \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma \\ \gamma & \gamma \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

MM reflected wave vector to SE vector (identical to above):

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} \gamma & \gamma \\ -\gamma & \gamma \end{bmatrix} \begin{bmatrix} b_{d1,2} \\ b_{c1,2} \end{bmatrix}$$

In a more formal way:

$$a_{MM} = M a_{STD} \qquad a_{STD} = M^{-1} a_{MM} = M^{t} a_{MM}$$

$$b_{MM} = M b_{STD} \qquad b_{STD} = M^{-1} b_{MM} = M^{t} b_{MM}$$

S-parameter matrix undergoes 2-side transformation:

$$S_{MM} = M S_{STD} M^{t} \qquad S_{STD} = M^{t} S_{MM} M \qquad (2)$$

Mode conversion for a pair of ports

Y and Z parameters: $I = Y\,V$, $V = Z\,I$,
$I_{mm} = Y_{mm}\,V_{mm}$, $V_{mm} = Z_{mm}\,I_{mm}$

Theoretically, mode conversion for Y and Z types could be made exactly same way as for S-parameters, with same matrices. However, we proceed from existing definitions of the mixed mode voltage and current:

Single ended voltage to MM voltage:

$$\begin{bmatrix} v_{d1,2} \\ v_{c1,2} \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

MM voltage to SE or standard voltage vector:

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0.5 & 1 \\ -0.5 & 1 \end{bmatrix} \begin{bmatrix} v_{d1,2} \\ v_{c1,2} \end{bmatrix}$$

Single ended current MM current (differs from above!):

$$\begin{bmatrix} i_{d1,2} \\ i_{c1,2} \end{bmatrix} = \begin{bmatrix} 0.5 & -0.5 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

MM current vector to SE vector:

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 & 0.5 \\ -1 & 0.5 \end{bmatrix} \begin{bmatrix} i_{d1,2} \\ i_{c1,2} \end{bmatrix}$$

In a formal way:

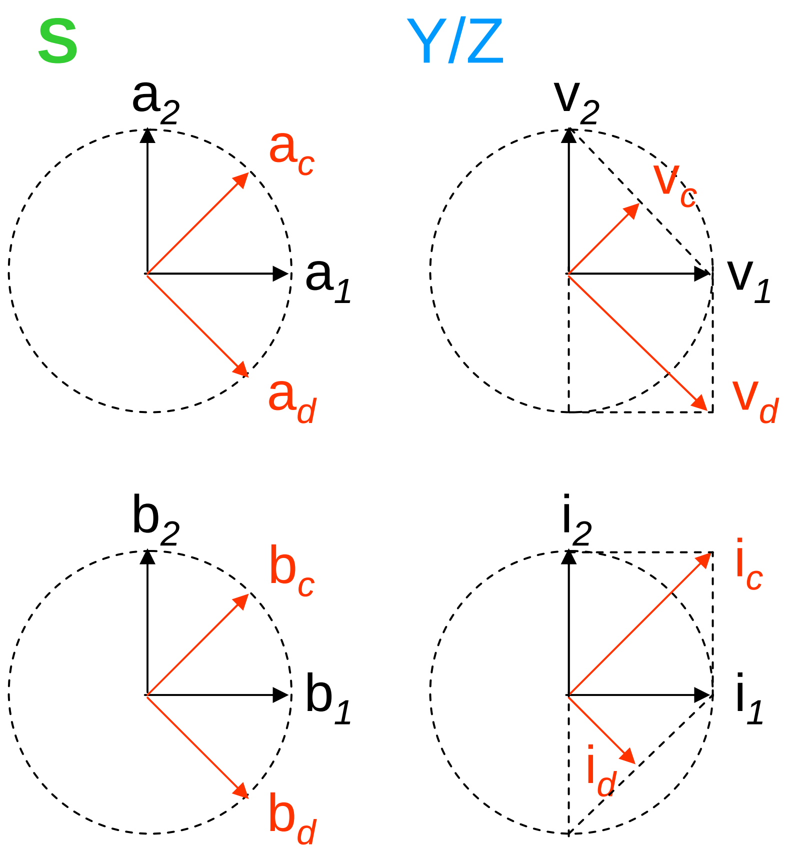
$$V_{MM} = (KM)V_{STD} \qquad V_{STD} = (M^t K^{-1})V_{MM}$$

$$I_{MM} = (K^{-1}M)I_{STD} \qquad I_{STD} = (M^t K)I_{MM}$$

where:

$$M = \begin{bmatrix} \gamma & -\gamma \\ \gamma & \gamma \end{bmatrix} \qquad K = \begin{bmatrix} 1/\gamma & 0 \\ 0 & \gamma \end{bmatrix} \qquad \gamma = \frac{1}{\sqrt{2}}$$

What's the difference between S and non-S type conversion?



Geometrical interpretation of mode conversion

- S parameter style conversion between STD and MM is represented by rotation
- Y or Z parameter conversion between STD and MM is represented by rotation and scaling
- Scaling does not affect effective power from the common and differential components
- The ratio between modules of voltage and current vectors in STD and MM form differs by factor 2 and ½. From here, it follows that reference impedance of differential mode is twice the STD impedance, common is half of it.
- Theoretically, the conversion similar to S-parameters can also be applied to Y and Z parameters. However, as we stick to conventional definitions of differential and common mode voltage and current, we have to use different conversion formulas

Mode conversion for a pair of ports

S parameters:

$$S_{MM} = MS_{STD}M^t \qquad\qquad S_{STD} = M^t S_{MM} M \tag{2}$$

Y and Z parameters:

$$Y_{MM} = (K^{-1}M)Y_{STD}(K^{-1}M)^t \qquad\qquad Y_{STD} = (KM)^t Y_{MM}(KM) \tag{3}$$

$$Z_{MM} = (KM)Z_{STD}(KM)^t \qquad\qquad Z_{STD} = (K^{-1}M)^t Z_{MM}(K^{-1}M) \tag{4}$$

Similar to standard parameters, we have:

$$Z_{MM} = Y_{MM}^{-1} \qquad\qquad Y_{MM} = Z_{MM}^{-1}$$

All types of STD/MM conversions are made by congruent transformations with matrices whose abs(eig()) all equal 1.

Mode conversion for a pair of ports
What happens if two ports have identical properties?

If single ended ports 1 and 2 have identical properties, 2x2 standard mode parameter matrix has identical diagonal components. This translates into purely diagonal MM matrix.

$$S_{STD} = \begin{bmatrix} \eta & \nu \\ \nu & \eta \end{bmatrix} \qquad S_{MM} = \begin{bmatrix} \eta - \nu & 0 \\ 0 & \eta + \nu \end{bmatrix}$$

$$Y_{STD} = \begin{bmatrix} \eta & \nu \\ \nu & \eta \end{bmatrix} \qquad Y_{MM} = \begin{bmatrix} \frac{1}{2}(\eta - \nu) & 0 \\ 0 & 2(\eta + \nu) \end{bmatrix}$$

$$Z_{STD} = \begin{bmatrix} \eta & \nu \\ \nu & \eta \end{bmatrix} \qquad Z_{MM} = \begin{bmatrix} 2(\eta - \nu) & 0 \\ 0 & \frac{1}{2}(\eta + \nu) \end{bmatrix}$$

How to convert multi-port parameters into standard mode

1. By inspection, as shown on slide 10, create permutation matrix P_{mm} that collects all pairs of differential and common mode ports together. Apply two side multiplication to the parameter matrix, as in (1)
2. Apply appropriate mode conversion, as in (2)-(4), by multiplying the matrix on two mutually transposed matrices from left and right. In case of several (N) pairs of ports, the conversion matrices consist of N diagonal 2x2 blocks
3. After all mixed mode pairs are converted into standard parameters, they should be properly re-ordered, according to the port numbers. This requires additional permutation by two side multiplication on matrix P_{std}.

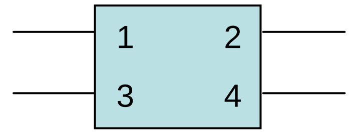
For example:

$$S_{STD} = \underbrace{P_{STD}(\underbrace{M^t(\underbrace{P_{MM}S_{MM}P^t_{MM}}_{\text{First perm}})M}_{\text{Mode conversion}})P^t_{STD}}_{\text{Second permutation}}$$

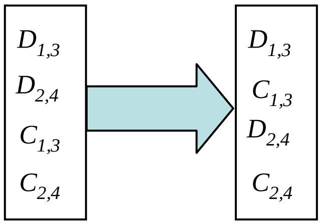
More details in Touchstone File format specification, Appendix A

4-port example

[Mixed-Mode order]
D1,3 D2,4 C1,3 C2,4



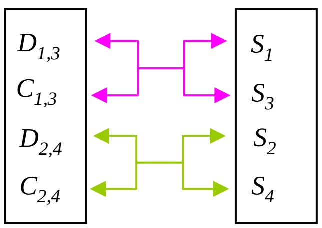
1. MM permutation



$$P_{mm} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_{MM} S_{MM} P_{MM}^t$$

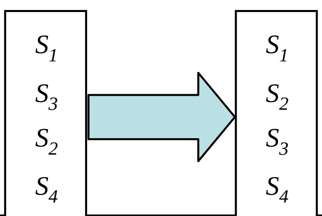
2. Mode conversion



$$M = \begin{bmatrix} \gamma & -\gamma & 0 & 0 \\ \gamma & \gamma & 0 & 0 \\ 0 & 0 & \gamma & -\gamma \\ 0 & 0 & \gamma & \gamma \end{bmatrix}$$

$$M^t (P_{MM} S_{MM} P_{MM}^t) M$$

3. STD permutation



$$P_{std} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S_{STD} = P_{STD} (M^t (P_{MM} S_{MM} P_{MM}^t) M) P_{STD}^t$$

Consistency of the matrix conversion

As follows from considered relations, including (1)-(4), and the fact that transformation matrices are not frequency dependent, conversion from mixed mode into standard and reverse preserves such basic properties as:

- Matrix symmetry (reciprocity of the system): $A_{i,j} = A_{j,i}$

- Passivity:

$$eig(E - SS^*) \geq 0$$
$$eig(Y + Y^*) \geq 0$$
$$eig(Z + Z^*) \geq 0$$

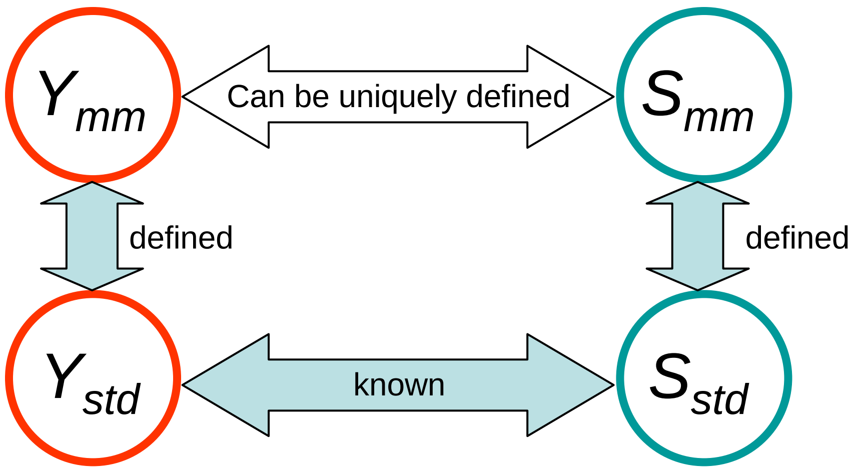
- Causality: every matrix component obeys e.g. Kronig-Kramers relation
- Geometrical symmetry (between port 1 and 2) creates diagonal sub-matrices in MM

The procedure outlined above is a convenient illustration that serves as a guideline. Practical implementations may include some modifications (i.e. finding a single matrix, combining permutation and mode conversion, etc.)

More details in Touchstone File format specification, Appendix A

Transformation between parameter types (S/Y/Z) in mixed mode

1. We do not define here direct transformations between different parameter types in mixed mode
2. These transformations have been defined for the standard mode parameters, including the case of different reference impedances
3. Given these transformations, and conversions between MM and STD modes shown in this document, it is straightforward to derive transformations between different parameter types in MM
4. Due to different scaling in STD to MM conversions, the transformations between S/Y/Z parameters in mixed mode are formally different from similar transformations in standard mode



Thank you!