Model Order Reduction for Design with Signal Integrity

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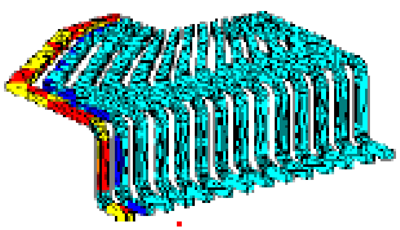
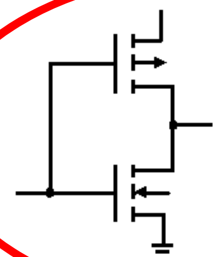
IBIS Talk
January 26, 2022

Interconnect Structures

Crosstalk
Couplings
Reflections
Losses
Dispersion

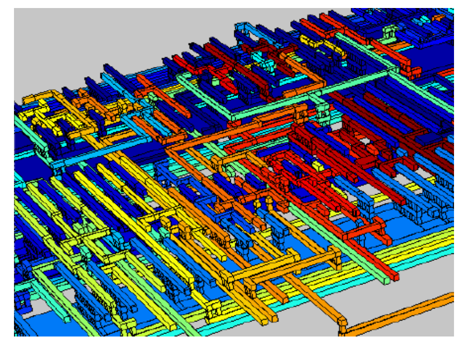


Packages

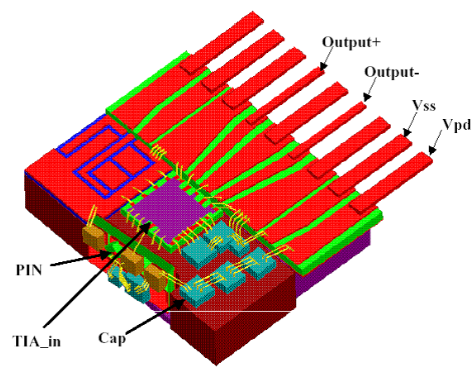


Circuit Simulator

Ground Noise
Nonlinear effects
Radiation, EMI

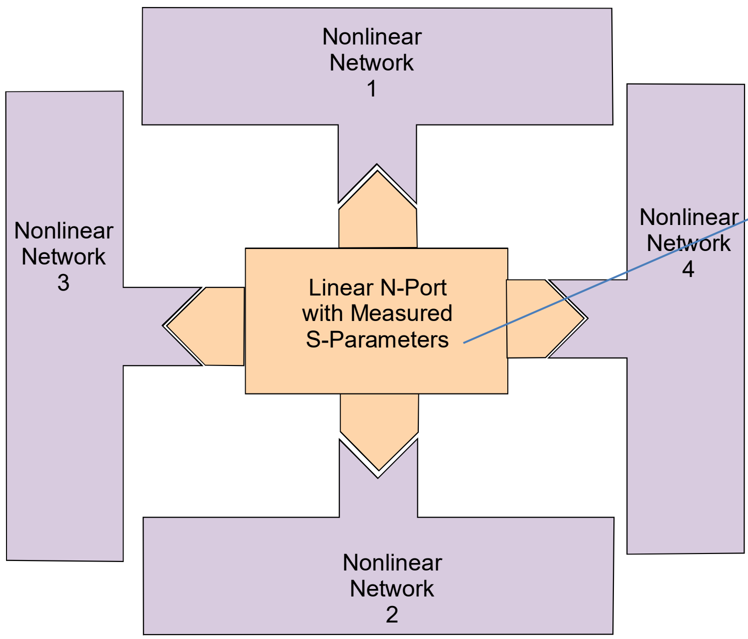


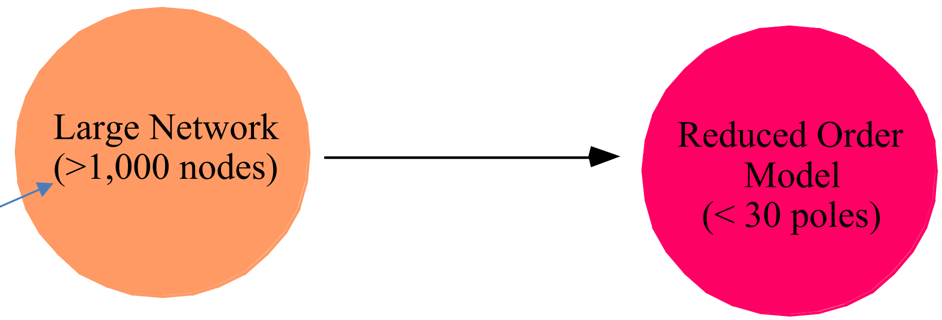
Interconnects



Courtesy of http://www.ansoft.com/hfworkshop03/Weimin_Sun_Vitesse.pdf

Model-Order Reduction





SPICE

$Y(t)\,v(t) = i(t)$

$$\tilde{Y}(\omega) = \left[A_1 + \sum_{i=1}^{L} \frac{a_{1i}}{1 + j\omega / \omega_{c1i}} \right]$$

$Y(\omega)\,V(\omega) = I(\omega)$

↓

Order Reduction

↓

$Y(\omega) \simeq \tilde{Y}(\omega)$

↓

Recursive Convolution

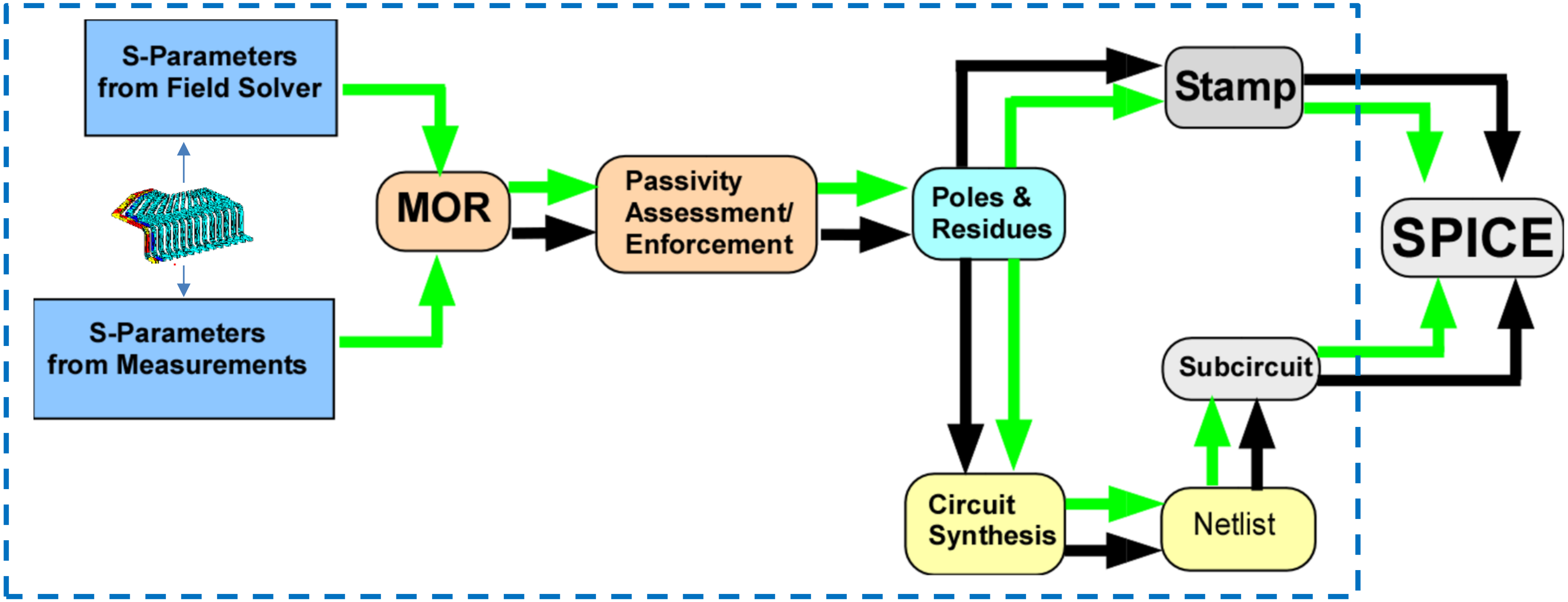
↓

$\tilde{Y}(t)\,v(t) = i(t)$

- **Strategy**: Use reduced order model to minimize computation time.

$$\tilde{Y}(\omega) \ll Y(\omega)$$

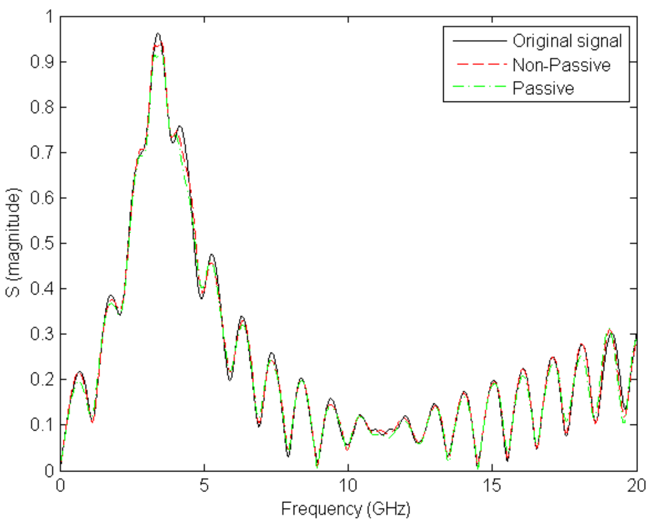
Model-Order Reduction



- **Objective**: Incorporate frequency dependence into time-domain simulator
- **Approaches**: 1) Direct integration of code into SPICE – 2) Generation of SPICE-compatible netlist

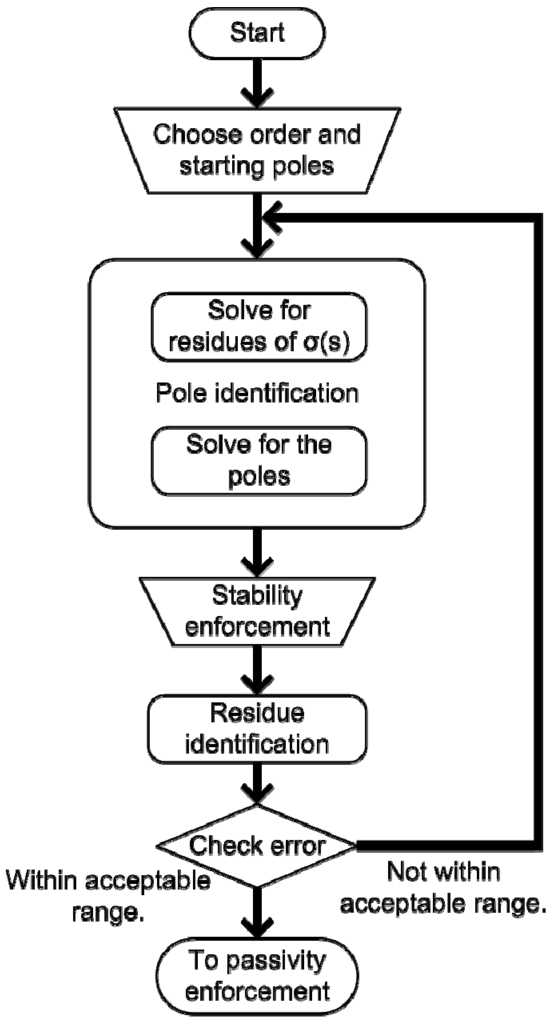
Model-Order Reduction

- ➔ **Start with S parameters from field solver**
- ➔ **Use vector fitting to get poles & residues**
- ➔ **Perform assessment via Hamiltonian**
- ➔ **Enforcement: Residue Perturbation Method**
- ➔ **Simulation: Recursive convolution➔Fast**

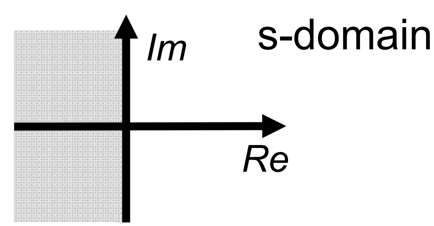


Number of Ports	Order	CPU-Time
4-Port	20	1.7 secs
6-port	32	3.69 secs
10-port	34	8.84 secs
20-port	34	33 secs
40	50	142 secs
80	12	255 secs

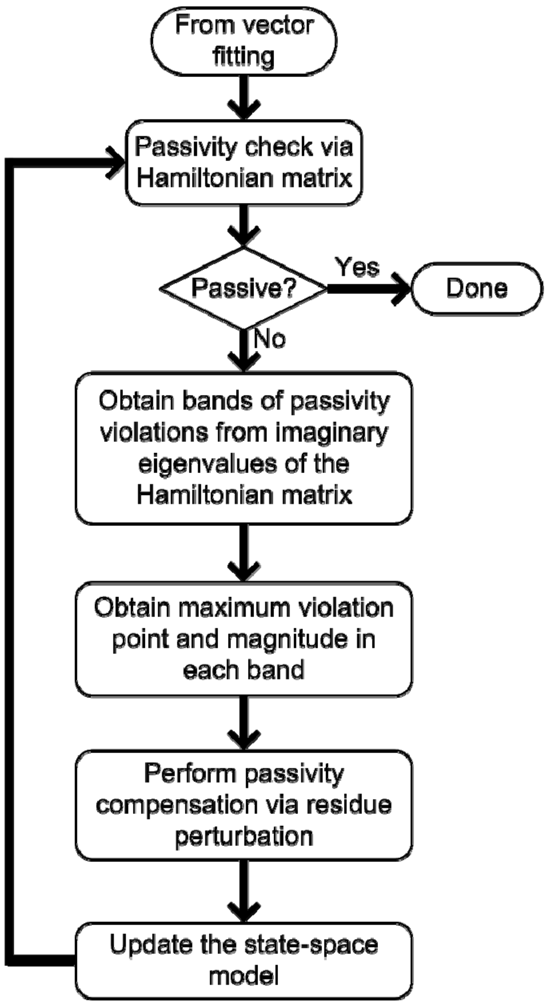
MOR via Vector Fitting



- Rational function approximation: $$f(s) \approx \sum_{n=1}^{N} \frac{c_n}{s-a_n} + d + sh$$
- Introduce an unknown function *σ(s)* that satisfies: $$\begin{bmatrix} \sigma(s) f(s) \\ \sigma(s) \end{bmatrix} \approx \begin{bmatrix} \sum_{n=1}^{N} \frac{c_n}{s-\tilde{a}_n} + d + sh \\ \sum_{n=1}^{N} \frac{\tilde{c}_n}{s-\tilde{a}_n} + 1 \end{bmatrix}$$
- Poles of *f(s)* = zeros of *σ(s)*: $$f(s) \approx \frac{\sum_{n=1}^{N} \frac{c_n}{s-\tilde{a}_n} + d + sh}{\sum_{n=1}^{N} \frac{\tilde{c}_n}{s-\tilde{a}_n} + 1} = \frac{\prod_{n=1}^{N+1} (s-z_n)}{\prod_{n=1}^{N} (s-\tilde{z}_n)}$$
- Flip unstable poles into the left half plane.



Passivity Enforcement



- State-space form:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

- Hamiltonian matrix:

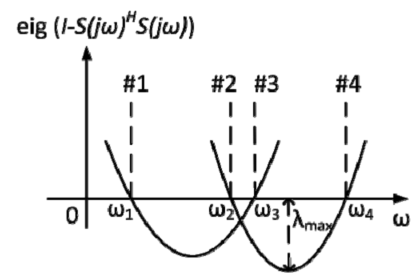
$$M = \begin{bmatrix} A + BKD^T C & BKB^T \\ -C^T LC & -A^T - C^T DKB^T \end{bmatrix}$$

$$K = \left(I - D^T D\right)^{-1} \quad L = \left(I - DD^T\right)^{-1}$$

- Passive if *M* has no imaginary eigenvalues.

- Sweep:

$$eig\left(I - S(j\omega)^H S(j\omega)\right)$$



- Quadratic programming:
  - Minimize *(change in response)* subject to *(passivity compensation).*

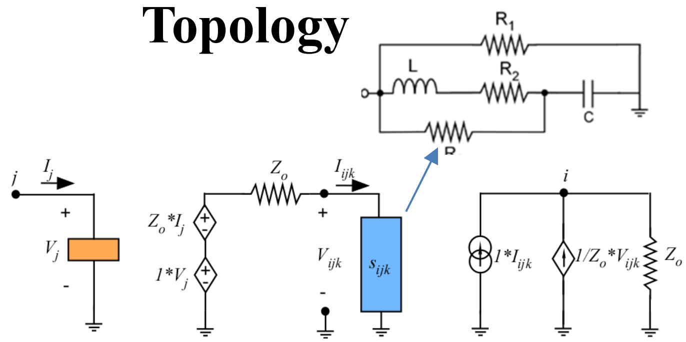
$$\min\left(vec(\Delta C)^T H\, vec(\Delta C)\right) \quad \text{subject to} \quad \Delta\lambda = G \cdot vec(\Delta C).$$

SPICE Netlist Synthesis

- Goal is to generate (using pole/residue information) a circuit netlist that will exhibit the same (frequency-dependent) behavior as that of the S-parameters of connector under study

Passive Poles/Residues from MOR

Circuit Cell Topology

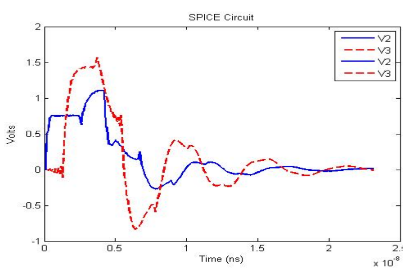


Synthesis Algorithm

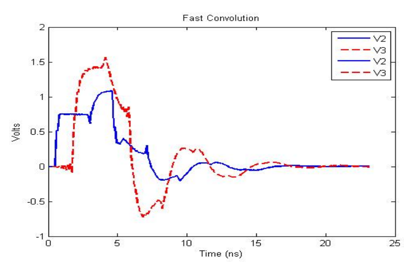
Netlist

Simulation

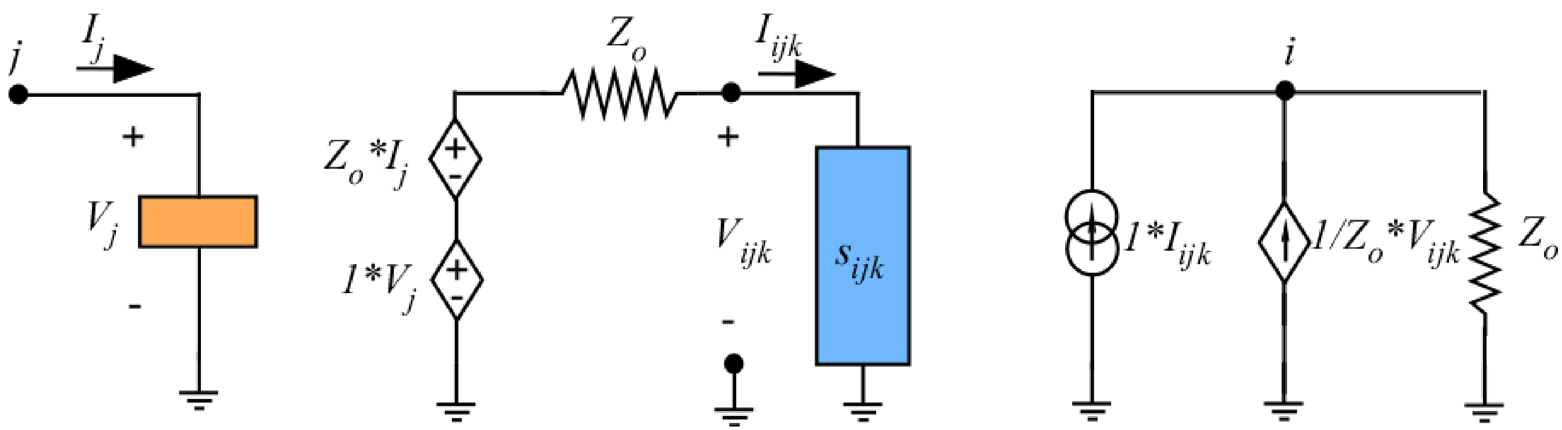
SPICE simulation from MOR generated Netlist



Direct Convolution



S-Parameter Circuit Synthesis



$$A_i(\omega) = \frac{1}{2}\left[V_i(\omega) + Z_o I_i(\omega)\right]$$

$$B_i(\omega) = \frac{1}{2}\left[V_i(\omega) - Z_o I_i(\omega)\right]$$

Need equivalent circuit for S_{ijk}

Strategy

For a given circuit, a relationship between the input admittance $Y_{ijk}(s)$ of the circuit and the associated one-port S-parameter representation $S_{ijk}(s)$ can be described by

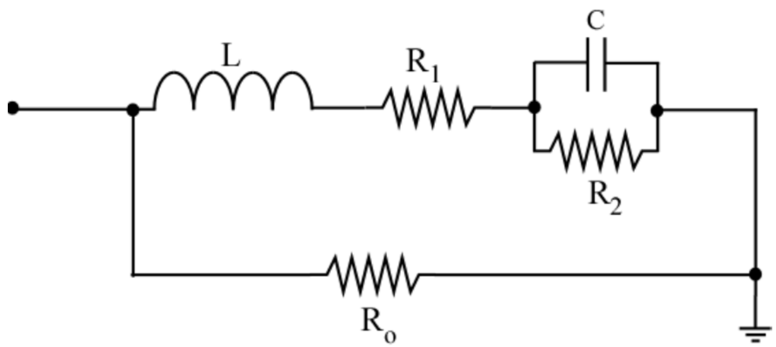
$$S_{ijk}(s) = \frac{Y_o - Y_{ijk}(s)}{Y_o + Y_{ijk}(s)} \qquad Y_{ijk}(s) = Y_o \frac{1 - S_{ijk}(s)}{1 + S_{ijk}(s)}$$

Y_o is the reference admittance

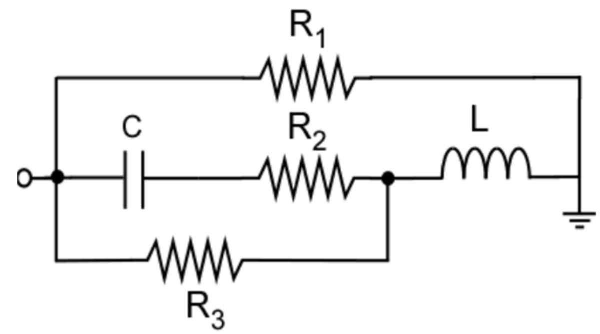
Realization – Complex Poles

There are several circuit topologies that will work

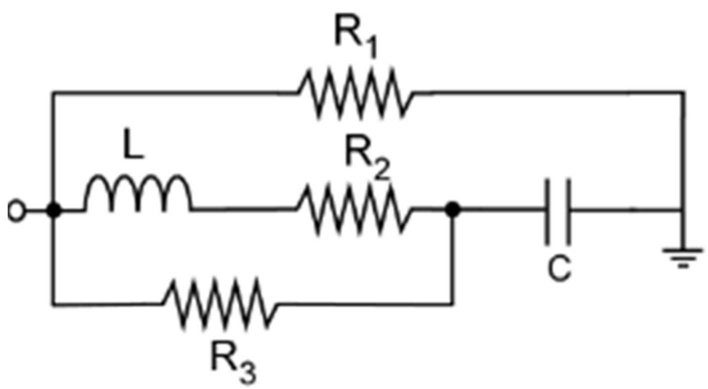
Model 1



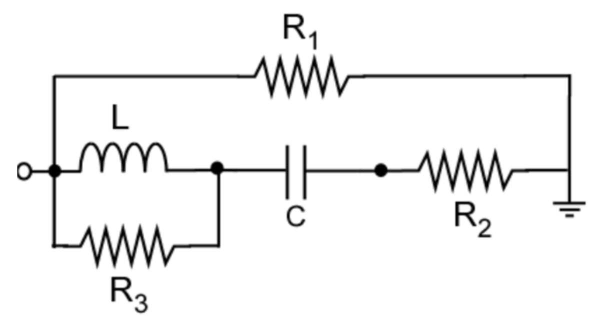
Model 8



Model 9



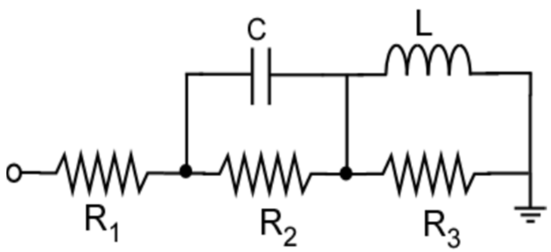
Model 10



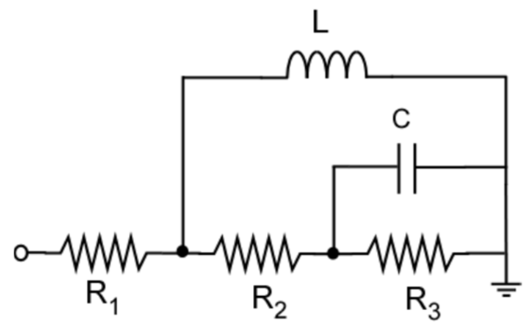
Realization – Complex Poles

More circuit topologies that will work

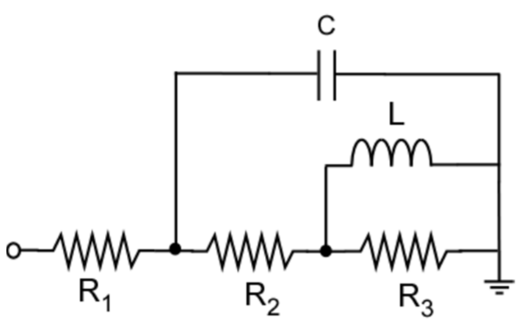
Model 11



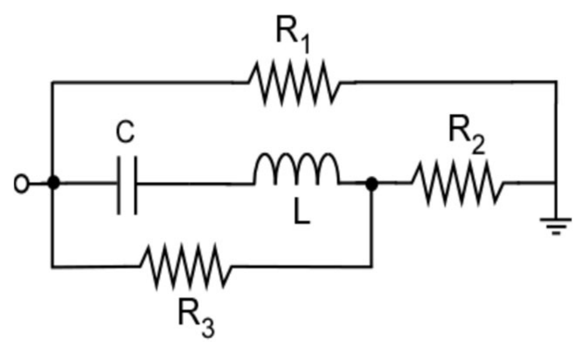
Model 12



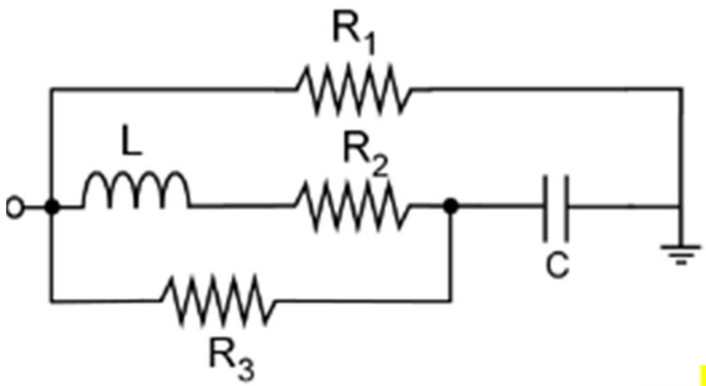
Model 13



Model D



Complex Poles – Model 9



$$R_1 = \frac{F}{BH}$$

$$C = \frac{(-BD + AF)H}{F^2}$$

$$R_2 = \frac{(-BD^2 + BF + ADF - F^2)}{(B^2 - ABD + BD^2 + A^2F - 2BF - ADF + F^2)H}$$

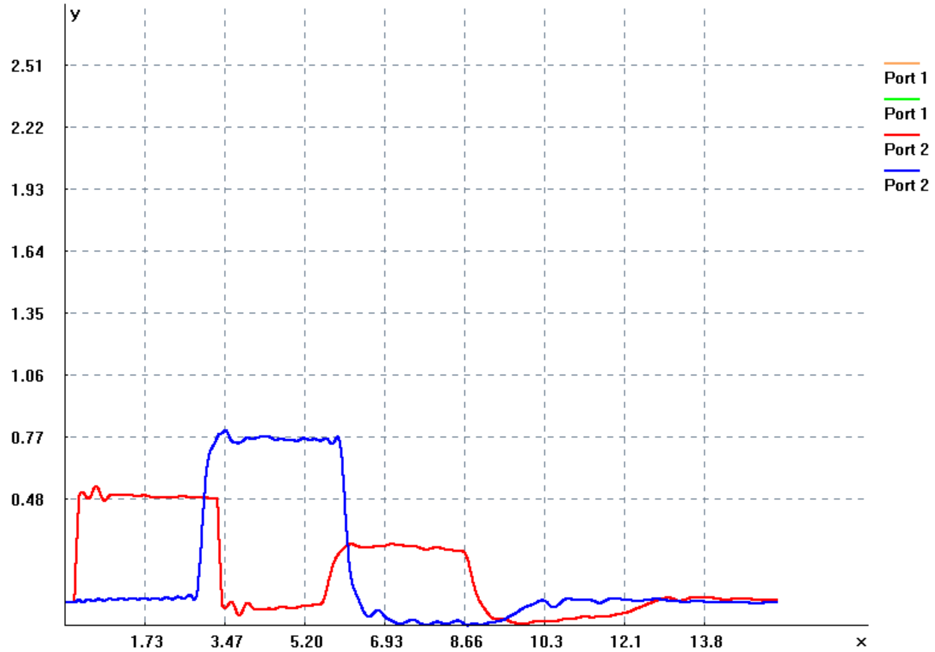
$$R_3 = \frac{F}{(-B + F)H}$$

$$L = \frac{-BD + AF}{(B^2 - ABD + BD^2 + A^2F - 2BF - ADF + F^2)H}$$

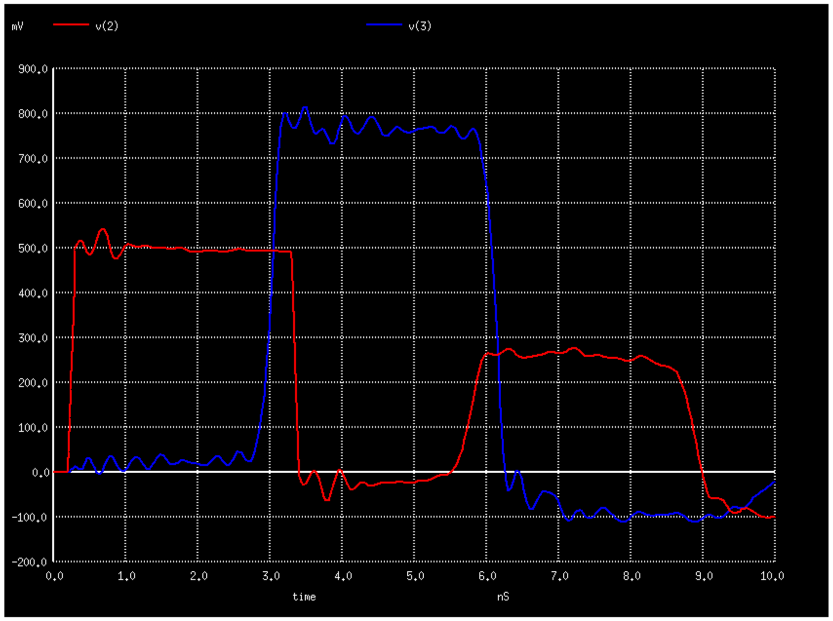
Netlist from Poles & Residues

```
*Myckt   2-port S-parameter circuit model
* 14 -pole approximation

.subckt Myckt  42000  56000
vsens42001 42000 42001 0.0
vsens56001 56000 56001 0.0

*subcircuit for s[1][1]
*complex residue-pole pairs for S[1][1] at k=  1 -> 1st pole: -4.8961e+00 3.6506e+01 residue: 2.1006e-01 -2.8971e-01
*                                           -> 2nd pole: -4.8961e+00 -3.6506e+01 residue: 2.1006e-01 2.8971e-01
*circuit type =   9
elc1  1 0 42001 0 1.0
hc2  2  1 vsens42001 50.0
rtersc3  2  3 50.0
vp4  3  4 0.0
r1cd5  4  0 5.17406e+01
l1cd5  4  5 -1.25500e-08
r2cd6  5  6 -1.30103e+03
c1cd6  6  0 -7.19920e-15
r3cd6  4  6 1.48633e+03
*complex residue-pole pairs for S[1][1] at k=  2 -> 1st pole: -1.3039e+00 2.7679e+01 residue: -4.3856e-01 -1.9087e+00
*                                           -> 2nd pole: -1.3039e+00 -2.7679e+01 residue: -4.3856e-01 1.9087e+00
rtersc9  8  9 50.0
:
:
gs196    0 56001 196 0 0.020
rnort42001 42001 0 5.00000e+01
rnort56001 56001 0 5.00000e+01
.ends Myckt
*main circuit
rgen 1 2 50.0
x1  2 3 Myckt
vin 1 0  pulse (0 1  0.20000ns  0.10000ns  0.10000ns  2.00000ns  6.00000ns)
rport2 3 0 50000.0000000
.tran   0.00039ns  7.00000ns
.end
```

2-Port Network



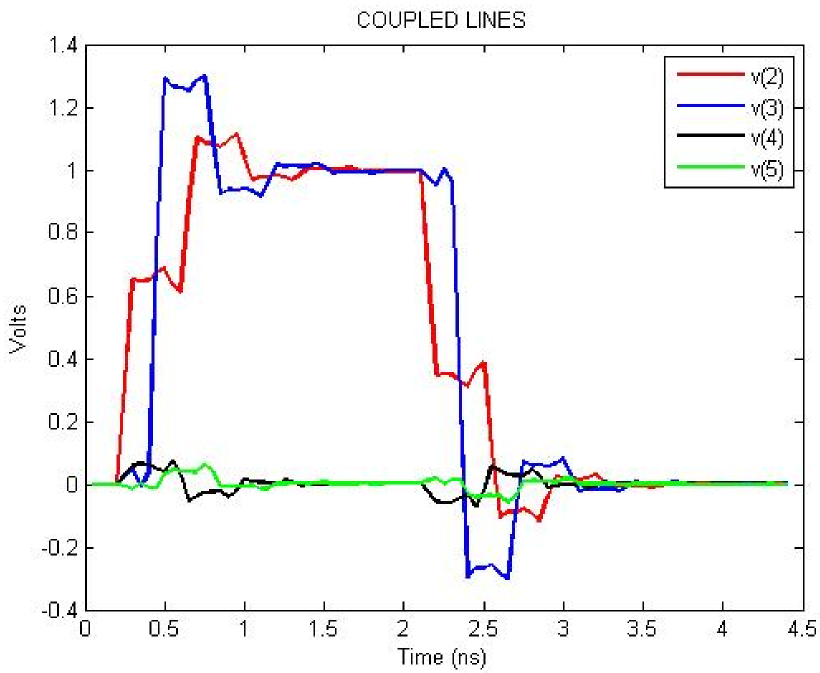
Recursive convolution



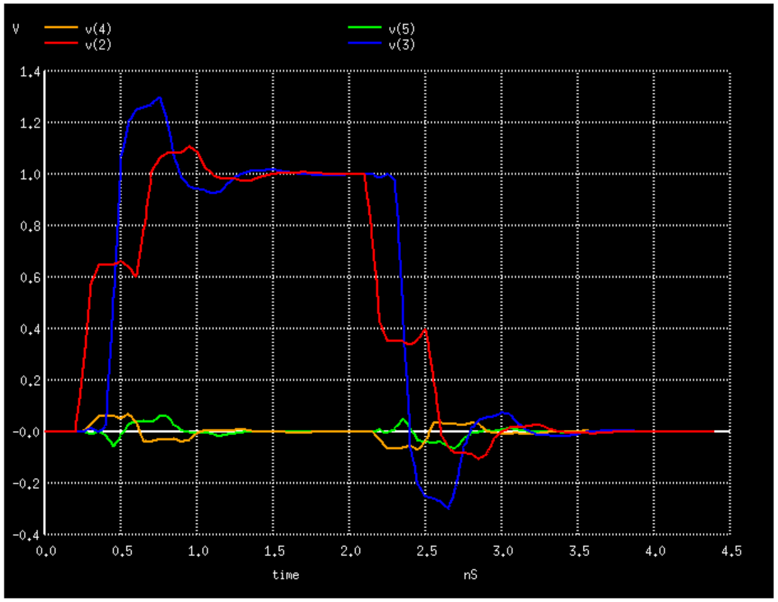
SPICE realization

4-Port Network

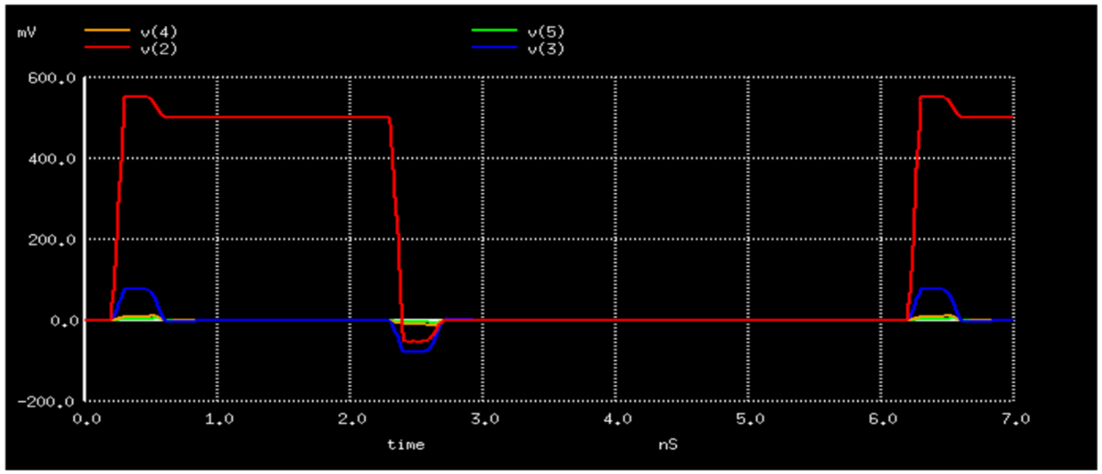
Direct



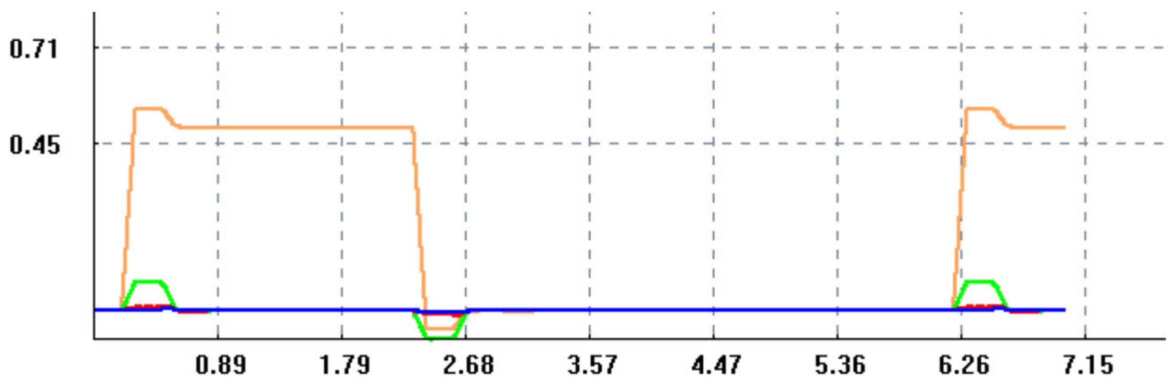
SPICE simulation Using generated netlist (Method 2)



8-Port Network

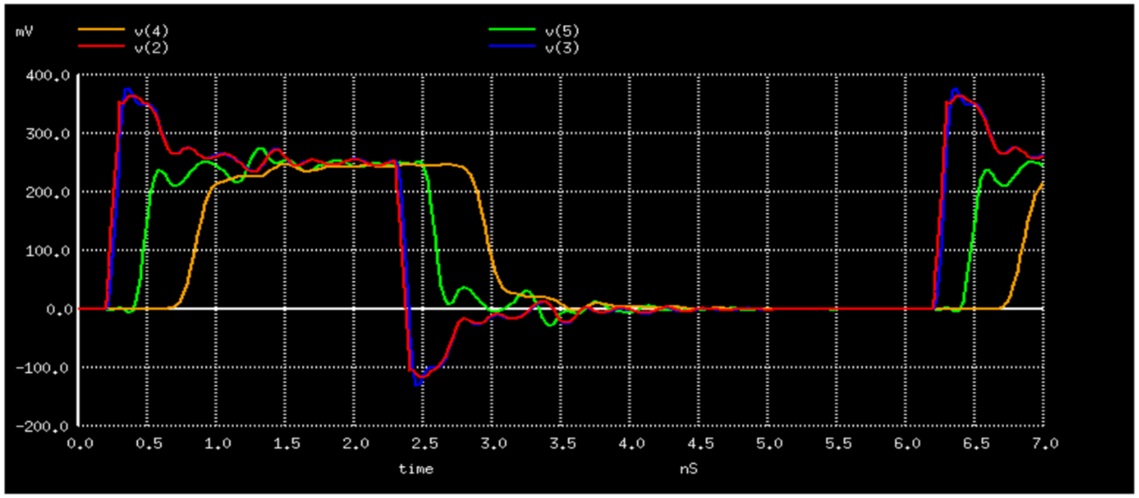


SPICE simulation Using generated netlist

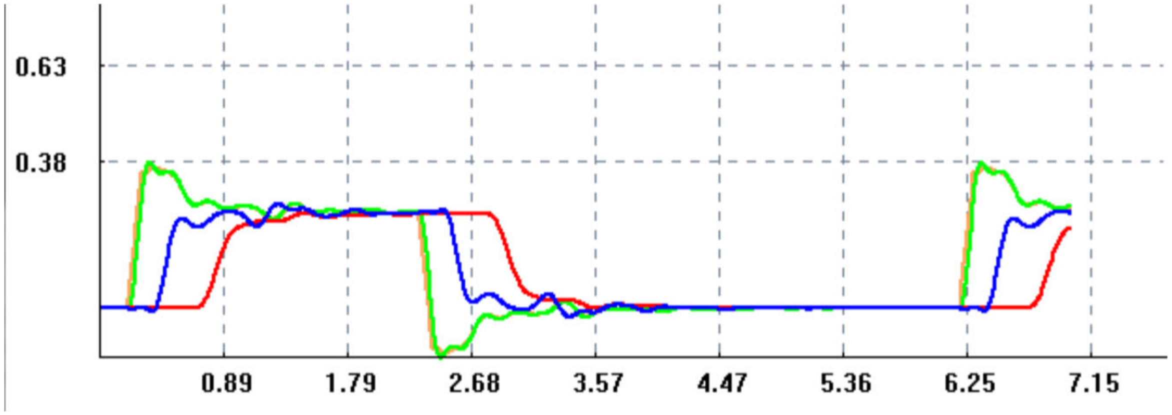


Direct

10-Port Network

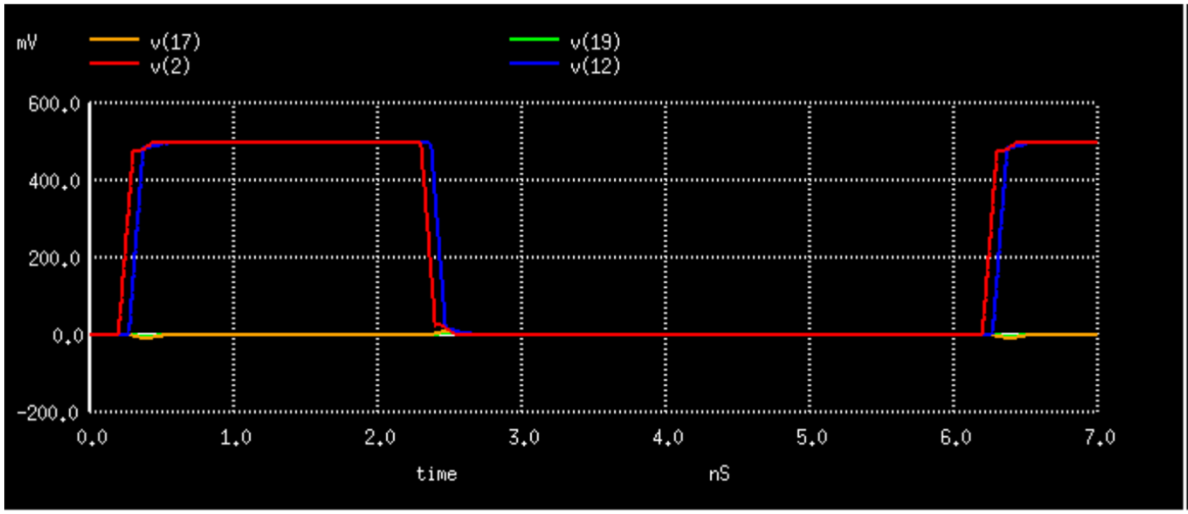


SPICE simulation Using generated netlist

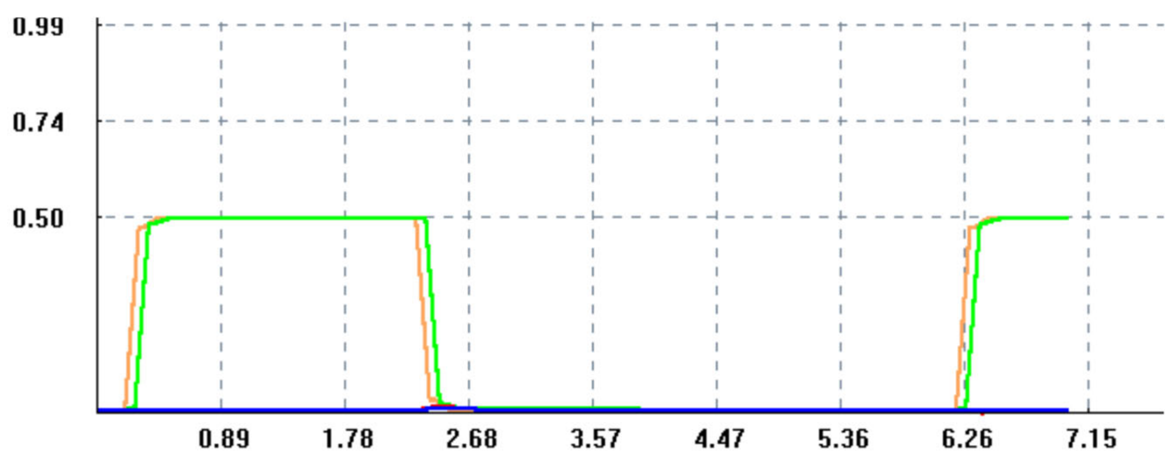


Direct

20-Port Network



SPICE simulation Using generated netlist



Direct

Conclusion

- S-parameter based SPICE circuits are robust
- Several topologies can be used for complex poles
- 20-port networks have been successfully tested
- Good agreement with direct simulations