Circuit Synthesis of Multiport Networks from Passive Poles and Residues

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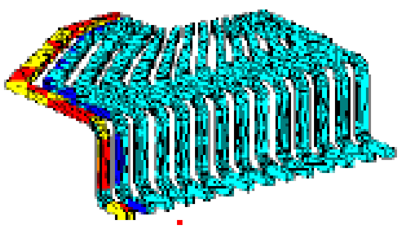
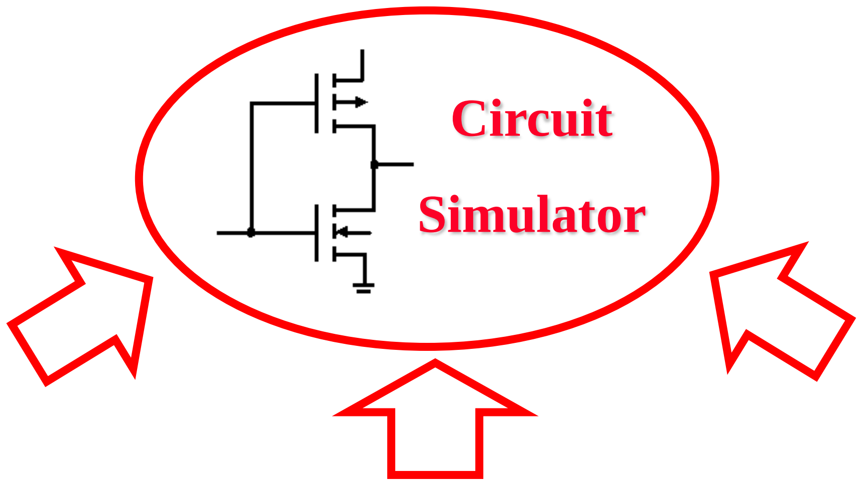
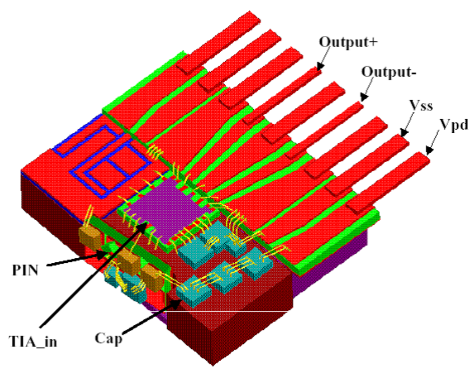
Interconnect Structures

Crosstalk
Couplings
Reflections
Losses
Dispersion

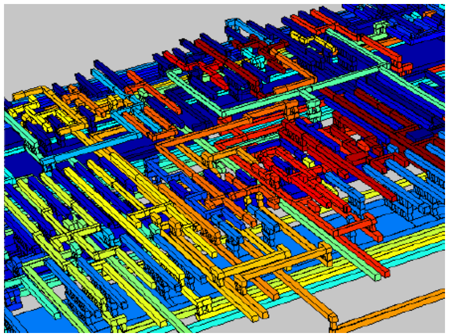


Packages



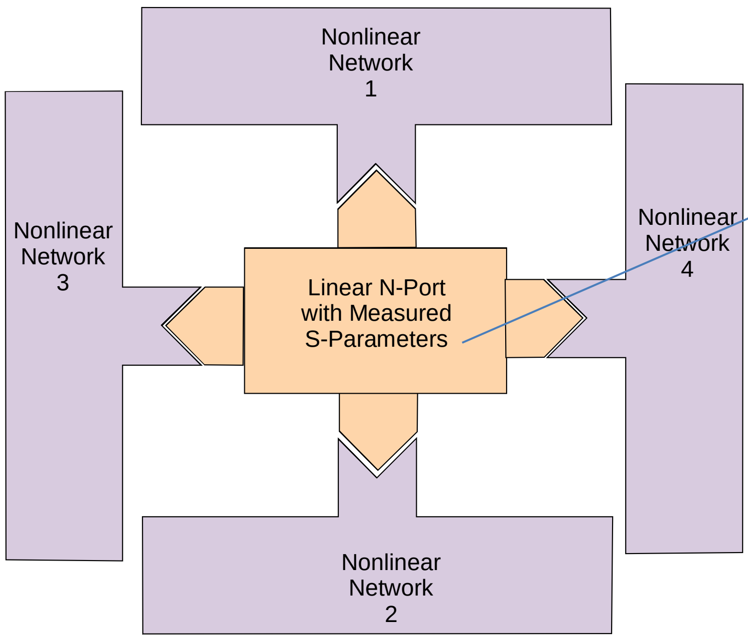


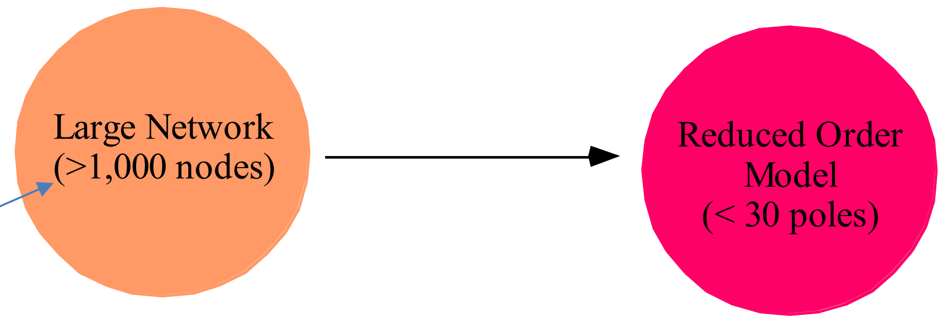
Ground Noise
Nonlinear effects
Radiation, EMI



Interconnects

Model-Order Reduction





SPICE

$Y(t)\, v(t) = i(t)$

$$\tilde{Y}(\omega) = \left[A_1 + \sum_{i=1}^{L} \frac{a_{1i}}{1 + j\omega / \omega_{c1i}} \right]$$

$Y(\omega)\, V(\omega) = I(\omega)$

↓

Order Reduction

↓

$Y(\omega) \simeq \tilde{Y}(\omega)$

↓

Recursive Convolution

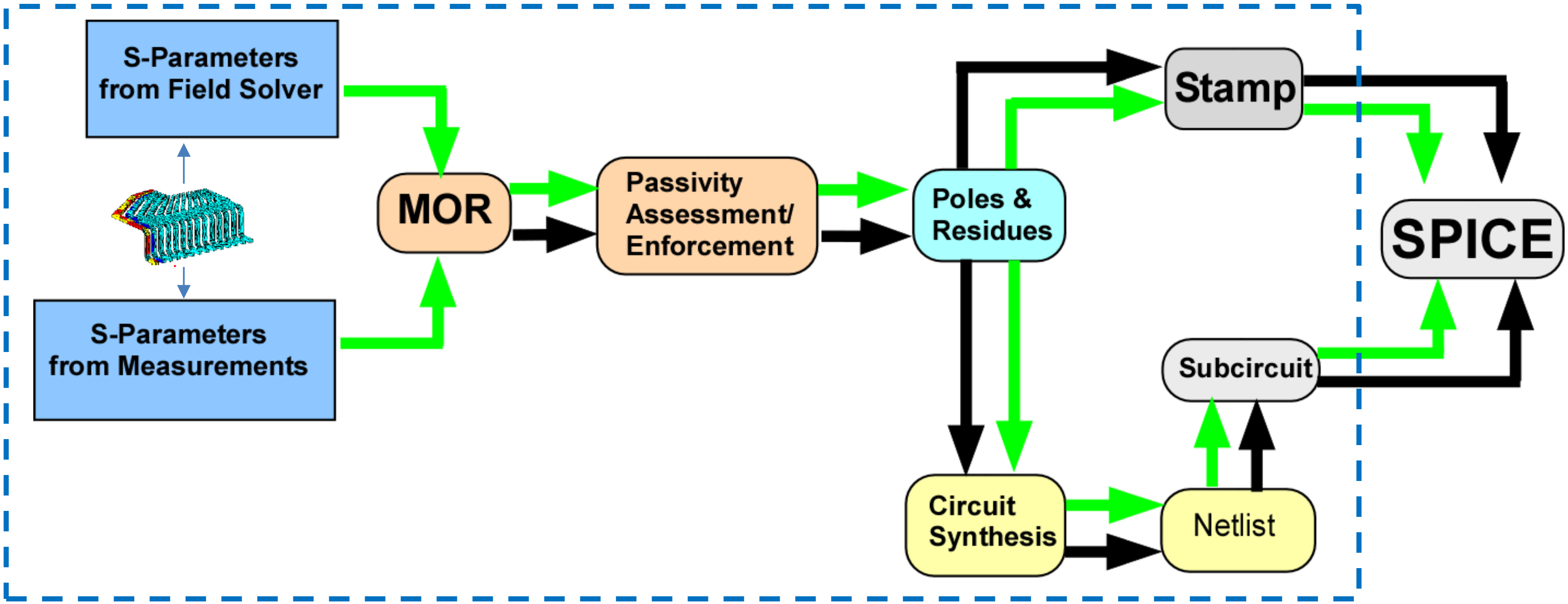
↓

$\tilde{Y}(t)\, v(t) = i(t)$

- **Strategy**: Use reduced order model to minimize computation time.

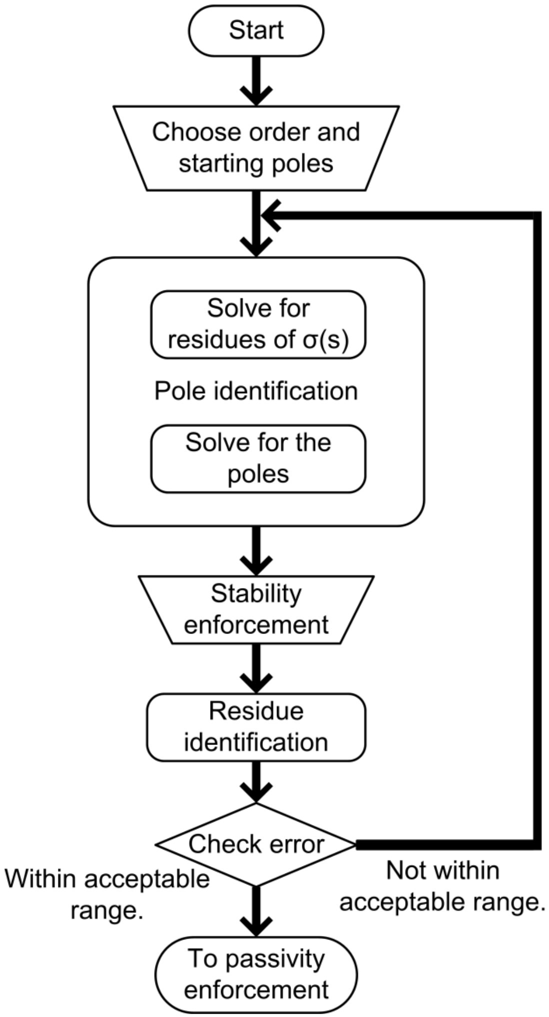
$$\tilde{Y}(\omega) \approx Y(\omega)$$

Model-Order Reduction



- **Objective**: Incorporate frequency dependence into time-domain simulator
- **Approaches**: 1) Direct integration of code into SPICE – 2) Generation of SPICE-compatible netlist

MOR via Vector Fitting



- Rational function approximation:

$$f(s) \approx \sum_{n=1}^{N} \frac{c_n}{s-a_n} + d + sh$$

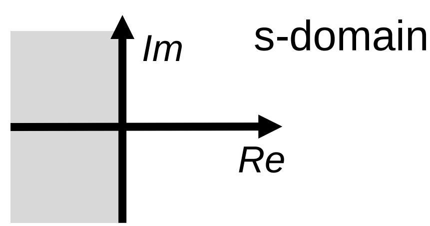
- Introduce an unknown function *σ(s)* that satisfies:

$$\begin{bmatrix} \sigma(s) f(s) \\ \sigma(s) \end{bmatrix} \approx \begin{bmatrix} \displaystyle\sum_{n=1}^{N} \frac{c_n}{s-\tilde{a}_n} + d + sh \\ \displaystyle\sum_{n=1}^{N} \frac{\tilde{c}_n}{s-\tilde{a}_n} + 1 \end{bmatrix}$$

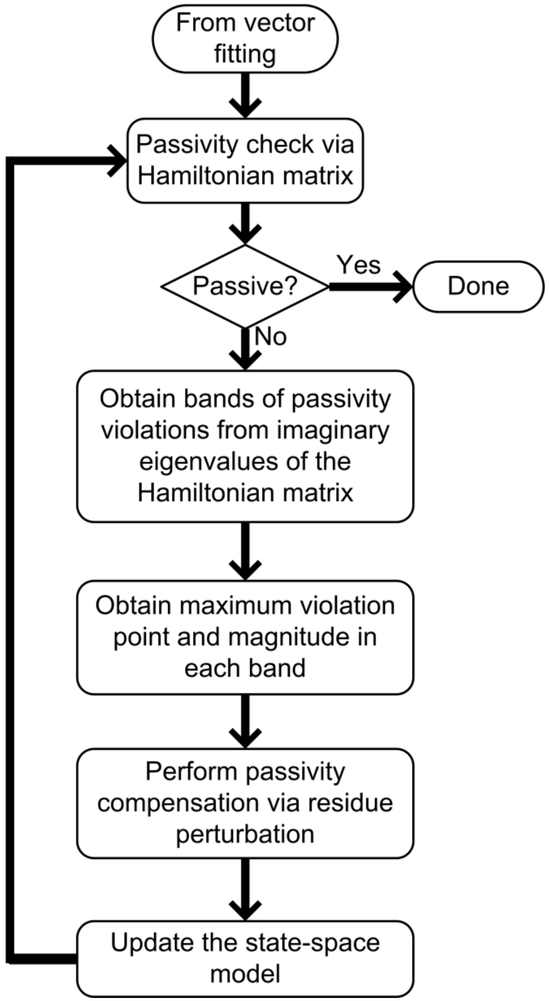
- Poles of *f(s)* = zeros of *σ(s)*:

$$f(s) \approx \frac{\displaystyle\sum_{n=1}^{N} \frac{c_n}{s-\tilde{a}_n} + d + sh}{\displaystyle\sum_{n=1}^{N} \frac{\tilde{c}_n}{s-\tilde{a}_n} + 1} = \frac{\displaystyle\prod_{n=1}^{N+1} (s - z_n)}{\displaystyle\prod_{n=1}^{N} (s - \tilde{z}_n)}$$

- Flip unstable poles into the left half plane.



Passivity Enforcement



- State-space form:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

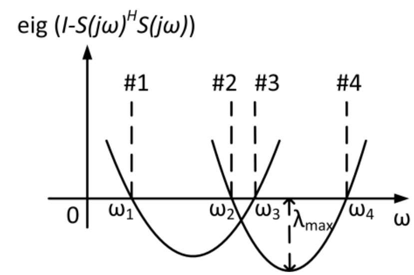
- Hamiltonian matrix:

$$M = \begin{bmatrix} A + BKD^T C & BKB^T \\ -C^T LC & -A^T - C^T DKB^T \end{bmatrix}$$

$$K = \left(I - D^T D\right)^{-1} \quad L = \left(I - DD^T\right)^{-1}$$

- Passive if *M* has no imaginary eigenvalues.

- Sweep: $eig\left(I - S(j\omega)^H S(j\omega)\right)$



- Quadratic programming:
  - Minimize *(change in response)* subject to *(passivity compensation).*

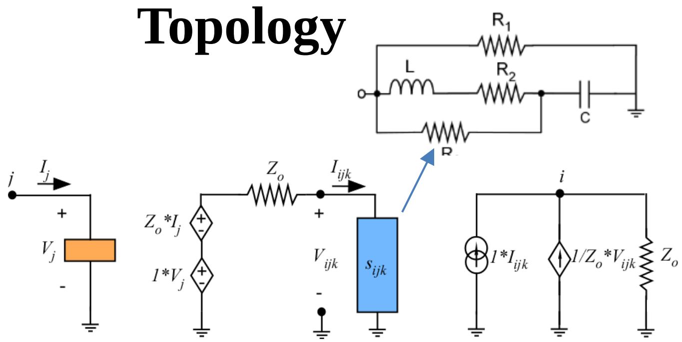
$$\min\left(vec(\Delta \mathbf{C})^{\mathbf{T}}\mathbf{H}\, vec(\Delta \mathbf{C})\right) \quad \textbf{subject to} \quad \Delta\lambda = G \cdot vec(\Delta \mathbf{C}).$$

SPICE Netlist Synthesis

- Goal is to generate (using pole/residue information) a circuit netlist that will exhibit the same (frequency-dependent) behavior as that of the S-parameters of connector under study

Passive Poles/Residues from MOR

Circuit Cell Topology

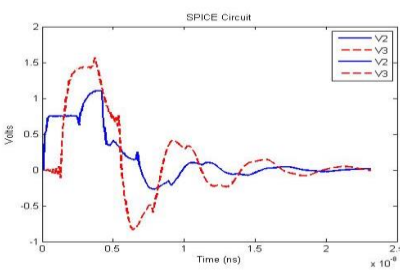


Synthesis Algorithm

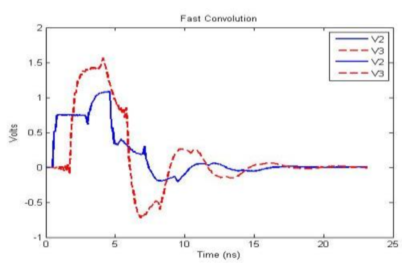
Netlist

Simulation

SPICE simulation from MOR generated Netlist



Direct Convolution



Equivalent-Circuit Extraction

Macromodel is curve-fit to take the form

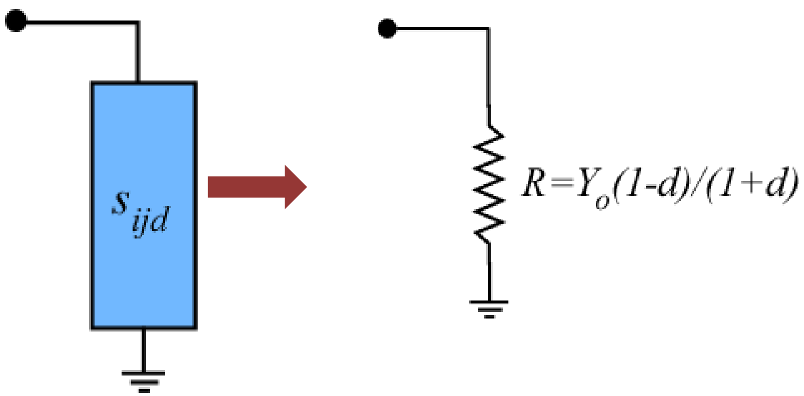
$$S(s) = d + \sum_{k=1}^{L} \frac{r_k}{s - p_k}$$

Need to find equivalent circuit associated with

- Constant term d
- Real Poles
- Complex Poles

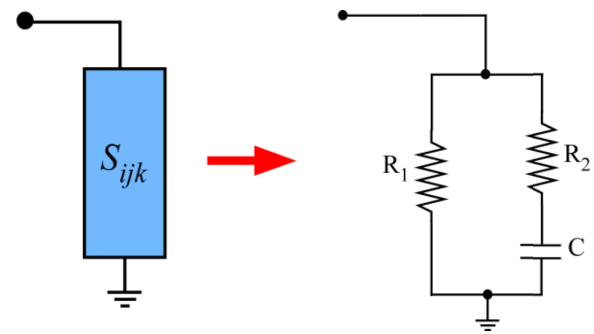
Equivalent-Circuit Extraction

Constant Term



$$R = Y_o\left(\frac{1-d}{1+d}\right)$$

Real Poles



$$R_2 = \frac{-1}{bC}$$

$$R_1 = -R_2 - \frac{1}{aC}$$

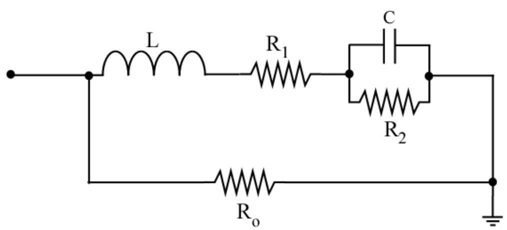
$$C = -\frac{(b-a)}{b^2 Z_o}$$

$$a = p_k + r_k, \quad \text{and} \quad b = p_k - r_k$$

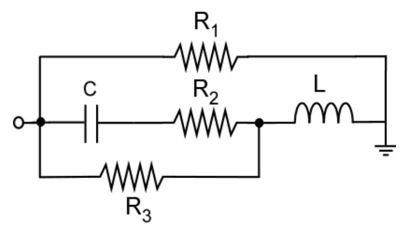
Realization – Complex Poles

There are several circuit topologies that will work

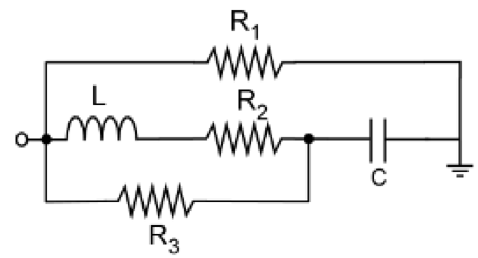
Model 1



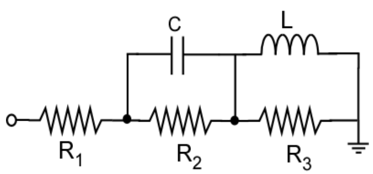
Model 8



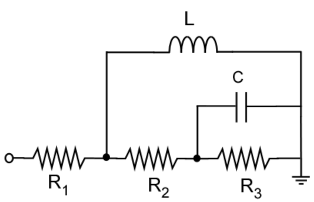
Model 9



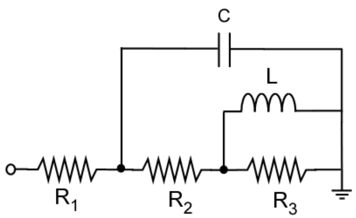
Model 11



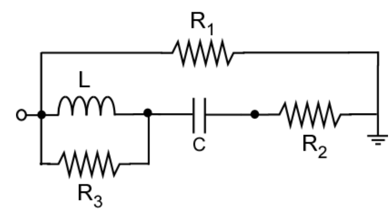
Model 12



Model 13



Model 10

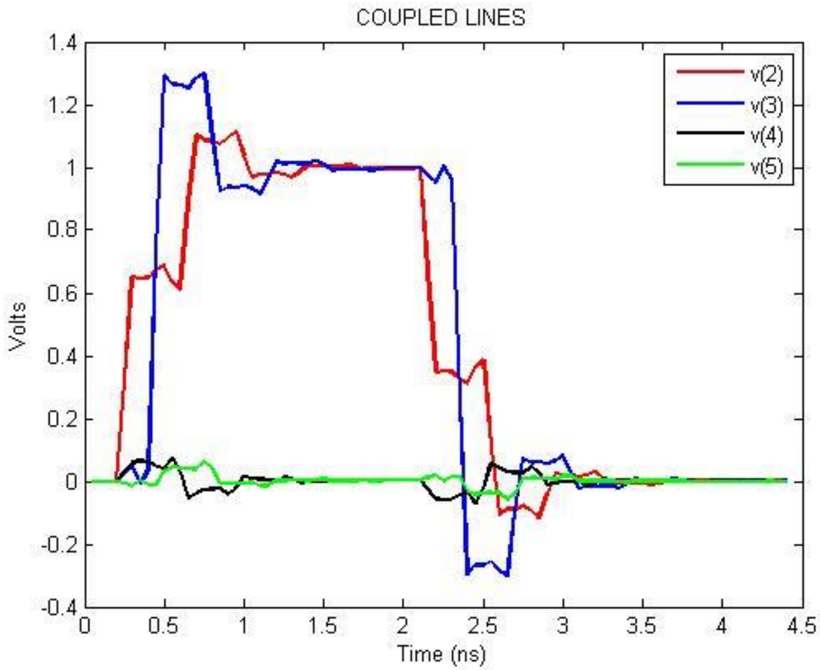


Netlist from Poles & residues

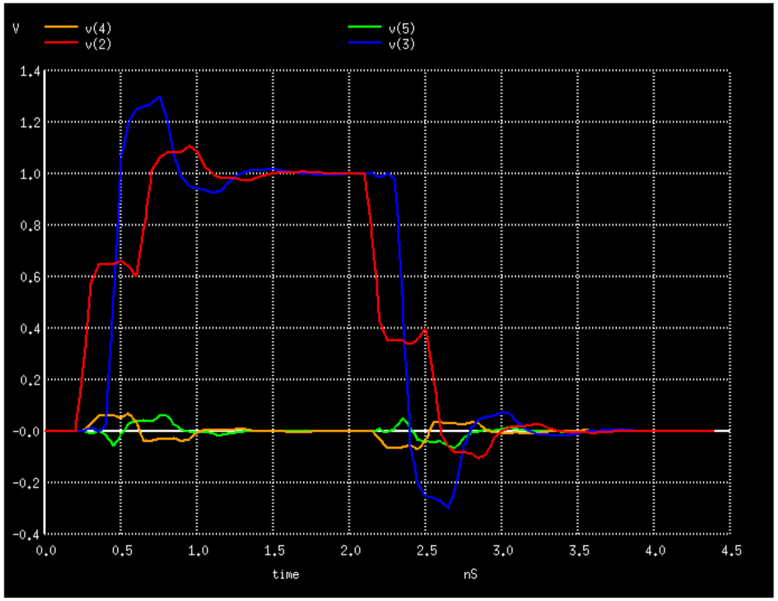
```
*Poll   2-port S-parameter circuit model
* 14 -pole approximation

.subckt Poll  42000  56000
vsens42001 42000 42001 0.0
vsens56001 56000 56001 0.0

*subcircuit for s[1][1]
*complex residue-pole pairs for S[1][1] at k=  1 -> 1st pole: -4.8961e+00 3.6506e+01 residue: 2.1006e-01 -2.8971e-01
*                                              -> 2nd pole: -4.8961e+00 -3.6506e+01 residue: 2.1006e-01 2.8971e-01
*circuit type =   9
elc1  1 0 42001 0 1.0
hc2  2  1 vsens42001 50.0
rtersc3  2  3 50.0
vp4  3  4 0.0
r1cd5  4  0 5.17406e+01
l1cd5  4  5 -1.25500e-08
r2cd6  5  6 -1.30103e+03
c1cd6  6  0 -7.19920e-15
r3cd6  4  6 1.48633e+03
*complex residue-pole pairs for S[1][1] at k=  2 -> 1st pole: -1.3039e+00 2.7679e+01 residue: -4.3856e-01 -1.9087e+00
*                                              -> 2nd pole: -1.3039e+00 -2.7679e+01 residue: -4.3856e-01 1.9087e+00
rtersc9  8  9 50.0
:
:
gs196    0 56001 196 0 0.020
rnort42001 42001 0 5.00000e+01
rnort56001 56001 0 5.00000e+01
.ends Poll
*main circuit
rgen 1 2 50.0
x1  2 3 Poll
vin 1 0  pulse (0 1  0.20000ns  0.10000ns  0.10000ns  2.00000ns  6.00000ns)
rport2 3 0 50000.0000000
.tran   0.00039ns  7.00000ns
.end
```

4-Port Network

Direct

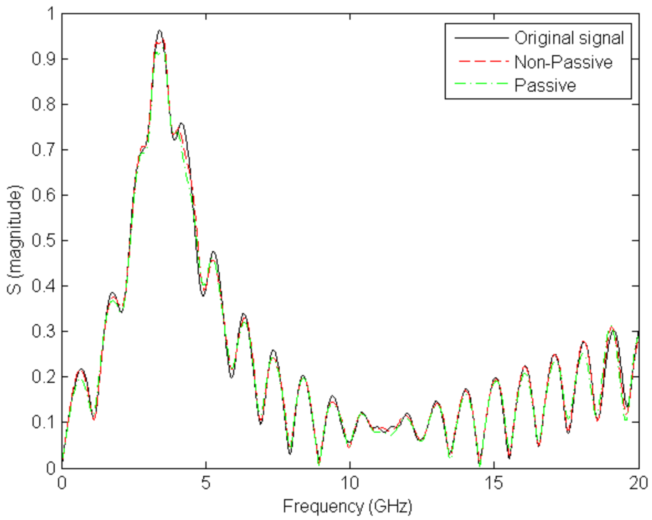


SPICE simulation Using generated netlist (Method 2)



Model-Order Reduction

- ➔ Start with S parameters from field solver
- ➔ Use vector fitting to get poles & residues
- ➔ Perform assessment via Hamiltonian
- ➔ Enforcement: Residue Perturbation Method
- ➔ Simulation: Recursive convolution➔Fast



Number of Ports	Order	CPU-Time
4	20	1.7 secs
6	32	3.69 secs
10	34	8.84 secs
20	34	33 secs
40	50	142 secs
80	12	255 secs

Review of some classic synthesis approaches for S matrix in pole-residue form*

1. (PI network for Y matrix)
2. PI network for S by Y + VCVS + CCVS
3. State-space S
4. State-space S-to-Y then PI
5. Pole-residue S-as-Y
6. Direct pole-residue specification

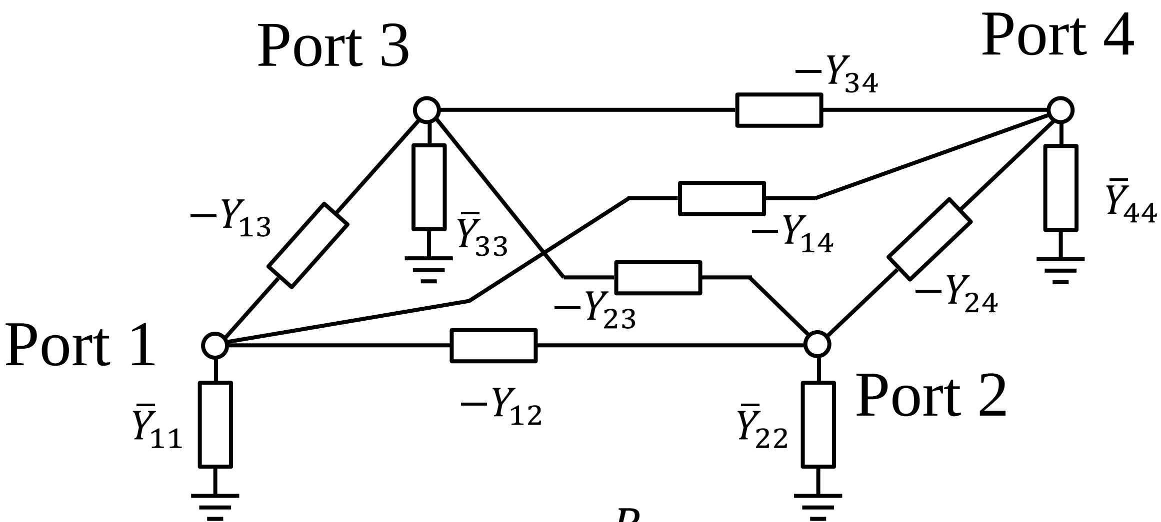
$$S_{ij} = d^{(ij)} + \sum_{k=1}^{N} \frac{r_k^{(ij)}}{s - p_k^{(ij)}} = d^{(ij)} + \sum_{k=1}^{N} S_{ij,k}$$

* Chiu-Chih Chou, José E. Schutt-Ainé, "Equivalent Circuit Synthesis of Multiport S Parameters in Pole–Residue Form", *IEEE Transactions on Components, Packaging and Manufacturing Technology,* Volume 11, Issue: 11, pp. 1971-1979, 2021, November 2021

Model 1. PI network for Y matrix (1/2)

$$Y_{ij} = d^{(ij)} + \sum_{k=1}^{N} \frac{r_k^{(ij)}}{s - p_k^{(ij)}}$$

Pole-residue Y matrix ➔ PI model (direct correspondence)

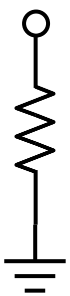


$$\bar{Y}_{ii} = \sum_{j=1}^{P} Y_{ij}$$

Model 1. PI network for Y matrix (2/2) (no controlled sources, but have negative elements)

Constant

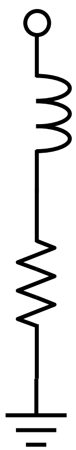
$$Y = d$$



$$R = \frac{1}{d}$$

A real pole

$$Y = \frac{r}{s-p}$$

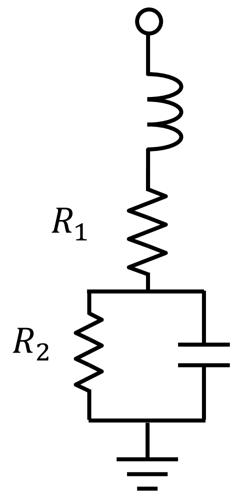


$$L = \frac{1}{r}$$

$$R = \frac{-p}{r}$$

A pair of complex conjugate poles (RLCR)

$$Y = \frac{r}{s-p} + \frac{r^*}{s-p^*}$$



$$L = \frac{1}{2r_r}$$

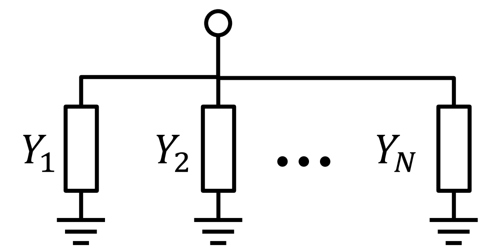
$$R_1 = \frac{-p_r r_r + p_i r_i}{2r_r^2}$$

$$C = \frac{-r_r}{(p_r r_r + p_i r_i)} \cdot \frac{1}{R_2}$$

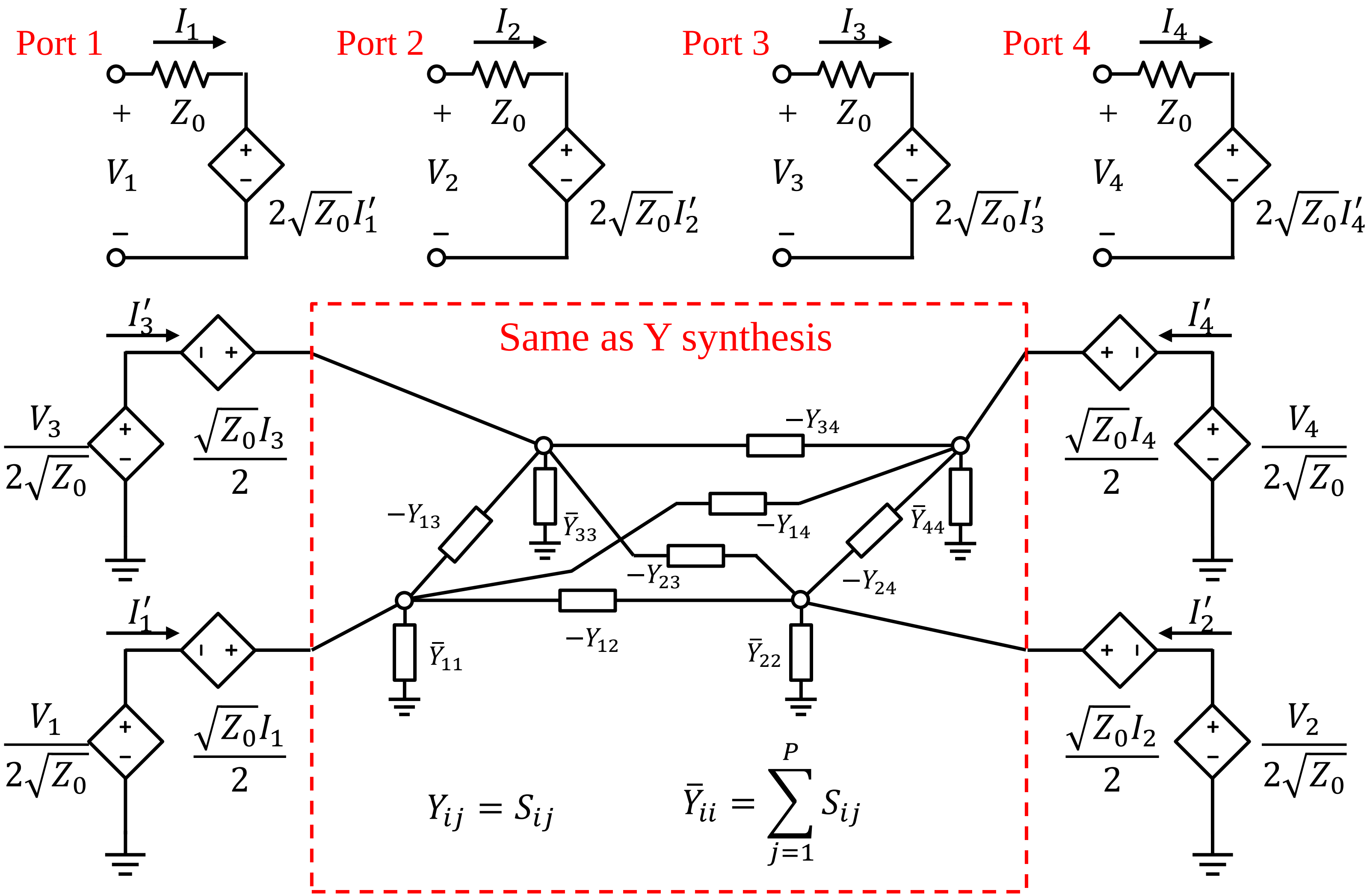
$$R_2 = -\frac{p_r^2 + p_i^2}{2(p_r r_r + p_i r_i)} - R_1$$

N pole-residue pairs

$$Y = \sum_{k=1}^{N} Y_k$$



Model 2. PI network for S by Y + VCVS + CCVS

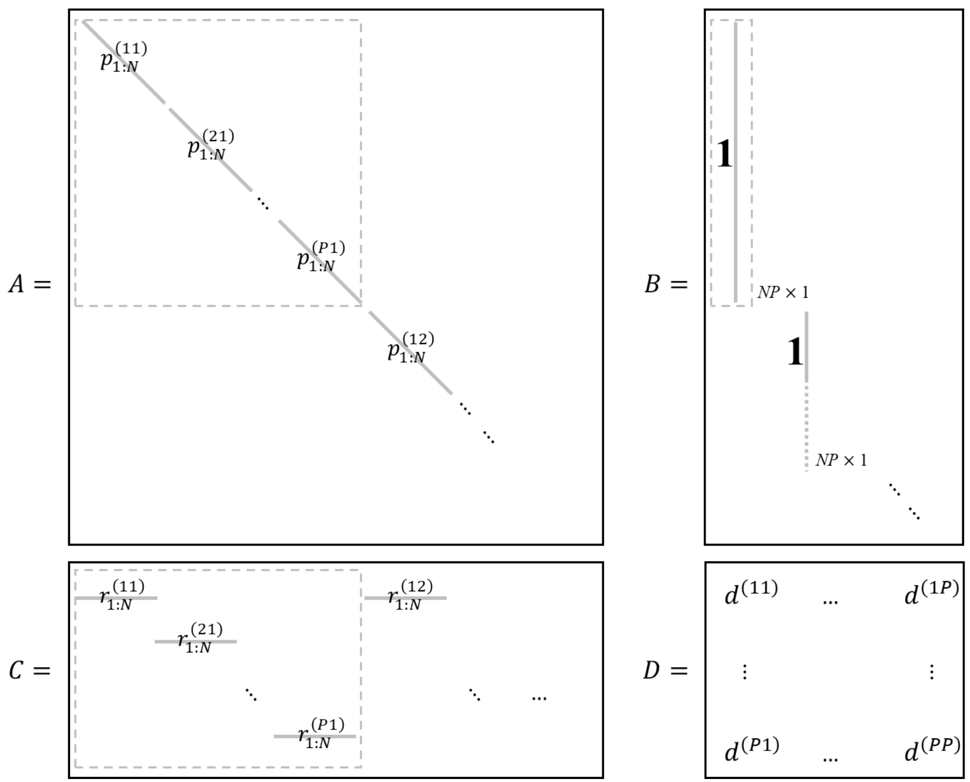


Model 3. State-space S (1/2) (a common cross-platform topology)

$$S_{ij} = d^{(ij)} + \sum_{k=1}^{N} \frac{r_k^{(ij)}}{s - p_k^{(ij)}}$$

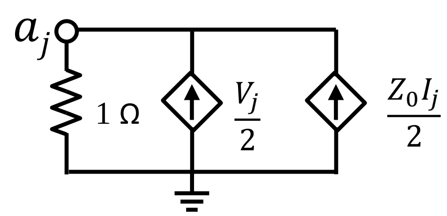
Pole-residue
to state-space

$$\begin{cases} A = \operatorname*{diag}_{\substack{j=1\ldots P \\ i=1\ldots P \\ k=1\ldots.N}} p_k^{(ij)} \\ B_{nj} = 1_{\{(j-1)NP<n\le jNP\}} \\ C_{in} = r_k^{(ij)} \cdot 1_{\{n\,=\,(j-1)NP+(i-1)N+k\}} \\ D_{ij} = d^{(ij)} \end{cases}$$



Model 3. State-space S (2/2)

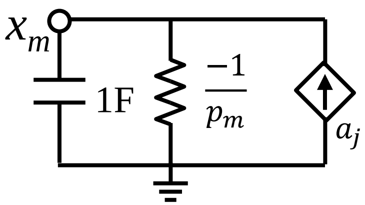
Input equations (incident wave):



State equations:

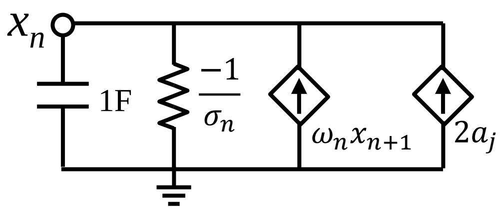
real pole p_m

$\dot{x}_m = p_m x_m + a_j$

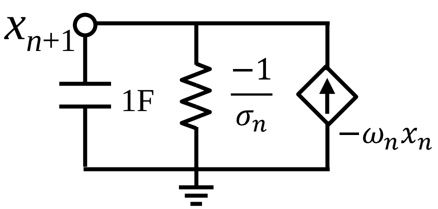


complex-conjugate poles $p_n = \sigma_n + j\omega_n, \quad p_{n+1} = \sigma_n - j\omega_n$

$\dot{x}_n = \sigma_n x_n + \omega_n x_{n+1} + 2a_j$

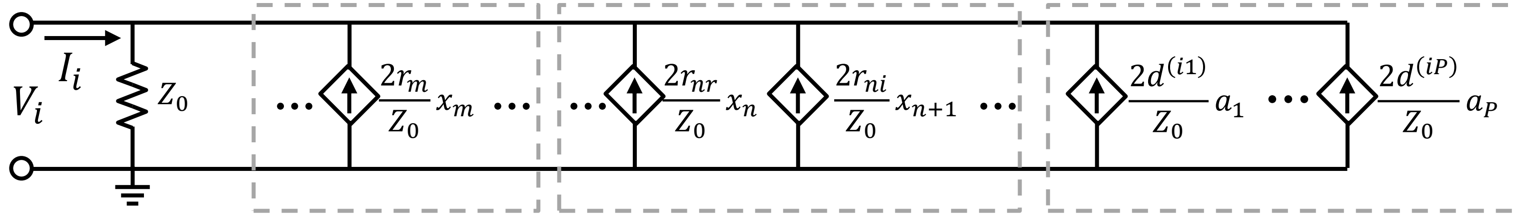


$\dot{x}_{n+1} = \sigma_n x_{n+1} - \omega_n x_n$



Output equations:

Port *i* | real poles | complex-conjugate poles | direct coupling terms



Model 4. State-space S to Y then PI (no controlled sources needed)

State-space for S matrix

$$\begin{cases} A = \operatorname{diag}\limits_{\substack{j=1\dots P \\ i=1\dots P \\ k=1\dots N}} p_k^{(ij)} \\ B_{nj} = 1_{\{(j-1)NP<n\le jNP\}} \\ C_{in} = r_k^{(ij)} \cdot 1_{\{n\ =\ (j-1)NP+(i-1)N+k\}} \\ D_{ij} = d^{(ij)} \end{cases}$$

State-space for Y matrix

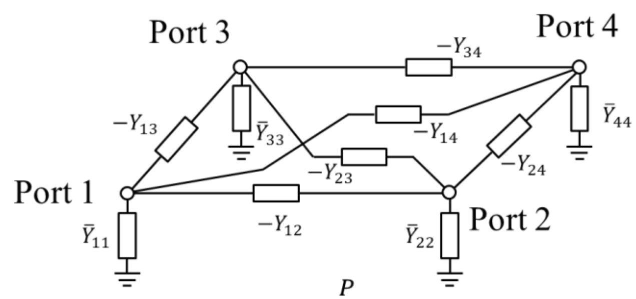
$$\begin{cases} A' = A - B(I+D)^{-1}C \\ B' = \dfrac{1}{\sqrt{Z_0}} B(I+D)^{-1} \\ C' = \dfrac{-2}{\sqrt{Z_0}} (I+D)^{-1}C \\ D' = \dfrac{1}{Z_0}(I-D)(I+D)^{-1} \end{cases}$$

Diagonalization

Pole-residue for Y matrix

$$Y_{ij} = d^{(ij)} + \sum_{k=1}^{N} \frac{r_k^{(ij)}}{s - p_k^{(ij)}}$$

PI model

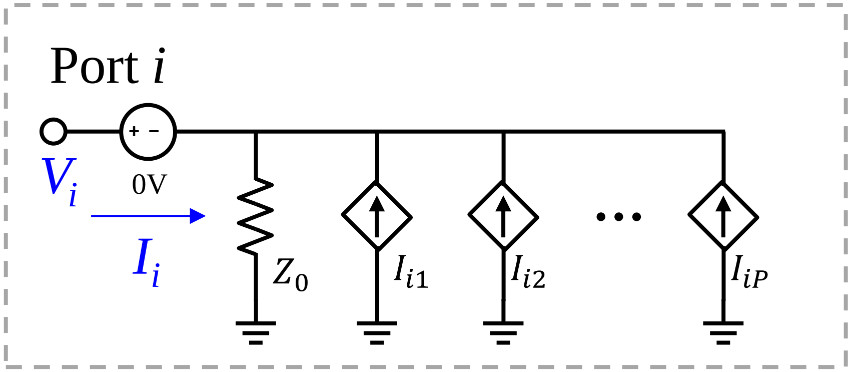


$$\bar{Y}_{ii} = \sum_{j=1}^{P} Y_{ij}$$

Problem: $(I + D)$ may be singular!

Model 5. Pole-residue S-as-Y (minimized for SISO pole)

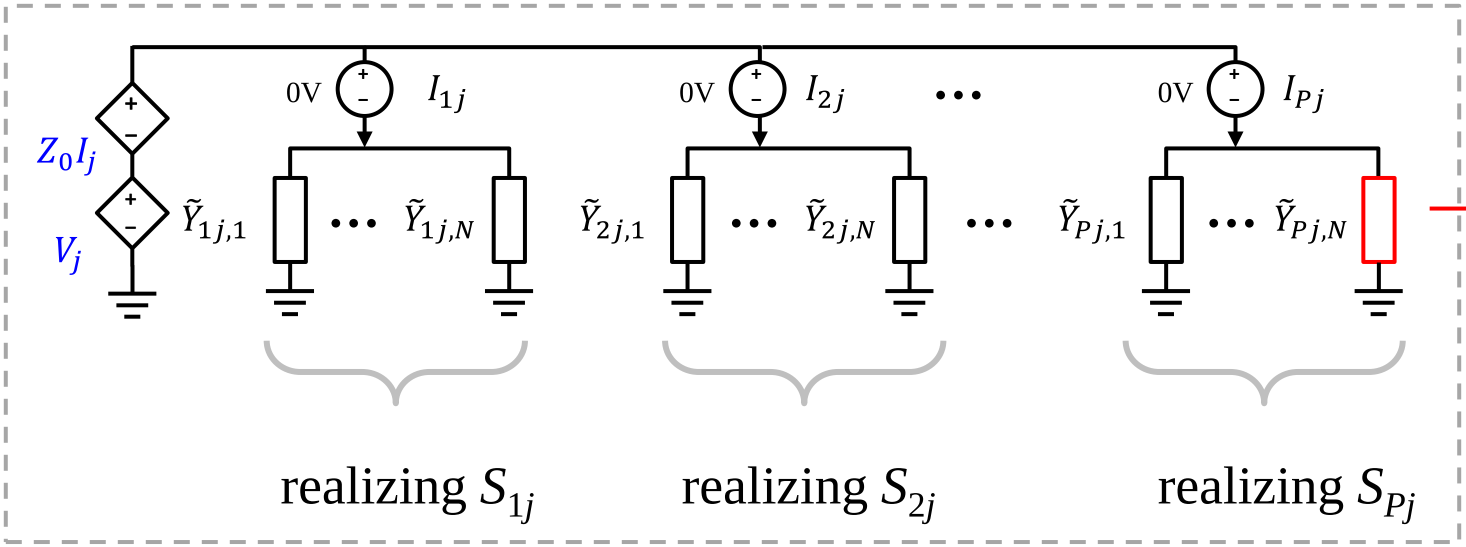
Port sections: $i = 1, 2, \ldots, P$



Use the same approach as model 1 (Y branch)

$$\tilde{Y}_{ij,k} \triangleq \frac{1}{Z_0} S_{ij,k}$$

Intermediate stages: $j = 1, 2, \ldots, P$

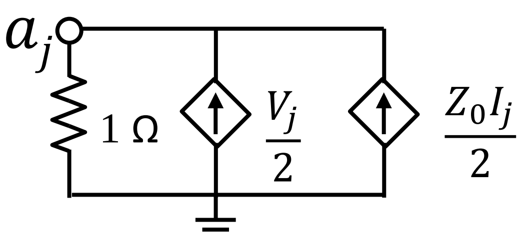


Model 6. Direct pole-residue specification (permit recursive convolution)

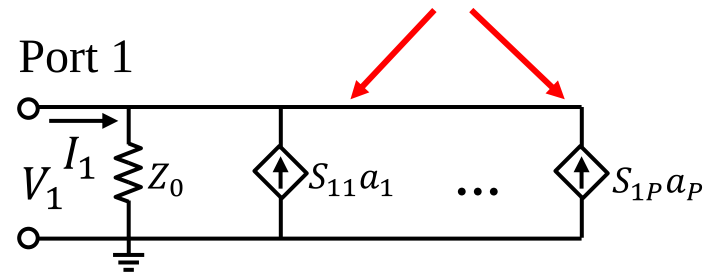
$$S_{ij} = d^{(ij)} + \sum_{k=1}^{N} \frac{r_k^{(ij)}}{s - p_k^{(ij)}}$$

Not all simulator support this format!

Incident wave calculator for each port



Foster G element




```
Gs_1_1  gnd_0  p_1  FOSTER  inc_1  0  -5.69243475541412 6e-03  0
+ ( -2.43915043185622 2e+04, 0 ) / ( -4.96047604161921 0e+07, 0 )
+ (  6.16073177854365 6e+07, 0 ) / ( -1.71417039466460 3e+09, 0 )
+ ( -1.89478842252053 8e+07,  5.66229376645860 2e+07 ) / ( -1.67218
+ (  8.71396634829680 2e+07, -8.13962212929225 4e+07 ) / ( -1.84733
+ (  2.61717341198152 7e+08,  1.35846443493299 4e+08 ) / ( -2.17708
+ (  1.63354379245143 9e+07,  6.27157348818774 0e+08 ) / ( -2.57889
+ ( -7.92545322499179 8e+08, -3.73145747944773 3e+07 ) / ( -2.81115
+ ( -1.39341694575600 4e+08, -3.07839808832204 8e+08 ) / ( -2.26115
+ (  3.61228306947034 6e+06, -4.99390841438674 2e+06 ) / ( -1.35763
+ ( -1.35975093314697 7e+07, -2.87575264814606 5e+08 ) / ( -2.39584
```

Comparison

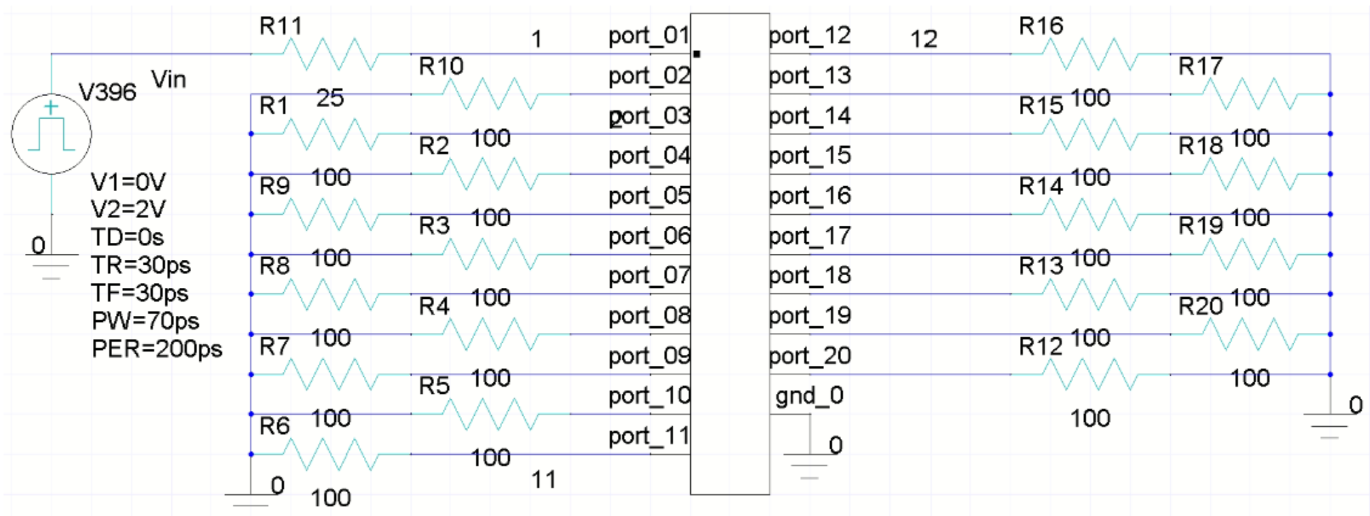
	Model 2	Model 3	Model 4	Model 5	Model 6
	PI network for S by Y + VCVS+CCVS	State-space S	State-space S to Y	Pole-residue S-as-Y	Direct pole-residue specification
Controlled sources	Yes	Yes	No	Yes	Yes
Negative RLC	Yes	No	Yes	Yes	No
Recursive convolution	No	No	No	No	Yes
Cross platform	Yes*	Yes	Yes*	Yes*	No

* if negative RLC permitted

Offered by many commercial EDA tools.

Example: macromodel of 20 ports and 110 MIMO poles

(10 coupled microstrips)



TRAN
Tstep = 1ps
Tstop = 100ns

Total simulation time (s)

	State-space S (model 3)	PI network (model 2)	Foster (model 6)
EDA Tool A	338	815	190
EDA Tool B	295	383	94
Ngspice	200	788	

Conclusion

- Many different ways to synthesize equivalent circuits for S-parameters in pole-residue form
- Considerations for choosing circuit topology
  - Want recursive convolution?
  - Want cross platform exchangeability?
  - Controlled source and negative RLC acceptable?
  - SISO or MIMO pole-residue model?