

From Vladimir Dmitriev-Zdorov

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There are a few reasons we (Mentor/Siemens) chose the format below for S-parameters instead of the Foster format:

- 1) It is problematic to write the expression that defines the S-parameter matrix component in the Foster form because it may require hundreds of complex and real poles. The format we use has a fixed number of columns (4), which are the real/imaginary part of frequency and the real/imaginary part of the coefficient). This greatly simplifies the parser and the writer code.
- 2) We'd like to see actual contribution from each pole (or conjugate pair) into the transfer function. In equation (2) below the factor A shows how much a particular pole contributes into DC value. The coefficients A, B have the same dimension as the parameters, i.e., dimensionless for S, or impedance/admittance for Z/Y, but denominator is dimensionless. In the Foster format, denominator is measured in rad/sec therefore the factors in the numerator bear this unit as well and are not normalized. That is, in the Foster format, the coefficients at the larger poles are typically very large as well, so that their contribution into DC equals their ratio.
- 3) In the PLS format we define pole frequencies in Hz, not rad/s, similar to the sampled Touchstone data. This simplifies interpretation of the values. Since the denominator is dimensionless, the poles could be easily rescaled (into rad/s) without touching the coefficients, which is not possible in Foster format.
- 4) There is no difference in the format between the real pole or a complex conjugate pair of poles (except that for a real pole the imaginary parts of the pole frequency or the coefficient are zero), but the Foster format assumes different number of summands.
- 5) Delay multiplier could be added by defining it as a single value, e.g., an insertion loss.

Of course, the Foster format could always be converted into PLS format, and in many cases an inverse is possible as well. The main reason was convenience and performance when reading and writing complicated PLS files. Some of these reach tens of MB of disk space, especially for multi-port S-parameter models. Ultimately, the spec must be convenient to the users / EDA companies and allow flexibility. The IBIS Interconnect Task Group may decide which format to use.

The PLS format is closely related to the Foster formula, which defines partial fraction expansion of the frequency dependence. The Foster format defines the function of the complex frequency s as:

$$H(s) = k_0 + k_1 s + \sum_m \frac{K_m}{s - P_m} \quad (1)$$

(1) is a sum of the constant component k_0 , the linearly growing imaginary component $k_1 s$, and the sum of partial fractions each one being a ratio of the complex coefficient K_m and the difference $(s - P_m)$ between the complex frequency and the pole P_m , which could be real or complex.

Note that (1), by its structure, doesn't enforce realness and stability of the corresponding time domain responses. **Realness** requires that the summands corresponding to the complex poles always come in complex conjugate pairs, which makes the corresponding time domain response real. **Stability** means that with bounded input the system produces a bounded output. The latter requires that the real part of the poles in (1) is negative.

We propose a format that enforces these conditions. In particular, the component that corresponds to a pair of the complex conjugate poles, is represented as:

$$H_m(s) = \frac{1}{2} \left[\frac{C_m}{1 + s/Q_m} + \frac{C_m^*}{1 + s/Q_m^*} \right] \quad (2)$$

Where $C_m = A_m + jB_m$ and $Q_m = \alpha_m - j\omega_m$ are complex numbers that define the normalized residue and the complex pole, and C_m^* , Q_m^* are their complex conjugates. We can find the Foster form or $H_m(s)$ as follows:

$$H_m(s) = \frac{(1/2)C_m Q_m}{s - (-Q_m)} + \frac{(1/2)C_m^* Q_m^*}{s - (-Q_m^*)} = \frac{K_m}{s - P_m} + \frac{K_m^*}{s - P_m^*} \quad (3)$$

where $K_m = \frac{1}{2}[(A_m + jB_m)(\alpha_m - j\omega_m)] = \frac{1}{2}[(A_m\alpha_m + B_m\omega_m) + j(B_m\alpha_m - A_m\omega_m)]$ and

$$P_m = -Q_m = -\alpha_m + j\omega_m$$

Expression (2) can define a single real pole when $\text{imag}\{C_m\} = 0$ and $\text{imag}\{Q_m\} = 0$, which makes

$$P_m = -\alpha_m \text{ and } K_m = \frac{1}{2} A_m \alpha_m$$

Conversion of parameters:

PLS to FOSTER: $K_m = (1/2)C_m Q_m$, $P_m = -Q_m$

FOSTER to PLS: $C_m = -2K_m / P_m$, $Q_m = -P_m$